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THE

ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

VOLUME XI

JANUARY—JUNE, 1900

CHICAGO

The University of Chicago Press

1900

PRINTED BY
The University of Chicago Press
CHICAGO

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THE SPIRAL NEBULA *H I, 55 PEGASI.*

By JAMES E. KEELER.

THE small nebula *H I, 55 Pegasi* (*G. C.* 4892) is not in itself very remarkable, as compared with many other objects of the same class, but considerable interest attaches to it on account of the diverse forms which have been ascribed to it by different observers.

Herschel describes this nebula as pretty bright; irregularly round; resolvable; elongated between two stars; containing two or three stars. In another observation it is described as much elongated, its length being 2′.1 and its breadth 30″. The drawing[1] shows a narrow, spindle-shaped nebula, which accords with this description.

D'Arrest's drawing[2] much resembles that of Herschel, but the width of the nebula is greater, and the condensation toward the center is shown. The conventionalized, lozenge-shaped outline gives the drawing a somewhat unnatural appearance.

[1] *Phil. Trans.*, 1833, Pl. XIV, Fig. 39.

[2] *Instrumentum magnum aequatoreum*, Pl. II, Fig. 6.

Lord Rosse, who observed the nebula many times, was uncertain whether to class it as an annular or a spiral nebula. His drawing[1] shows a spindle-shaped nebulous mass, somewhat like Herschel's, with spiral convolutions surrounding a star of about the fifteenth magnitude on the preceding side.

In Volume I of *Himmel und Erde* there is a drawing[2] by Tempel, which was communicated to that journal with some interesting remarks on the subject of unconscious personal tendencies in drawing faint objects. All the above-mentioned drawings are brought together, for comparison, in Plate III, and they are reproduced here with the acknowledgments of the Editors of the ASTROPHYSICAL JOURNAL.

In Tempel's drawing the spiral convolutions of Lord Rosse are represented by a diffuse patch of nebulosity, in which no structure is shown. The writer of the article in *Himmel und Erde* observes that Tempel's failure to see a spiral form could not have been due to insufficient optical power, since he has shown a faint new nebula which escaped the telescope of Lord Rosse; but this remark loses all its force when it is shown (as it is shown farther below), that Tempel's supposed nebula certainly does not exist. There is no doubt that Lord Rosse saw the nebula to much greater advantage than Tempel.

A photograph of this nebula has been made by Dr. Roberts, and is reproduced on plate 5a of his *Selection of Photographs of Stars, Star-Clusters and Nebulae*. The description there given is as follows:

"The photograph shows the nebula to be elliptical, with a dense, broad line of nebulosity, curved at both ends, forming the major axis, which has a star of about fifteenth magnitude in its center, and there is also a slightly fainter star in the center of the *preceding* semi-ellipse. No structure is visible in it, such as that shown on the drawing by Lord Rosse, but the semi-ellipse on the *following* side is shown on the photograph, though it is not on the drawing."

[1] *Phil. Trans.*, 1850, Pl. XXXVI, Fig. 4.

[2] Opp., p. 133.

So far as the general character of the nebula is concerned, Dr. Roberts seems to have misinterpreted the details shown by his photograph, owing doubtless to the small scale of the negative. The exposure was amply sufficient to bring out the fainter parts of the nebula.

Two photographs of *H I, 55 Pegasi* have been taken with the Crossley reflector, which has a focal length more than twice as great as that of Dr. Roberts' telescope, and is correspondingly better adapted to the photography of these small nebulae. A plate was exposed on August 9, 1899, for two hours, and another on August 28 for three hours and thirty minutes, yielding an excellent negative. From this a positive was made on glass, with an enlargement of 7.5 diameters. It is reproduced in the accompanying plate. The slight ellipticity of the star-disks is in this case due, not to imperfect guiding, but to aberration, the nebula having been photographed at some little distance from the center of the plate.

A glance at the photograph shows that the nebula is a two-branched, left-handed spiral, with a nucleus or condensation near the point of inflection. The preceding branch is strong and single, but the following branch is split into two, which cross where their curvature is greatest, at some distance from the center of the spiral, and unite again at their extremities. This appearance in the components of the following branch, and the fact that the ends of both branches curve around so as to approach the center more closely than do the intermediate parts, are doubtless effects of projection, the plane of the spiral lying obliquely to the line of sight. Proceeding from the following branch are also numerous streams of faint nebulosity, which very likely may not appear in the reproduction.

In the center of the space which is nearly enclosed by the preceding branch is the fifteenth magnitude star drawn by Lord Rosse. The greater part of this space is filled with faint nebulosity, but a narrow dark bay extends inward from the opening on the north, and the star is situated in the middle of the head of this bay. The dark space and the faint nebulosity which

borders it are concentrically disposed with respect to the star.[1] It would be of great interest to know whether this singular position of the star is accidental or whether the star and the nebula are physically connected; and if so, in what way the star was left in its present position during the process of contraction. On the first of these questions an investigation of the spectrum, which will be made in due time, may throw some light. Assuming, for the present, that the star is physically connected with the nebula, it seems to me possible that the proximity of this star may account for the unsymmetrical appearance of the spiral which may be due to an actual difference in the dimensions of the two branches, or to their lying in differently inclined planes.

A comparison of the photograph with the figures in the plate illustrates in a very interesting and instructive manner, the personality of the draftsman which is commented upon by Tempel, and from which his own drawing is by no means exempt.

The most obvious tendency of the draughtsman, which is seen in this and in similar comparisons, is to prolong a line or curve beyond the point at which it actually stops. Thus all the observers of *H I, 55 Pegasi*, regarding the central part of the spiral as a straight, elongated nebula, imagined that they saw it extended in both directions beyond its real limits, and consequently represented it as having the spindle-shaped outline shown in the figures. The preceding branch of the spiral was recognized in its true character by Lord Rosse alone. The following branch can hardly be detected visually. It is barely visible with the Crossley reflector on a fine night, and its spiral structure would certainly not be suspected.

There is also a natural tendency in drawing to emphasize the details caught by the eye, while others, which are missed, may be nearly or quite as prominent. Thus the drawing and the photograph differ.

The faint nebula drawn by Tempel a little east of *H I, 55 Pegasi* does not appear on the Crossley photograph, although it

[1] It is quite likely that these details also will fail to appear in the reproduction. The largest star of the drawings, south of the nebula, is just outside the limit of the photograph.

PLATE II.

PHOTOGRAPH OF THE NEBULA *H I. 55 PEGASI*
Made with the Crossley Reflector

PLATE III.

DRAWINGS OF THE NEBULA *H I. 55 PEGASI*

is hardly necessary to say that with an exposure of three and one half hours this instrument will photograph nebulae beyond the reach of any visual telescope whatever. Near the place of the supposed nebula there are several small stars, whose combined effect may possibly have produced an appearance of nebulosity, though they are all below the sixteenth magnitude. In the case of Lord Rosse's drawing it is probable that the numerous streaks and patches represent a manner of drawing, rather than an attempt to depict extremely minute details.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
December 1899.

ON A NEW SYSTEM FOR SPECTRAL PHOTOMETRIC WORK.

By D. B. Brace.

In the comparison of intensities of the spectra of different radiants or of the different parts of the same spectrum, the greatest sensibility of the eye is attained when the determination between these elements in the field of view is the sharpest, so that in a setting for a match the boundary can be made to vanish abruptly and completely, thus giving a perfect uniformity in the intensity of the field. This latter must also be sufficiently bright to give to the eye the most favorable conditions in such a comparison. It must also be possible to measure, to at least as high a degree of accuracy, in a simple way, the variations in the intensities by which this is attained and maintain constant sources, which may be readily reproduced for comparison.

In many of the instruments these three conditions are not realized. In some we find the means for measuring a variation in intensity very accurately attained, but at the expense of the amount of the original light, so that an accurate setting cannot be obtained on account of the weakness of the field of view. When sufficient light is available, a dark line or (bright line) is usually visible between the two parts of the field, which seriously reduces the sensibility of the eye in determining a match.

In the instruments devised by Govi, Vierordt, Crova, and others, the two spectra are made contiguous by means of double slits or total reflecting prisms which, if they do not overlap, give a line of separation of perceptible width and of less intensity than the adjacent spectra or, if they do overlap, a region of variable intensity of finite width which confuses the eye in determining a match. In the polarizing instruments where double image prisms are used to affect the same thing, this gradual transition from the true intensity of one spectrum to that of the other is of finite extent. In addition to this difficulty, the

brightness of the field is reduced too much to make accurate comparisons of radiants of low intensity, as in the types used by Glan, Crova, Glazebrook, König, and others. In the instruments of Cornu and Kundt where the variation in intensity is attained by diaphragming the objective sectorially, the two spectra must be brought together by systems having the above inherent defects, which, with the liability to irregular distribution oj intensities over the objective and consequent error of measurement, render them open to serious objection.

Photometers like those of Wild, Trannin, and others, which depend upon the vanishing of the interference bands of two distinct sources when properly superposed, after a method used by Babinet, are not capable of great sensibility and are decidedly objectionable on account of the severe strain upon the eye in determining the exact vanishment of the bands. The accuracy of measurement of the intensities is very high, far more so than the sensibility of the eye in making a match. The degree of accuracy to which an instrument is graduated for measuring changes of intensities should not be mistaken for the accuracy with which the eye can make the comparisons as seems to have been done in the case of one of these latter instruments and generally quoted by authorities of high experience. The constructors of such an instrument have readily obtained a graduation accurate to one tenth per cent. but have scarcely attained a setting with the eye accurate to one per cent., so that they have replaced it by several instruments which give superior results. It is doubtful if a true sensibility of the value so generally quoted for interference photometers has ever been attained to even one tenth part. It is only in one or two later types of spectrophotometers, that the sensibility in the settings by the eye have reached or exceeded the means of measuring the intensities as well as of maintaining their uniformity, notwithstanding the various types of radiants proposed as photometric standards. While much attention has been given to this latter question no radiant, with possibly the exception of the Reichsanstalt platinum standard, has been proposed which will maintain its constancy

throughout the spectrum, for an extended period of time, to within one fourth of one per cent.,—the sensibility now obtainable by the eye with a simple prismatic viewing screen. If we consider the complexity of the optical systems in some of the spectral photometers, particularly in the polarizing types, the impossibility of collating the results of different observers with one another or of obtaining absolute spectrophotometric data accurate to within one per cent. is apparent.

In the photometer cube of Lummer and Brodhun[1] we have the conditions of great accuracy in comparisons, since here the boundary between the fields compared is very sharp and vanishes for equal intensities. They have adopted different means for obtaining this condition, two of which are now in use. The first, which is more available for white light, is attained by etching sharply away a part of the surface forming the diagonal of a cube consisting of two prisms, so that total reflection from the diagonal face of the second prism abruptly terminates at the boundary of the etching, thus giving the required condition. In the second, which is suitable for spectral comparison, the prisms of the cube are cemented together after a part of one of the diagonal surfaces has been silvered to a sharp limit, so that internal reflection abruptly ceases at this line. An analogous method proposed by the writer, for obtaining abrupt transitions from several successively parallel reflecting surfaces has been used by Doubt[2] in making comparisons of color mixtures.

In order to avoid diffraction, which would destroy the sharpness of the boundary, the line of delimitation must be chosen so that it is parallel to the dispersion at each point. It is found by so doing that all diffracting phenomena disappear for a match.

In these several screens no dispersion takes place. This is effected by the usual dispersing prism which forms a part of the optical train. In the photometer here described the dispersion takes place within the prismatic viewing screen itself, thus simplifying the optical system and reducing the number of refracting

[1] *Zeitschrift für Instrumentenkunde*, February 1892; April 1892.

[2] T. E. Doubt, *Phil. Mag.*, 46, 216, 1898.

surfaces and the number of adjustments necessary to eliminate diffraction and obtain a perfectly sharp boundary.

The compound prism, Fig. 1, is essentially an equilateral prism bisected by a plane *A D* through one of its refracting edges, thus making the vertical angles at *A* of each prism equal. The face *A D* of one of the prisms, *A D C* say, is then silvered, and this silvering carefully removed, all except a strip whose length is normal to the bisected edge and whose height is about one third the field of view. The edges of the strip *S S* must be perfectly sharp and regular. This is generally effected by carefully treating with acid and finishing the bounding lines with a sharp ivory chisel. The vertical angles *B* and *C* should be the same. The two prisms are then placed together with some cement or liquid between the common surfaces *A D*. For prisms

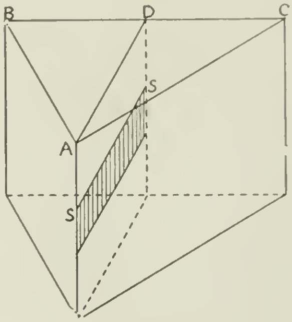

Fig. 1

of low dispersion balsam is used, and α-monobromonapthaline in a prism of high refractive index. This latter substance is very constant under such conditions and does not attack the silver and is further colorless and very transparent to the blue and violet rays. Both substances however must be cemented in to prevent evaporation, the latter especially being quite volatile and gradually disappearing at the edges, thus contracting the field of view. If gelatine, which is not attacked by it, is placed over the edges, the evaporation can be checked and the prism made permanent. In this arrangement the prism can readily be taken apart and cleaned and refilled with new fluid, if necessary, without injuring the silver strip. On account of the slight thickness of this strip and the difference in the refractive indices between the cementing fluid and the glass, some light may be reflected from other portions of the interface than *S S* which, when this difference is marked, will produce the phenomenon of interference bands for thin films. Under certain conditions

these can be seen, but, as will be shown later, they disappear entirely when the eye observes directly the interface and a match is made. In general, prisms of high dispersive power are preferable especially when sufficient light is available, in order that comparisons may be made with the field as nearly monochromatic as possible. For low intensities, such as in comparisons of stellar spectra, crown glass may be advantageous. The glass selected for higher dispersion is that from the factory of Messrs. Schott and Co., Jena, Dense Silicate Flint O 102 $n_d =$ 1.6527, $\left(\frac{\Delta n}{n_d - 1}\right)_{c-f} = 0.01950$. The corresponding index of a-monobromonapthaline is $n_d = 1.6582$ @ 20° C. This combination gives no appreciable internal reflection or interference and a monochromatic field of great uniformity when viewed directly by the eye through a slit of quite sufficient dimensions to obtain the proper intensities for comparisons. The above fluid is now much used for optical purposes and is readily obtained. When this is not at hand Canada balsam may be used with this glass, although interference bands are generally present when viewed with the eyepiece, but vanish when viewed directly with the eye. The results however are not as satisfactory as when the indices are nearly the same, and it is difficult to separate the prisms after a considerable length of time without destroying the silver strip. Crown-glass prisms cemented with balsam give less dispersion but greater spectral intensities and a fairly monochromatic field, particularly in the upper portions of the spectrum. For low intensities these may be used to advantage, for example, in comparing stellar spectra of the fainter stars.

Heretofore in making spectral comparisons of two sources sent from different directions through the same dispersing prism into the field of view,—notably in the well-known color instrument of von Helmholtz,—the adjacent spectra extend in opposite directions across the field. On account of this change in direction of the color gradient a perceptible change in tint at the extremities occurs, while at the same time a black vertical line is always seen separating the two halves of the field. From the

well-known optical principle that an odd number of reflections produces a reversion and an even number a rectification of the image, it will be seen that in the combination here considered we have one reflection and a direct transmission giving the same color gradient to the spectra of the two sources.

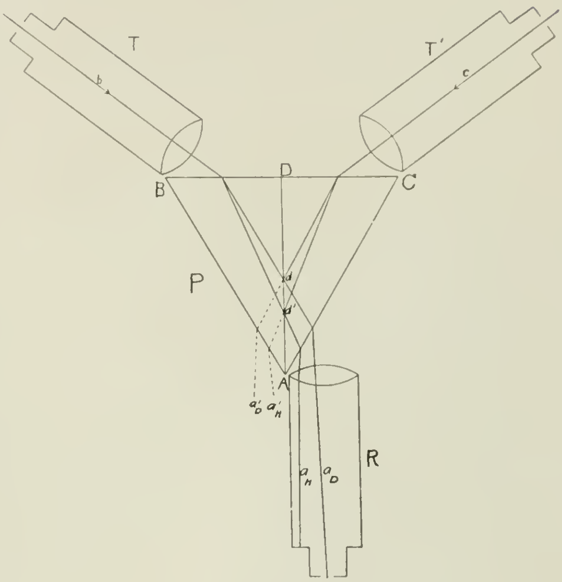

Fig. 2

In Fig. 2, *A B C* is the compound prism, *T* and *T′* are the collimating telescopes and *R* is the observing telescope. A ray *b*, for example, from the slit of *T* is refracted at the face *B D* after which each color component meets the interface *A D* at *d . . . d′*, those rays meeting the silvered strip being reflected out through the prism in the direction $a_d' \ldots a_h'$, while the remainder pass on through the prism into the observing telescope *R* in the direction $a_d \ldots a_h$, certain color components passing through the ocular slit into the eye, which sees, consequently, the interface *A D* illuminated with this color, excepting the portion of the field covered by the silvered strip. Similarly the collimating telescope *T′* is so adjusted that a ray *c* from its slit is refracted at the surface *D C* at such an angle that its corresponding color components meet the interface at *d . . . d′*, some passing on through the prism and emerging in the direction $a_d' \ldots a_h'$, while those meeting the silvered strip are reflected on out of the prism into the observing telescope along the direction $a_d \ldots a_h$, like color components to the first ray *b* thus passing

through the slit R to the eye, which sees illuminated, with the same color, the part of the field covered by the silvered strip, the boundary between the two portions being thus abrupt and sharply defined. On varying the intensity of the components this is made to vanish completely, so that the eye sees the interface $A\ D$ illuminated uniformly with the same color and without any perceptible light or dark lines. The field may be stopped down to a suitable size by a circular diaphragm on R. If now the telescope R be shifted, the field appears illuminated successively with all the different colors of the spectrum.

Since it would introduce mechanical complications, and the error in color intensity would be inappreciable, if observations were made for minimum deviation in every case, the prism is adjusted for the mean color of the direct ray b. This is effected by inserting the eyepiece in R and observing with the cross hairs the sodium line from the slit of T. The compound prism need have but slight movement and is to be permanently fastened to its base. The collimating telescope T' is then adjusted for the same line and then clamped. In order that the different color components of the same original ray may pass through the same point in the telescope R, its axis of rotation should be about the radiant axis of color C, while its optical axis should intersect the prism one fourth the distance from A to C. A like condition applies to the collimating telescopes T and T'. For most purposes the collimators may be permanently fixed, the telescope only being mounted for rotation. The distance from the slit of R to the interface should be approximately 25 cm,—that of distinct vision. In the adjustment with the eyepiece, secondary images or ghosts are seen. These are produced by interference of residual reflections at the interface when the cementing fluid is of a different index from the glass. When the glass and the cementing fluid have the corresponding indices stated above the difference between the internal losses of the two sources arising from these and other causes is small. When the cementing fluid is removed and only the surface of the prism which contains the silver strip illuminated by the collimator T', the two parts of

the field lighted by the strip and by total reflection appear approximately of the same intensity, showing that the reflection from silver in this way is approximately total for all colors.

Dispersing prisms of higher indices may be used, but the internal absorption becomes marked. Furthermore experience has shown that, with denser glass, surface oxidization takes place after a time, thus changing the constant of absorption for different colors, a factor which must be invariable when the absolute color gradient is being determined.

It is evident at once that this construction may serve, with the use of an eyepiece, with one collimator, as a simple, and with two collimators, as a comparison spectroscope. On the other hand, from the symmetry of the optical system, a single prism spectroscope or a spectrometer may be immediately transformed into a spectral photometer by replacing the ordinary prism with such a compound prism and adding a second collimating telescope.

This type of direct viewing spectrophotometer is especially suited for making comparisons of very narrow or line spectra, such as stellar spectra, lunar or solar spectra of very limited areas, and of spectra from point or line radiants, etc. In color comparisons, a finite field of view must be presented to the eye in order to make accurate comparisons. This may be accomplished by placing a diffusing screen in the focal plane to receive the spectra, in which case there is a very great loss of intensity, only a small fraction of the rays reaching the eyes. It may also be obtained by using a high power eyepiece or a cylindrical lens to broaden the spectra, but serious distortions and consequent variation in uniformity of the field result.

Fig. 3 shows an instrument attached to a large telescope for spectrophotometric observations of the heavenly bodies. It is evident that all the light of any color gathered by the astronomical objective from one point reaches the retina; and the narrowest line spectra, when observed directly by the eye through the slit of the viewing telescope of the photometer, produce a full field of perfect uniformity. On removing the

collimator slits and sending the light from two stars directly through the respective collimators, comparisons of their spectra can at once be made with a field of view of sufficient aperture to give the eye the conditions of greatest sensibility. Observations on the spectrum of Capella with a 4-inch telescope and its attached single prism spectroscope with the slit thrown wide and the eyepiece removed showed a field of more than sufficient intensity to make good comparisons; from which it is estimated that accurate stellar spectral measurements can be made down to the fifth magnitude with a 40-inch objective. As will be referred to later, comparisons have been made with intensities representing a range from one to two millions. On examination, the Moon also showed abundant light to determine the craterous luminosity gradient in spectral elements. Atmospheric absorption may also be determined for specific parts of the spectra of different luminous bodies.

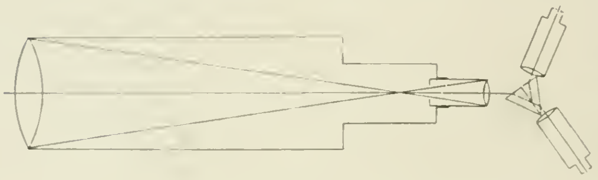

FIG. 3.

Fig. 4 (Plate IV) shows a spectrophotometer as constructed by Schmidt & Haensch, of Berlin, for physical purposes, one ninth actual size. The collimator T is fastened to the body of the instrument. The collimator T' and the telescope R are mounted on separate radial arms so that each may be displaced independently about the axis of the instrument. Each arm is mounted with a micrometer screw and clamp which engages the ring Q so that any setting may be attained and the finer adjustments made with the micrometer screws N and N' respectively. M is a vernier and scale for determining the position of R and the part of the spectrum observed. The collimator T may be mounted with a unilateral slit, but the slit of T' should be bilateral and graduated to at least one two-hundredths of a millimeter. Both slits are mounted with total reflecting prisms for comparison spectral work, and for photometric work with the eyepiece after the method of Vierordt. The total aperture of

PLATE IV.

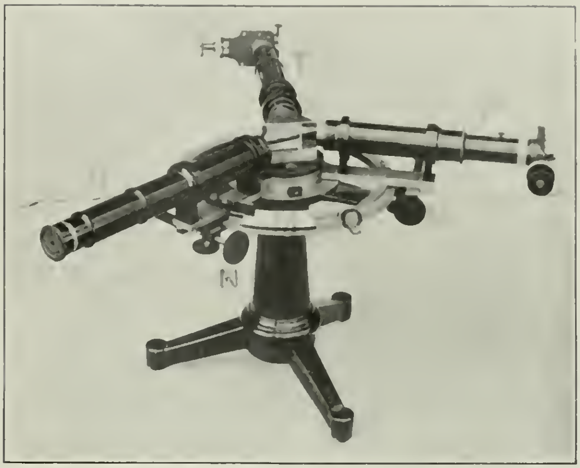

Fig. 4.

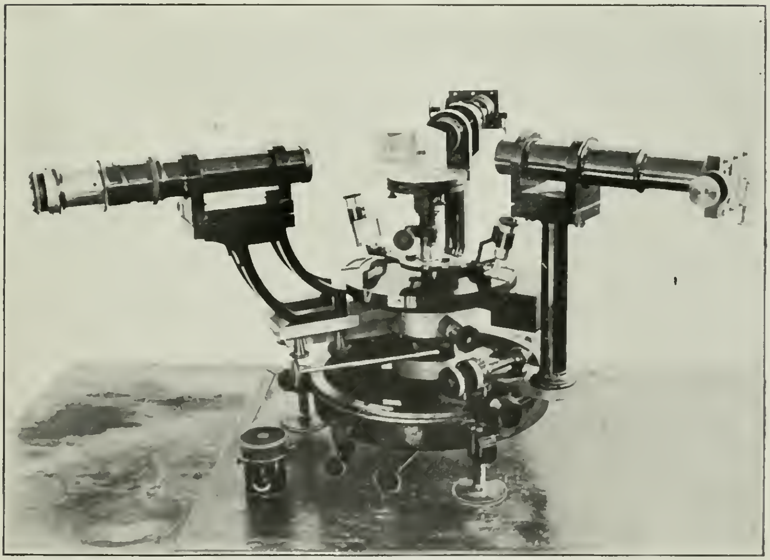

Fig. 5.

the slits need not exceed two millimeters. The telescope R is provided with an eyepiece E which fits over a double adjustable slit in its focal plane and from which it can be readily removed and the eye applied directly. The compound prism P within which may be seen the silver strip, is mounted above the axis of the instrument on a plate which can be shifted laterally and then screwed to the instrument when the minimum deviation has been obtained. The prism and telescope objectives are carefully housed to avoid diffused light which may interfere seriously in delicate comparisons. Special care must be exercised to eliminate extraneous light. The silver next the glass upon which it is deposited is used as the reflecting surface since its outside face is subject to oxidization and is not as perfect. This strip is upon the prism next the collimating telescope T', carrying the bilateral slit. With this instrument, the setting for minimum deviation for sodium light is effected with the collimator T first, using the compound slit in the telescope or cross hairs in making the setting. P is then fastened permanently and R clamped; and T' then adjusted for minimum deviation and clamped. On removing E and illuminating both slits with the same amount of light, the eye should see the field of approximately uniform color and intensity when R is shifted about the axis of rotation through the spectrum.

The high sensibility of the instrument is readily seen by the sharp outlining of the horizontal strip against the circular field of view when the base is jarred slightly. The least unsteadiness of the illuminants is manifested in a similar way. The arc M may now be graduated for the different wave lengths by inserting the eyepiece E and observing the Fraunhofer lines. The width of the occular slit may vary from 0.5 mm to 1 mm depending on the dispersion of the prism P, and the height may be somewhat greater except when line spectra are being compared with one another or with a broad spectrum. In this case it should be stopped down to the width of the line spectra. This bilateral ocular slit may be rapidly and continuously adjusted by rotation of a spirally slotted disk which engages the lateral jaws. The

vertical jaws are independent and adjusted by hand. The slit may be turned through ninety degrees for adjustment for line spectra if desired. The aperture of the telescope *R* is stopped down to 15 mm, its focal length is 25 cm, and the width of the strip which is adjusted to the center of the field is 5 mm.

Fig. 5 (Plate IV) shows a spectrometer after a model by von Lang, as constructed by Schmidt & Haensch, Berlin, and fitted with an extra collimator. Each collimator has a bilateral slit and comparison prism and is mounted on an independent arm which is provided with a micrometer screw for fine motion, the circular base of the model being faced down to allow independent motion and settings. The observing telescope has also been fitted with an ocular slit and a circular stop at the objective. This instrument has been found to be equally efficient with the special instrument described above. These additional accessories add about 20 per cent. to its original price. (Cf. Schmidt & Haensch, Catalogue 1896, No. 55.)

The regular spectrophotometers already described may be had for about 560 M., and with some simplifications for about 500 M.; for example, if both collimators are fixed and the objectives reduced to an aperture of 15 mm, the observing telescope alone being provided with a micrometer screw and without vernier and scale. This is sufficient for the purpose of photometric comparison of different spectra. If comparisons of adjacent elements of the same spectra are desired for the purpose of determining the curve of luminosity, one or both collimators should be provided with graduated micrometer screws. Any two adjacent elements can then be thrown into the field in succession and "step by step" comparisons be made clear across the spectrum.

In the optical system which has just been described almost all the light from the slit reaches the retina and in a way to give the highest sensibility for comparison, so that the first and second conditions premised for photometric work have been realized. The third and fourth conditions, namely of measuring the intensities and of obtaining constant and reproducible sources have in part been realized by simple methods.

By the use of Nicol prisms, double image prisms, or other polarizing systems, small changes of intensity are very accurately determined. However, these introduce errors for depolarization and they also reduce the amount of light which reaches the retina so much (at least one half) that accurate readings cannot be made for low intensities. The direct variation of the distance from the slit or from a screen, is too cumbersome and inconvenient for permanent spectrophotometric work. The rotating sector, as used first by Talbot and latterly by Lummer and Brodhun at the Reichsanstalt, is perhaps the most reliable method when properly applied, but it is a tedious and slow process for permanent work. The method of Vierordt of measuring the width of a unilateral or a bilateral slit introduces errors of several per cent. in different parts of the spectrum for even a bilateral slit, as has been shown by Lummer and Murphy.[1] That this should be so is made evident by inspection of the curve of luminosity for any source.

Fig. 6 represents curves of luminosity of a normal trichromatic eye obtained by A. König[2] for different intensities, the source being a triple gas burner. The unit of intensity is that which the eye with a 1 sq. mm diphragm receives from a white screen of magnesium oxide illuminated by a 0.1 sq. cm of melting platinum (=1.7 candle power) at a distance of 1 meter, the intensity of the comparison color, $535\mu\mu$, being given in the diagram for the corresponding curves. In general, where the slope of the curve is not a straight line, the light entering the slit will not be proportioned to its width. For a unilateral slit it will only be true when the curve is horizontal. For a bilateral slit it will be only true for the parts of the curve which are straight lines. This error may be large, depending on the width of the slit and the part of the spectrum examined.

The further difficulty of using the slit-width, especially for narrow widths, is in determining the true zero. Errors in the screen also enter, as well as those due to selective absorption in

[1] This JOURNAL, 6, 1.

[2] *Beiträge zur Psychologie und Physiologie der Sinnesorgane*, 1891.

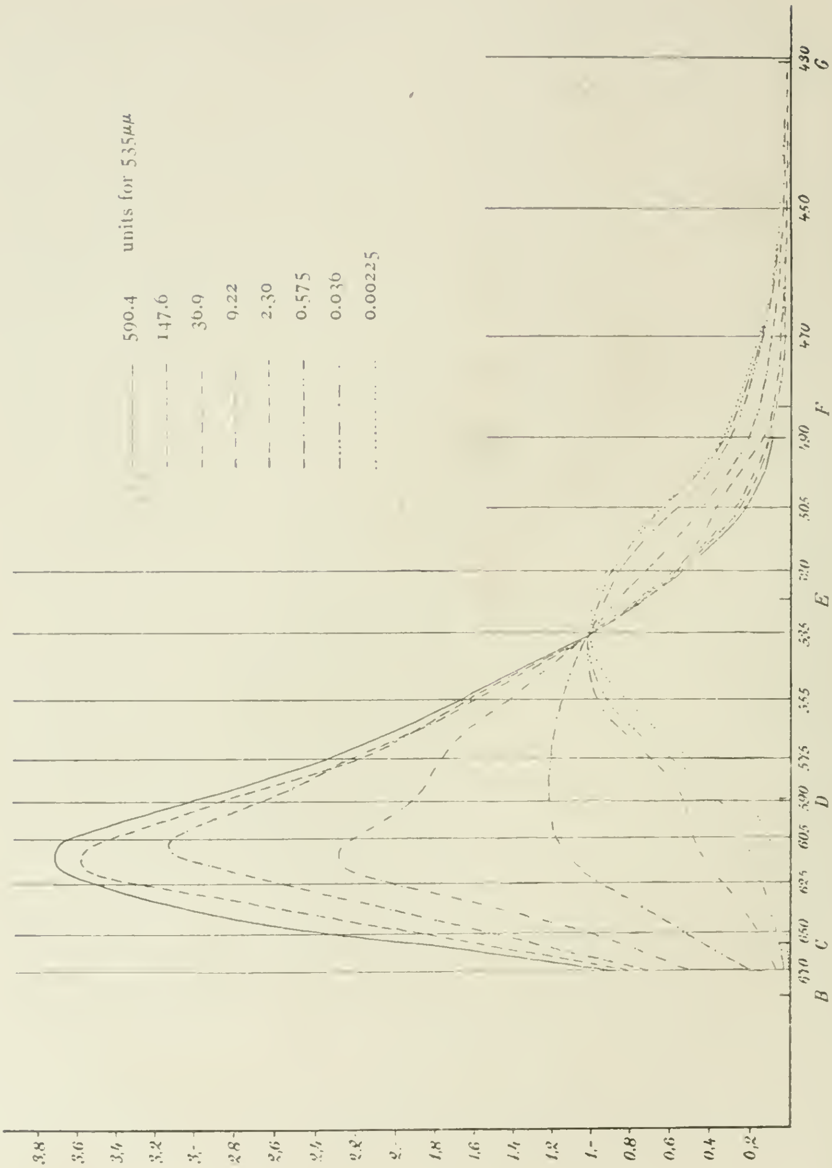
590.4 units for 535μμ
147.6
36.9
9.22
2.30
0.575
0.036
0.00225
B
C
D
E
F
G

FIG. 6.

the optical system of the instrument itself. All of the errors, however, may be eliminated by the direct optical calibration of one of the slits, a simple and exact method of the slit readings not heretofore used. With this calibration, observations can be made rapidly and as accurately as with the direct use of polarizing prisms or of the variable rotating sector. Both of these methods require elaborate and expensive accessories, while the latter is a tedious although accurate process. The means of calibration adopted as the most available and accurate is the rotating sector, which Lummer and Brodhun have shown to be the most reliable method. The sector should of course be driven considerably beyond the speed of perceptible flickering. Under these conditions the effect on the retina over sufficient ranges in intensity is simply proportional to the integral amount of light which reaches it. It is then only necessary to compensate for any known change of intensity by varying the slit-width to obtain its true optical value. To accomplish this, a simple cardboard disk, Fig. 7, is divided into any number of equal parts, for instance eight, and mounted on a whirling table or motor before one of the collimators of the spectrophotometer. This may be adjusted so as to bring any one of the sectors in front of the slit so that the intensity may be made to vary from one down to one eighth successively. If the slit to be standardized is mounted with T', Fig. 2, the sector may either be placed before it or before the slit of T. Having first obtained a match, the disk is set into rotation and the slit of T' adjusted again for a match. The slit reading gives the relative optical value corresponding to the sector used. For example, if the sector was half of the circumference, the slit reading is for one half its optical value if the rotating sector be before T, and twice its value if it be before T'. Successive values may thus be obtained dividing up the slit into proportional parts. The optical value for any other

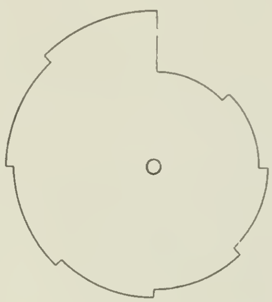

FIG. 7.

width or reading of the screw may be obtained, by interpolation, to any desired degree of accuracy. This calibration evidently eliminates all instrumental errors. This process may be repeated for all colors by shifting the telescope *R*. Of course, either of the methods of measurement may be used for calibrating and then dispensed with. Once the calibration curves are obtained the slit should never be closed and the setting should always be made for one direction of the screw.

The curve of luminosity depends on the nature of the source and on subjective conditions. A definite distribution of the radiant energy will produce a certain curve for any one eye. If each element be changed in the same ratio, the luminosity curve will not vary in the same way. This influence of the absolute intensities on the color gradient was first observed by Purkinje. The retinae of different eyes show a variation in the degree of sensation produced, not only for the same amount of energy for any one color but also different relative sensibilities for the different spectral elements. Thus trichromatic eyes may show greater discrepancies in their curves of luminosity than exist between one of them and a dichromatic (red, green or blue color blind) eye. A monochromatic eye will have its own curve of luminosity which will depend on the relative distribution of energy and its value for any one part of the spectrum.

Fig. 6, already referred to, illustrates these variations for a normal trichromatic eye, each ordinate being multiplied by a factor which makes the ordinates of all curves the same for the wave-length 535 $\mu\mu$.

Fig. 8 shows the curve of different eyes for the same distribution of energy when the intensity is high, and also these curves reduced to a common ordinate for wave-length 535 $\mu\mu$. The curve of a normal trichromatic eye for a uniform high distribution of energy throughout the spectrum is also given.

Fig. 9 gives the corresponding curves for very low intensities. These curves represent the changes produced by varying the intensity of the original source,—a triple gas burner. Other sources, such as incandescent, filaments, molten platinum, the arc

light, would give their own characteristic curve under these conditions. For low intensities the slope of the curves of different eyes are approximately the same and the calibration of the slit could be used by different persons without serious error, but for

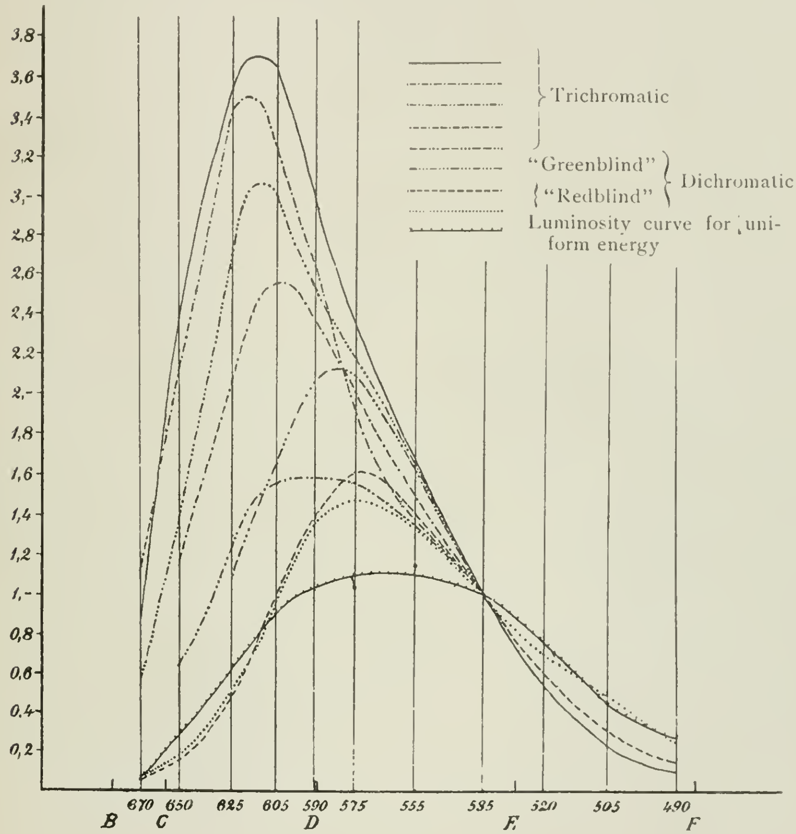

FIG. 8.

other intensities, the calibration should be made for each eye. This error becomes less and less as the width of the slit is reduced and the dispersion increased. With the flint glass recommended and a source readily available, the slit-width of 0.25 mm practically eliminates this error and gives sufficient light for the most sensitive comparisons in general work.

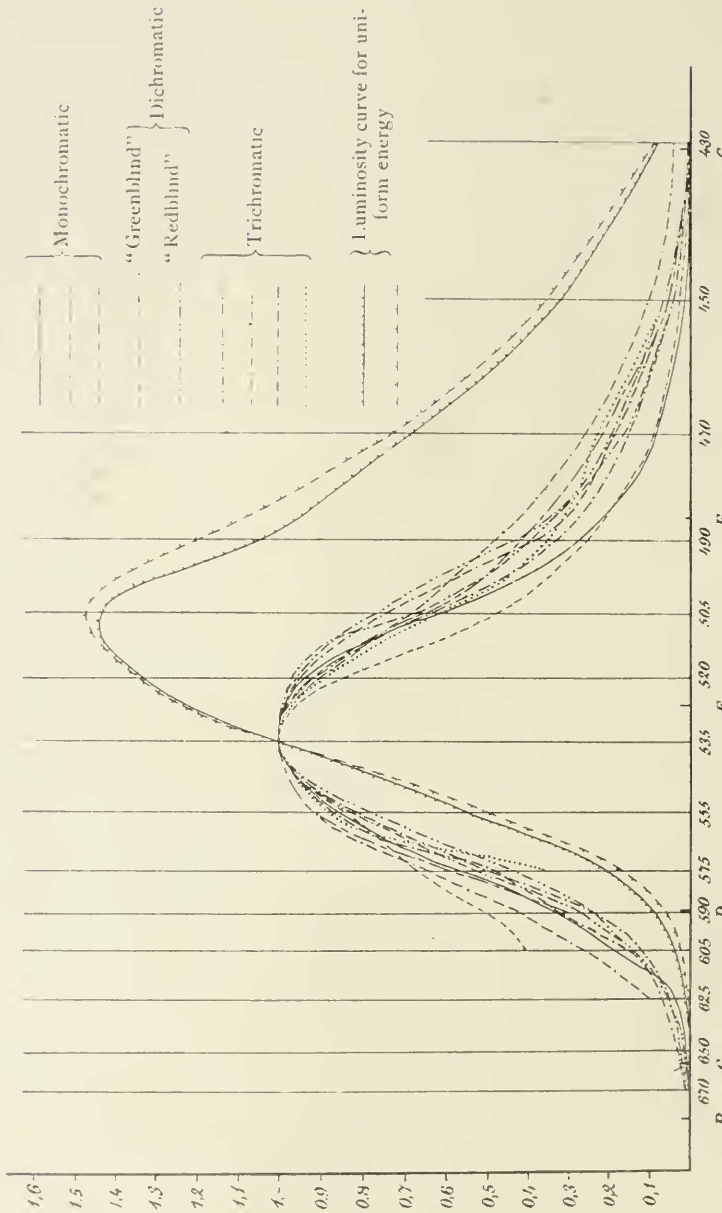
Monochromatic
"Greenblind"
"Redblind"
Dichromatic
Trichromatic
Luminosity curve for uniform energy
670 650 625 605 590 575 555 535 520 505 490 470 450 430
B C D E F G
1,6 1,5 1,4 1,3 1,2 1,1 1,- 0,9 0,8 0,7 0,6 0,5 0,4 0,3 0,2 0,1

Fig. 9.

Experiments upon the eye indicate that its curve does not change over long periods at a time ; hence if the source before the calibrated slit can be maintained uniform for such a period, the conditions of the problem are met. The only sources giving sufficient constancy have been found to be incandescent filaments maintained and regulated by electric currents. Filaments of carbon give approximately the same curve over considerable variations in voltage for slit-widths of 1 mm or less. These are usually placed in frosted globes (preferably in close spirals and near the glass). To obtain areas of uniform intensities additional ground glass plates are used when necessary. Under favorable conditions the instrument has given for the mean colors of the spectrum a variation of only 0.5 per cent. from a mean of ten settings, or a mean error of 0.25 per cent. for one setting. Obvious applications of the optical principle in this instrument may be made to other instruments of comparison, such as those for comparing mixtures of spectral elements.

Instead of using the slit as above outlined, incandescent line filaments at the focus may be used directly instead. These may be vertical, and the intensities determined for the different colors by previous comparison and "optical" calibrations of the rheostats in series with the filaments. Much greater intensity can be obtained in this way, and a perfectly uniform field is assured. For measurements of high absorption, this method may be utilized if the absorbing system can be interposed between the objective and the line radiant at the focus of the objective without disturbing the optical definition.

Good results have also been obtained by cutting down the height of the ocular slit and placing the filament horizontal and close to the collimator slit. By varying the resistance of the circuit, a match is obtained without using the readings of the screw. In these systems the errors to which the slit is subject are entirely avoided. The horizontal filament may also be maintained at a constant voltage and readings made with the slit. In this way a uniform field of high intensity is obtained, since when the globe is placed at a distance, the slit acts like a lens

to form an image of the filament within the compound prism or viewing screen which the eye sees directly, and any lack of uniformity becomes evident, so that additional ground glass plates are required, thus materially reducing the intensity.

For measurements where a standard of comparison is desired an incandescent platinum wire is to be occasionally substituted for one of the slits (or filaments) and checked up with the standardized slit and its radiant (or standardized filament). The platinum wire is to be of a definite section and its temperature is to be determined after the manner of the Reichsanstalt platinum photometric standard by making the ratio of the direct radiations to those transmitted by a quartz cell of water 1 cm thick some chosen value, for instance ten.

Physical Laboratory,
University of Nebraska,
December 1899.

CALIBRATION OF THE SLIT IN SPECTRAL PHOTOMETRIC MEASUREMENTS.

By E. V. Capps.[1]

The advantage of the slit in spectrophotometric measurements is the great rapidity with which observations can be taken, but, for its use, it becomes necessary to obtain reliable calibration.

The contents of this paper are the results of preliminary experiments which were made in connection with the study of absorption of light in aqueous solutions.

The intensity of the light at a given point in a spectrum formed from a slit of finite width, is the resultant of all the wave-lengths lying in a given space of a pure spectrum, so that it is determined by the width of the slit. By increasing the slit-width, light of greater or less wave-lengths, depending upon the direction of motion, is superimposed upon the original amount, and thus the increase of intensity will depend upon the ratio of the mean intensity of the wave-lengths added, to the mean intensity of the original wave-lengths.

The curve of luminosity of the spectrum is approximately represented in Fig. 1, the exact form depending upon the source of light.

It has been shown by D. W. Murphy (this Journal, 6, 1) by deduction from the energy curve of the spectrum and by experiment, that the intensity of the light which is received by the eye does not vary directly with the width of the slit. It follows, therefore, that with an unilateral slit the intensity would conform to the direct ratio law only where the curve is parallel to the axis. With a bilateral slit the direct ratio would hold at all points where the curve is a straight line, as the points *a*, *c*, *d*, and *f*. It also follows from the curve of luminosity that where the curve is convex to the axis as at *ab* and *ef*, the intensity of the

[1] Fellow in Physics, University of Nebraska.

light increases in a greater ratio than the width of the slit, the reverse being true for the concave portion *b*, *e*.

The experiments of Murphy were performed with a Lummer-Brodhun spectrophotometer. A revolving sector was used to weaken the light from one source. The sector was set to decrease the intensity of the light to one half of the original amount, and this was compared with the ratio of the corresponding slit readings. The results obtained show a variation from the direct ratio of 2 per cent. from the middle portion of the

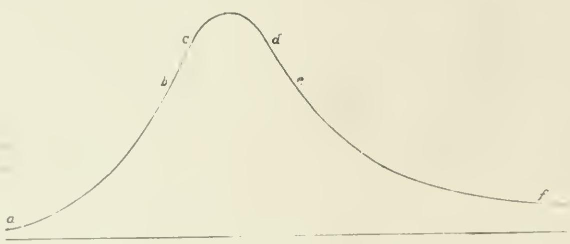

Fig. 1.

spectrum and nearly 10 per cent. for the red end. They also show that the intensity increases faster for the convex portions than the width of the slit, and slower for the concave portions, thus confirming the deductions from the luminosity curve. The present experiments were performed with a spectrophotometer designed and described by Professor Brace.[1]

The plan of the instrument is similar to the one employed by Professor Brace.[2] Two collimators, and fitted with bilateral slits, and graduated to hundredths of a millimeter, are mounted upon a circular base. Opposite to these, and fitted with a circular scale is the telescope. By means of a microscope it is possible to read its angular position to ten seconds. At the eye end there is an adjustable slit which limits the vision to a particular portion of the spectrum. When used as a spectrometer, the eyepiece is attached and the slit takes the place of the cross hairs. The prism consists of two right angle prisms which form, when clamped together, an equilateral triangle.

[1] See p. 6. [2] See p. 11, Fig. 2.

The space between is filled with some liquid whose refractive index is the same as that of the glass. At the middle of one of the internal surfaces and parallel to the base there is deposited a strip of silver 5 mm wide.

The prism when in position is set for minimum deviation. The light from one source passes through the collimator and is refracted by the prism into the telescope. The light from the second source passes into the prism until it meets the silver surface, where it is reflected into the telescope with light from the first slit. By adjusting the angular position of the second slit, the spectra from the two sources are made to coincide, line with line. A cap with a circular opening is fitted over the objective of the telescope, and upon viewing the field through the eye-slit there is seen a circular patch of light, crossed by a band of the same color. The setting is made by varying the width of the slit until the band vanishes.

The greatest difficulty encountered was to obtain a constant source of light, that at the same time would give great intensity and uniformity. The ordinary incandescent lamp, operated on a storage battery circuit, fulfilled the first condition, but in order to satisfy the third, it was necessary to place the lamp at least 25 cm behind a ground-glass screen, thus greatly reducing the intensity of the light. The spectrophotometer was originally set up with a crown-glass prism, using the above described source of light. This was satisfactory with the exception that measurements could not be made in the extreme violet with the absorbing liquid before the slit. It was also observed in studying the absorption of solutions that the spectrum was not sufficiently pure to obtain a match with respect to color, if the absorption exceeded 25 per cent. To overcome this difficulty, a flint-glass prism was substituted for the crown, and this necessitated an increase of intensity of the source of light to at least fifty times. After many unsuccessful attempts the author succeeded in constructing an incandescent lamp, having a flat filament 12 mm wide. This could be placed as far as 12 cm from the slit and still cover the field. When viewed directly the

intensity was nearly one hundred times as great as it was possible to obtain by the old method.

The many difficulties of observation were entirely removed by the adoption of the flint-glass prism and these special lamps, as sources of light. The dispersion of curves of the crown and flint-glass prisms are shown in Fig. 2.

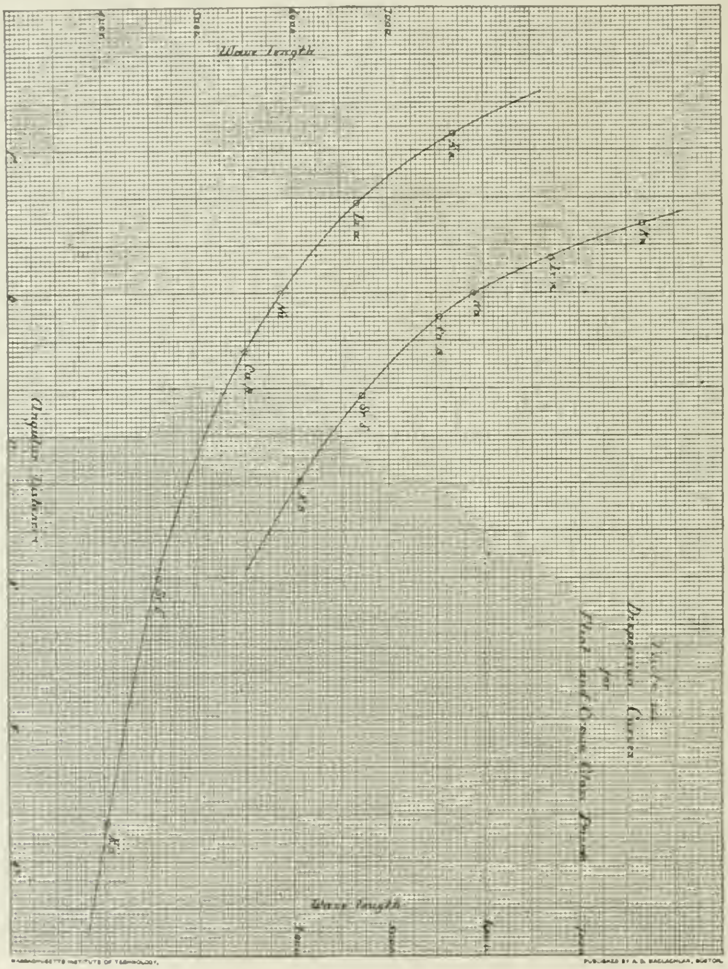

FIG. 2.

The method of making the calibration consisted in the use of a revolving disk notched at the periphery, as shown in Fig. 3, so that the intensity of the light is decreased one eighth of the whole for each notch. It was made of bristol board, and blackened with lampblack and shellac. The disk was fastened to a rotating wheel mounted upon a sliding base, which could be pushed along by a thumbscrew, the driving belt being made of spiral spring. It was driven with a small electric motor, and could be run at a high speed without vibration. The readings were taken as follows: Ten settings were made with the disk at rest. Then with the disk running, ten settings were taken for each successive notch in front of the slit, followed by ten more with the disk at rest. The mean value of the first and last ten was taken as the setting for 100 per cent. transmission, and the ratio of the settings for each notch of the disk to this value, was taken as the transmission as shown by the slit. Readings were taken for different slit-widths and different parts of the spectrum. The results give the calibration for both the flint and crown-glass prisms, subject to the conditions as previously explained.

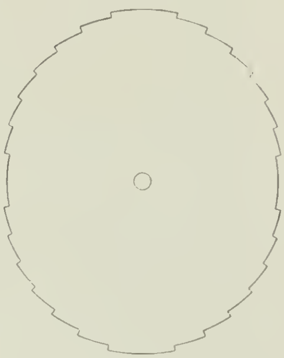

FIG. 3.

The accuracy with which the settings could be made is shown in Table I, which was chosen arbitrarily. The mean probable errors for the first column is seven tenths of 1 per cent., but the error can be reduced to one fourth of 1 per cent. under favorable conditions. The mean values of the first and last columns only vary from each other by three tenths of 1 per cent., the time interval between the two observations being thirty minutes. This shows how nearly the lights remain constant during the time interval of observation.

The results for the crown-glass prism are given in Table II, and the curves for the same in Figs. 4 and 5. The lines headed "per cent. by disk" are the transmissions as determined by the

notches of the disk. The lines headed "per cent. by slit" are the transmissions as found by the ratio of the slit readings. The curves show that for the red and blue parts of the spectrum, the intensity increases faster than the width of the slit, while the reverse is true for the yellow. This exactly confirms the

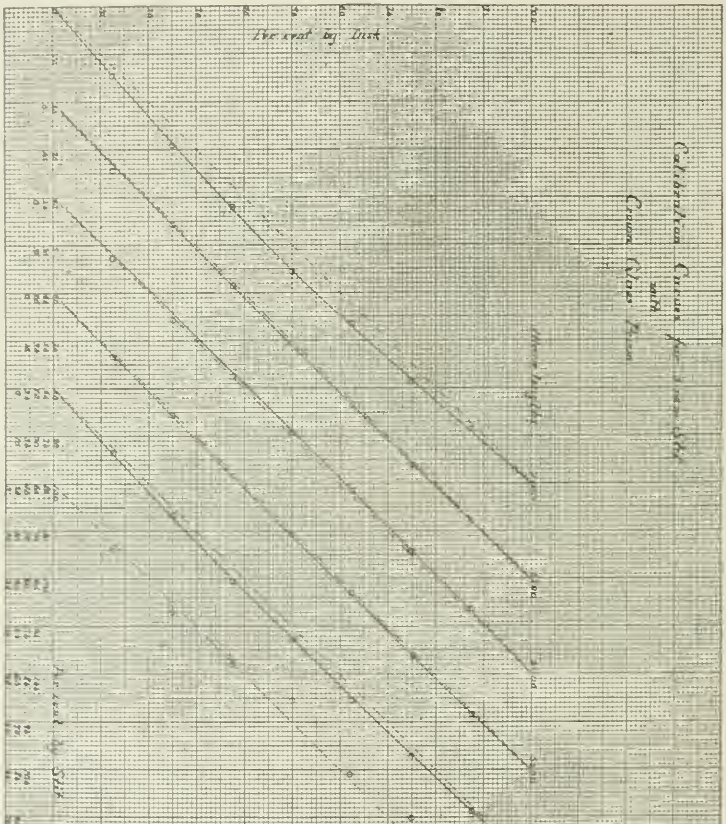

FIG. 4.

deductions made from the curve of luminosity. The effect of increasing the width of the slit is to increase the variation for all parts of the spectrum. The variation in the red for a 2 mm slit is seen to be very great.

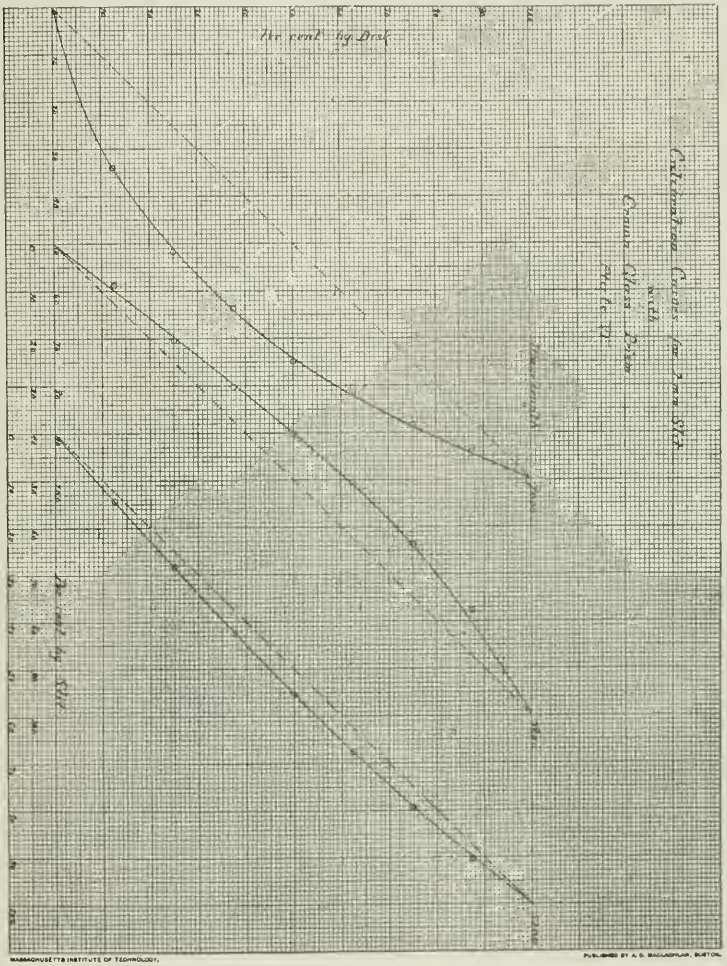

FIG. 5.

TABLE I.

CALIBRATION OF SLIT.

Slit 0.5 mm. Wave-length 0.7000 μ.

	Zero	12.5	25	37.5	50	62.5	75	87.5	Zero
Settings	.456	.026	.092	.151	.228	.275	.350	.400	.451
	.462	.028	.099	.160	.226	.286	.350	.403	.458
	.457	.028	.091	.162	.228	.278	.345	.404	.457
	.459	.029	.096	.152	.220	.280	.350	.402	.457
	.461	.030	.098	.153	.226	.280	.353	.409	.465
	.456	.029	.093	.153	.228	.2[illegible]3	.350	.400	.457
	.452	.029	.093	.152	.229	.278	.340	.403	.450
	.455	.030	.095	.152	.222	.280	.355	.409	.451
	.460	.031	.093	.155	.225	.276	.340	.404	.462
	.459	.027	.096	.161	.229	.276	.342	.405	.456
Mean	.4577	.0287	.0947	.1556	.2261	.2792	.3475	.4039	.4564

TABLE II. (Calibration for 0.5 mm slit.)

Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100	Wave length
Per cents. by slit	14.5	29.1	42.0	55.6	66.7	78.7	88.6	100	7000
	14.2	25.5	38.5	50.2	63.1	76.0	87.5	100	6500
	12.6	25.7	37.3	49.4	61.9	73.9	86.3	100	6000
	13.4	25.8	38.2	50.6	62.7	76.2	88.1	100	5500
	13.7	26.7	40.5	52.9	65.4	76.92	88.5	100	5000
	14.2	27.0	37.2	45.3	61.0	70.2	80.7	100	4500
	Slit 2 mm wide.								
	33.9	51.7	63.8	74.7	81.4	88.1	93.3	100	7000
	8.7	20.2	30.2	40.2	50.4	63.6	77.8	100	5800
	14.4	28.2	42.4	55.4	67.4	79.2	90.2	100	5200

TABLE III. (Wave-length 0.7000 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit[1]	13.8	27.1	39.3	53.5	64.4	77.9	89.3	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.9	28.8	42.5	55.3	68.6	78.1	90.4	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	16.8	31.9	45.4	58.4	69.9	81.1	90.4	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	19.2	36.2	50.8	63.5	74.7	82.7	90.5	100

TABLE IV. (Wave-length 0.6500 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.2	27.4	38.9	50.8	64.7	79.0	90.8	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.7	26.4	38.8	51.3	64.5	76.4	88.4	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.9	27.8	40.7	54.5	66.4	78.8	91.0	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.3	27.7	41.5	57.1	66.9	79.9	89.0	100

TABLE V. (Wave-length 0.6000 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.8	26.2	38.4	50.4	62.6	74.5	86.5	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.1	25.5	37.3	49.9	62.0	74.0	86.4	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	12.5	25.0	36.9	49.3	61.0	74.9	86.9	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	12.3	23.1	35.9	47.5	59.9	72.6	86.1	100

TABLE VI. (Wave-length 0.5500 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.2	26.2	38.2	50.9	63.0	75.5	87.6	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.6	26.3	39.1	51.8	63.5	75.8	87.8	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.6	26.4	38.9	52.1	63.8	76.3	87.9	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.9	26.5	39.2	52.3	64.1	77.1	88.5	100

TABLE VII. (Wave-length 0.5000 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.2	26.1	38.5	50.9	62.7	74.8	87.0	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.7	27.8	40.4	53.9	65.5	77.0	88.8	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.6	29.0	43.1	55.7	67.7	79.2	89.8	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	15.7	31.1	45.0	57.6	70.5	81.0	91.5	100

TABLE VIII. (Wave-length 0.4500 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.8	27.2	38.7	49.1	61.3	73.4	87.1	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.6	26.5	38.9	50.7	62.3	74.7	86.5	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.8	28.1	41.4	53.4	64.7	77.9	89.4	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.5	27.7	41.1	52.2	66.2	77.9	89.4	100

The results for the flint-glass prism are given in Tables III to VIII inclusive, the corresponding curves are plotted in Figs. 6 to 11. The general character of the results are the same as was obtained for the crown-glass prism with the exception that the variation is not nearly so large for the same slit-width and wave-length. The curves for the yellow and the violet, with a slit-width of 0.5 mm, cross the direct ratio line near its middle point. This may possibly be due to slight internal reflection, as nothing of this nature could be detected in the case of the crown glass. To test the relative constancy of the sources, one was dimmed by cutting down the current until the intensity was only one half of the other. The slit was then calibrated for the 50 per cent. notch, and the results were the same as previously obtained when

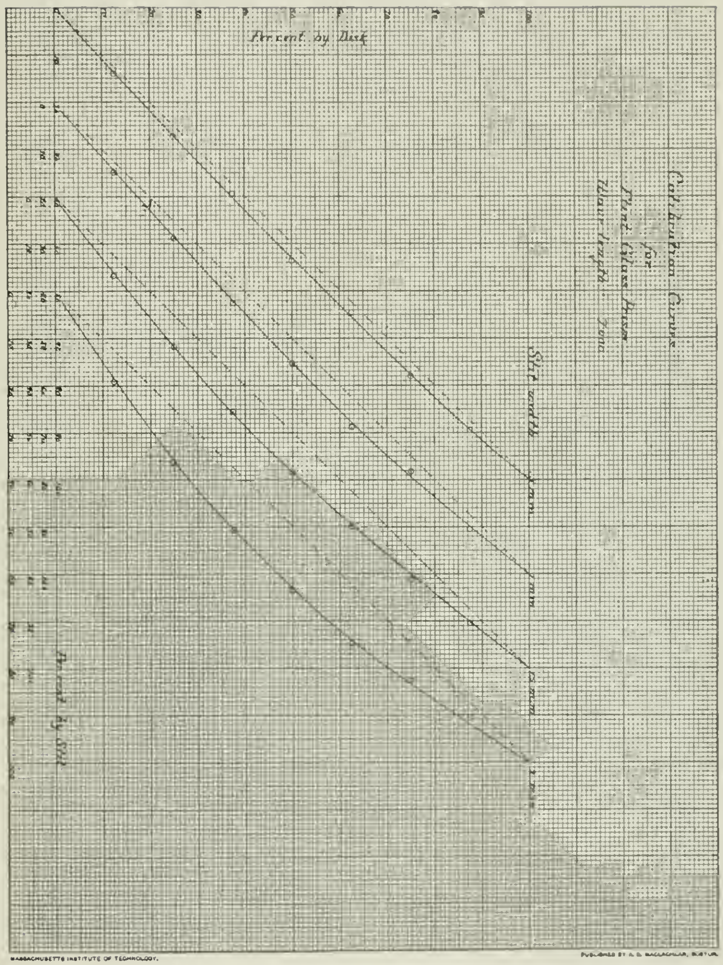

FIG. 6.

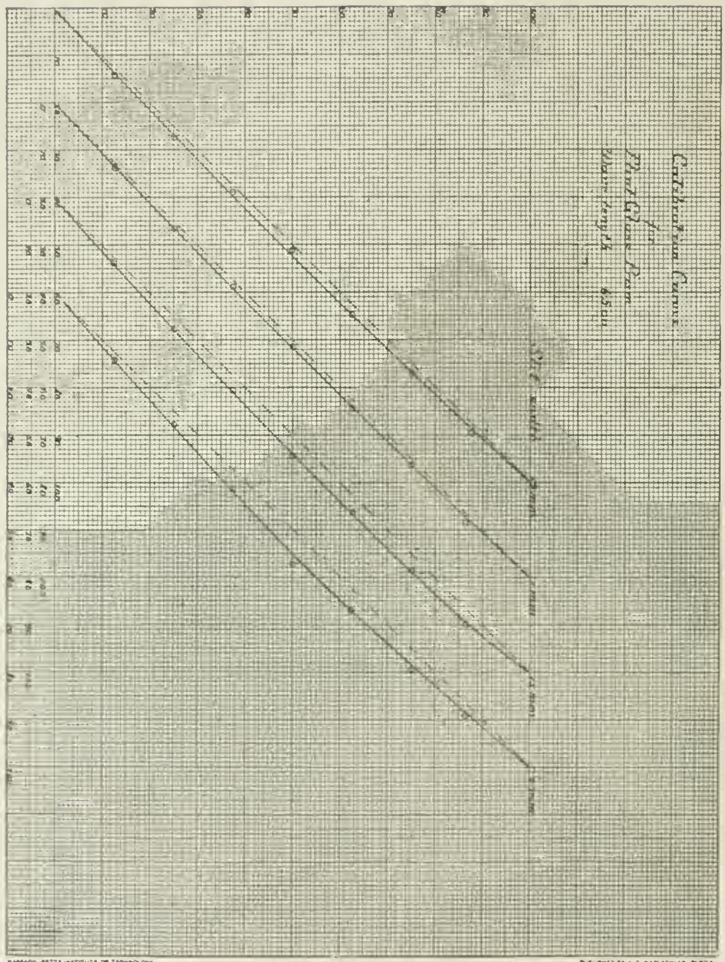

FIG. 7.

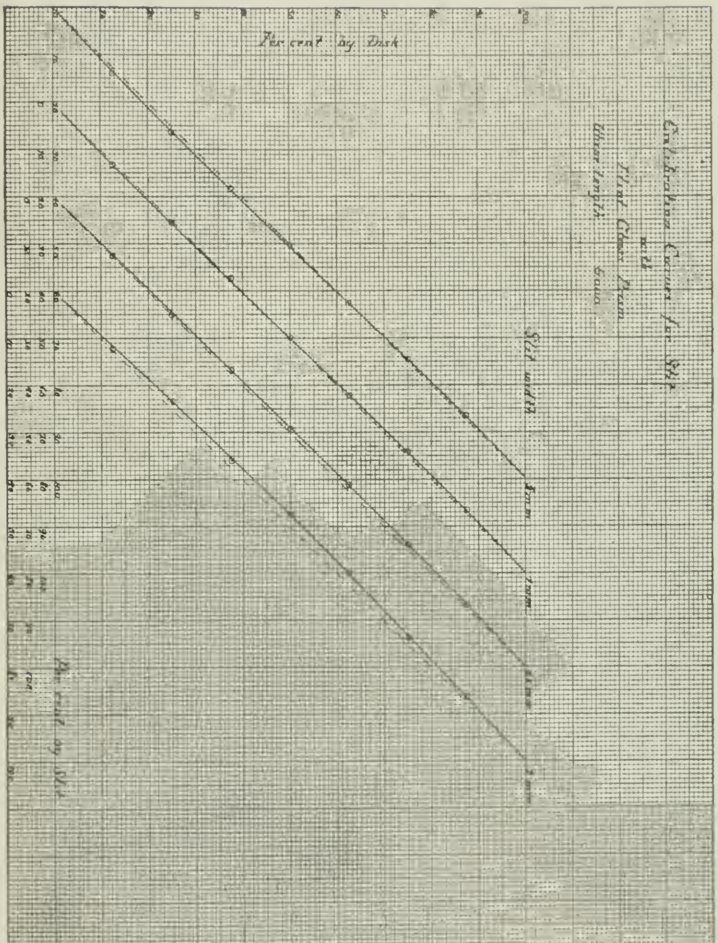

FIG. 8.

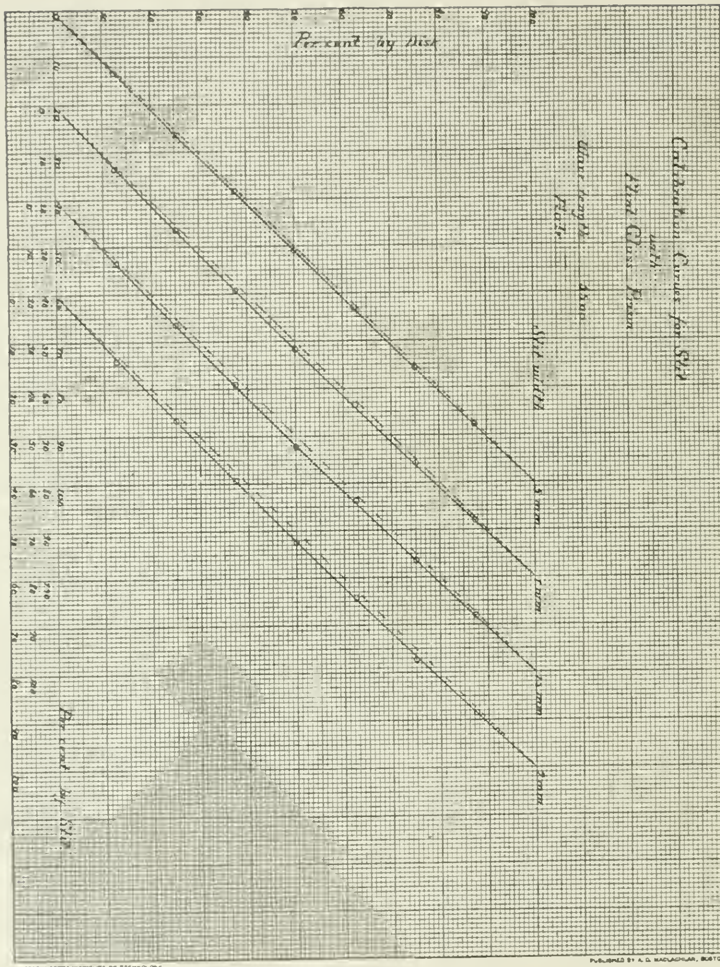
Calibration Curves for Slit
with
Flint Glass Prism
Slit width
Per cent by Disk
Per cent by Slit
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.
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FIG. 9.

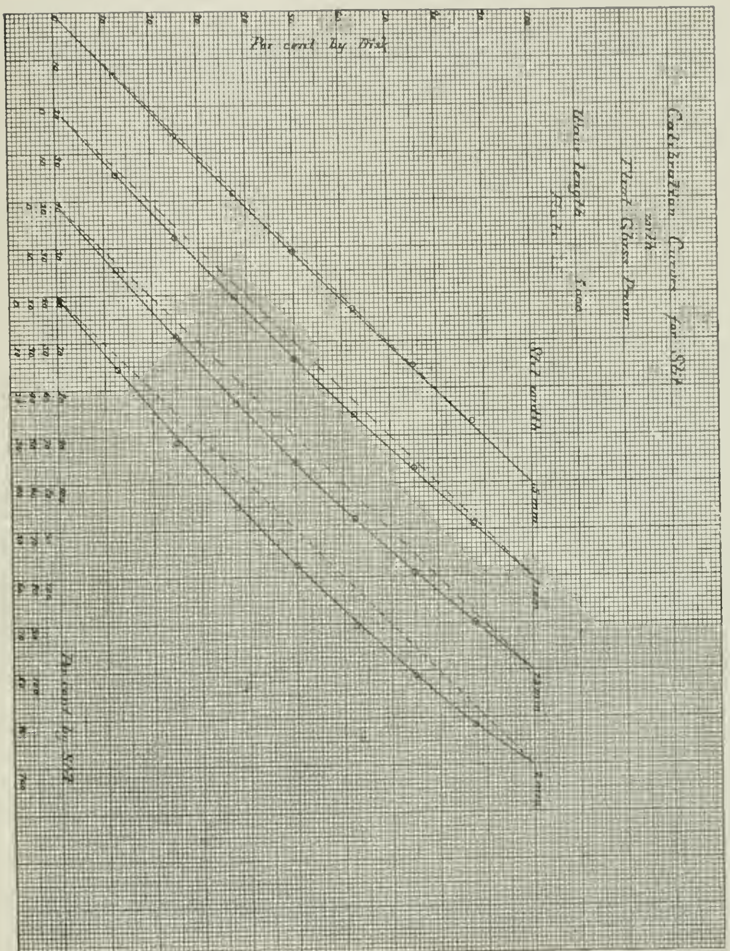
Per cent by Disk
Slit width
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.
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FIG. 10.

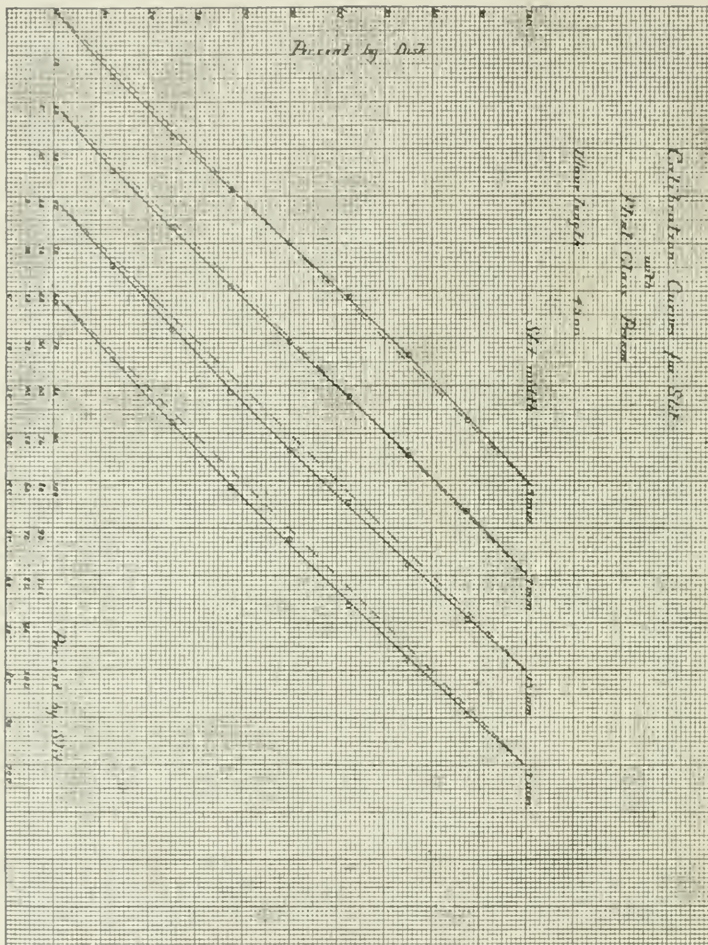
Percent by Disk
Calibration Curves for Slit
with
Flat Glass Plates
Slit width
Percent by Slit

FIG. 11.

both sources were of nearly the same intensity. When one lamp was replaced by a new one no change was produced in the calibration values.

The results may be summed up as follows:

1. The direct ratio law holds for only two points in the spectrum, corresponding approximately to wave-lengths 620 $\mu\mu$ and 570 $\mu\mu$.

2. The variation is greatest in the red and blue parts of the spectrum in the order named, and least and of opposite sign in the yellow.

3. It increases as the slit increases in width,

4. It decreases as the refractive power is increased.

5. It is practically independent of the relation of the two sources when they do not differ by more than 50 per cent.

6. The calibration curves are sufficiently reliable to be used to correct readings when the slit method is adopted for making photometric measurements.

In conclusion the author desires to thank Professors Brace and Moore for valuable suggestions that enabled him to carry out this work.

Physical Laboratory,
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December 1899.

ON THE ESCAPE OF GASES FROM PLANETARY ATMOSPHERES ACCORDING TO THE KINETIC THEORY.

By S. R. Cook.

The conditions for the existence of atmospheres on the planets, including the escape or retention of the lighter gases as hydrogen and helium by the Earth, are problems which have been recently discussed in a memoir by Dr. G. Johnstone Stoney.[1]

In these discussions Dr. Stoney has omitted the determination by the Kinetic Theory of the relative number of molecules which would have a velocity sufficient to enable them to escape from the Earth or planet, providing there be no retarding media. This velocity will be termed the critical velocity.

The purpose of the present paper is to apply Maxwell's distribution of velocities to the subject.

In order that any molecule may escape from the atmosphere of any member of the solar system, it must have a normal velocity equal to or greater than the velocity it would acquire in falling from infinity to its present position; and it must be so situated in the atmosphere that it will not be hindered by any impact while going from its present position to infinity.[2]

Since the mutual relations as to temperature, pressure, and density have not been sufficiently accurately determined for the limit of the Earth's atmosphere to allow satisfactory computations to be based on them, I shall compute the number of molecules that, with a critical velocity, will pass through the outer surface of a spherical shell surrounding the Earth and whose thickness is $r\lambda$, where λ is the mean free path of all the molecules in this layer, and $r\lambda$ that of all those escaping. This will be the number of molecules that will leave the atmosphere

[1] This Journal, 7, 25, January 1898, Royal Dublin Soc., Vol. VI, Part 13.

[2] It must be borne in mind that molecules may be moving in a medium which exerts a retarding influence.

under the assumption that this spherical shell is the outside limit of the atmosphere and that the temperature and density of the shell are constant.

For the Earth the number will be computed under the following conditions:

1. For a spherical shell at the Earth's surface at a mean temperature of 5° C.

2. For a spherical shell 200 km from the Earth's surface at a temperature of −66° C.

3. For a spherical shell 20 km from the Earth's surface at a temperature of −66° C.

4. For a spherical shell 50 km from the Earth's surface at a temperature of −180° C.

The conditions specified in (1) are obviously the most favorable for the escape of the molecules under the assumption that the spherical shell is the outside limit of the atmosphere. The conditions specified in (2) are those assumed in the memoir by Dr. Stoney.[1] The conditions specified in (3) are the results of recent experimental data from balloon ascensions at Paris and Berlin, kindly furnished me by Professor Cleveland Abbe. Those specified in (4) are based on results obtained from theoretical considerations by Ferrel.[2]

The number of molecules having a velocity between c and $c + dc$ is

$$d N_0 = \frac{4 N_0}{\bar{c} \sqrt{\pi}} \cdot \frac{c^2}{\bar{c}^2} \cdot \epsilon^{-\frac{c^2}{\bar{c}^2}} dc.$$

Where c is any velocity, $\bar{c}$ is the mean velocity, and N_0 the number of molecules.

The number of molecules having a velocity of $r\bar{c}$ or greater for

$$r > 1 \text{ is } N^1 = N_0 K,$$

where

$$K = 2 \epsilon^{-x^2} \left(x + \frac{1}{2x} - \frac{1}{4x^3} + \frac{3}{8x^5} - \frac{15}{16x^7} + \right)$$

and

$$x = \frac{2r}{\sqrt{\pi}}.$$

[1] *Loc. cit.*, p. 1.

[2] Ferrel, *Recent Advances*, p. 37.

For all calculation made for the Earth's atmosphere r is greater than 5, so that K becomes a rapidly converging series. The value of the series after the third term being less than 0.1 per cent.

If N is the number of molecules in the limiting spherical shell of thickness $r\lambda$, and if $r\bar{c} = c_1$ is the critical velocity, then NK will be the number of molecules having a velocity equal to or greater than c_1.

If n^1 is the number of molecules in unit volume and R_1 the radius of the spherical shell of thickness $r\lambda$,

$$N = 4\pi R_1^2 r \lambda n^1.$$

Since the critical velocity c_1 is equally probable in all directions, in order to find the number of molecules that will pass through the outer surface of the spherical shell with a velocity c_1 or greater, we determine $\bar{c}_1$, the mean velocity of the molecules having a velocity between c_1 and infinity. Since any molecule, whose component velocity normal to the shell is $c_1 = \bar{c}_1 \cos\theta$, will escape, the proportion of those that will escape during time t_1 will be $\phi : 4\pi$, t being the time of the mean free path $r\lambda$ of these molecules, and ϕ being the solid angle of aperture 2θ. Hence, to the first order of approximation, the number of molecules that will escape in time t_1 will be

$$N_2 = 2\pi R_1^2 r \lambda K n^1 (1 - \cos\theta) \qquad (5)$$

The number that will escape in any other time T will be

$$N_3 = 2\pi R_1^2 r \lambda K n^1 (1 - \cos\theta) \, . \, \frac{T}{t_1} \qquad (6)$$

This formula gives a maximum limit to the number of molecules that will pass outward from the limiting shell with a normal velocity equal to or greater than the critical velocity c_1.

The quantities on the right hand side of equation 6 can all be determined from the Kinetic Theory, except the critical velocity which can be obtained from the formula

$$c_1^2 = 2a \frac{R^2}{R_1}$$

where a is the total acceleration, R is the radius of the Earth, and R_1 is the distance from the center of the Earth to the molecule.

Since N_3 is a linear function of n^1, r_1, $1/t_1$, K, R_1^2 and $(1 - \cos\theta)$, the relative variation in N produced by these factors can readily be determined. The value of n^1 as determined by different observers varies from 6×10^{18} to 10×10^{18}, the latter value being used in the calculation. Similarly t_1 and λ will have a corresponding range of variation. The variation of R_1 arising from the different assumed heights of the limiting shell are contained in the tabulated results. Since $\theta = \cos^{-1}\frac{\bar{c}_1}{c_1}$. The variation in $(1 - \cos\theta)$ will depend upon the variation in $\frac{c_1}{\bar{c}_1}$, which, from inspection of the curve of distribution of velocities beyond the critical velocity, makes $\frac{c_1}{\bar{c}_1}$, approximately a constant ratio. However, the greatest value consistent with the problem was selected for $\bar{c}_1$, making the value of $(1 - \cos\theta)$ a maximum.

Since $c_0 = \frac{1}{3}\sqrt{\frac{p}{\rho}}$ and $\bar{c} = \bar{c}_0\sqrt{\frac{T}{273}}$, where $\bar{c}_0$ is zero temperature, any variation in $\bar{c}$ will depend on the variation in the pressure, p, the density, ρ, and the absolute temperature, T, for which $\bar{c}$ is computed. p and ρ are very accurately determined quantities. The absolute temperature T was selected for each assumed spherical shell, such that $\bar{c}$ would be a maximum value,[1] $c_1 = 2a\frac{R^2}{R_1}$, where a was taken as 981.5 cm, R as 6378 km, and R_1 varied from 6378 to 6578 km, depending on the height of the spherical shell. Since $r = \frac{c_1}{\bar{c}}$, the variation in n_3, produced directly by r will be small; but as r enters into K as an inverse exponent of the second degree, as a factor, and in the term used to the fifth degree, any slight variation in r will produce a very great variation in the value of K, and as the value of r has been

[1] The following table gives some of the results of recent balloon ascensions:

November	14, 1896,	height,	15,000	meters;	temperature,	−60° C.	Paris.
December	4, 1894,	"	10,000	"	"	−52° C.	Berlin.
October	20, 1895,	"	15,000	"	"	−70° C.	Paris.
March	29, 1896,	"	14,000	"	"	−63° C.	Paris.

determined in each case by the selection of the value of the absolute temperature and the distance of the spherical shell from the Earth, the value of K will depend upon their selected values. I have at all times selected such values of these factors as were consistent with a special problem and which would make N_3 a maximum.

In the following table, column one gives the conditions specified on page 2; the second column the critical velocity in kilometers per second; the third column the ratio of the critical velocity to the mean velocity; the fourth column the number of molecules that will escape during the time of the mean free path; the fifth column the number of cubic centimeters of the gas that will escape in one year; and the sixth column gives the number of cubic centimeters that will escape in 10^7 years.

The results which Professor Bigelow obtained by working over the data of all balloon ascensions gives in general a fall of 65° to 70° for 16,000 meters. I have taken the temperature —66° C. at a height of 20 km, and the value 1841 m sec. for $\bar{c}_0$ for hydrogen.

HYDROGEN.

Condition	c_1	r	N_3 $T=t_1$	N_3 in cc $T=1$ year	N_3 in cc $T=(10)^7$ years
1	11.	5.92	$80.24\,(10)^{19}$	$33.04\,(10)^{8}$	$33.04\,(10)^{15}$
2	10.5	6.55	$45.15\,(10)^{4}$	$23.58\,(10)^{-7}$	23.58
3	10.98	6.85	$72.61\,(10)^{9}$	$54.28\,(10)^{-2}$	$54.28\,(10)^{5}$
4	10.90	10.14	$66.8\,(10)^{-4}$	$43.5\,(10)^{-15}$	$43.5\,(10)^{-8}$

HELIUM.

1	11	8.37	19.8	$10.34\,(10)^{-11}$	$10.34\,(10)^{-4}$
2	10.5	9.27	$44.5\,(10)^{-18}$	$22.10\,(10)^{-31}$	$22.10\,(10)^{-24}$
3	10.98	9.78	$50.15\,(10)^{-19}$	$26.73\,(10)^{-30}$	$26.73\,(10)^{-23}$
4	10.90	14.5	$19.27\,(10)^{-53}$	$91.6\,(10)^{-93}$	$91.6\,(10)^{-86}$

The results here given were computed from equation (6). The values for N_3 are computed for a hydrogen and a helium atmosphere, and they represent the amount that would escape

from an atmosphere of hydrogen or helium under the specified conditions and bounded by the respective spherical shells.

Condition (4) represents most nearly the conditions of the outer limit of the atmosphere; $43.5\,(10)^{-8}$ cc of hydrogen would escape from a hydrogen atmosphere thus bounded and conditioned during 10^7 years, and $91.6\,(10)^{-8}$ cc of helium would escape from a helium atmosphere in the same time similarly conditioned. Since under these conditions a very small fraction of 1 cc of hydrogen would escape from a hydrogen atmosphere in 10^7 years, it is evident that the amount which is actually escaping from the Earth's atmosphere, if any, is insignificant. The amount of helium escaping would be zero.

There are approximately 10^{24} cc of air in the atmosphere, and under the most favorable condition less than 10^{10} cc of hydrogen would escape from an atmosphere of hydrogen whose outer layer was 5° C. and whose density was the density of hydrogen at atmospheric pressure, in one year. It would under these conditions take 10^{14} years for an amount of hydrogen equal to the Earth's atmosphere to escape. If we reduce the value of r from 5.92 to 5, we find approximately that 10^{15} cc of hydrogen would escape per year, and it would take 10^9 years for an amount equal to the atmosphere to escape under those conditions.

I shall now apply the results obtained for an atmosphere of hydrogen on the Earth to the atmospheres of the Moon and planets. I shall use the values for the minimum velocity of escape given by Dr. Stoney, as they will be sufficiently accurate for a very close approximation.

The relation between the molecular velocities and the absolute temperature is

$$\frac{c_1^2}{c_0^2} = \frac{T_1}{T_0}.$$

The following table gives the value of the temperature of the outer layer of the atmosphere which would enable an atmosphere to escape at the same rate as hydrogen would escape from an atmosphere of hydrogen whose outer layer is conditioned as in

(1) and whose critical velocity is $5\bar{c}_1$, the number of molecules in the atmosphere being the same as in the Earth's atmosphere, and the time allowed for the atmosphere to escape being 10^9 years.

TABLE II.

			Hydrogen		Air		Carbondioxide	
	c_1	$\bar{c}_t$	t	r_1	t	r_1	t	r_1
Moon......	2.380	.476	—256	1.24	— 10°	4.7	274	6.6
Mercury....	4.468	.8936	—209°	2.4	894	9.2	1371	12.4
Venus......	9.546	1.909	20 .5	5.185	5031	19.27	7403	26.5
Mars.......	4.803	.960	—195°	2.66	1139	9.9	1807	13.3
Earth......	10.5	2.100	291°	5.7	9937	21.7	14447	29.17

c_1 is the critical velocity in kilometers per second; $\bar{c}_t$ is the mean velocity of the molecule in km sec. $=c_1/5$; t is the temperature of the outer layer of the atmosphere in centigrade degrees, and r is the ratio of the critical velocity to the mean velocity.

This table shows that on the Moon an atmosphere of hydrogen would escape with its outer layer at $-256°$ C.; an atmosphere of air at $-10°$ C., and an atmosphere of carbon dioxide at 274° C. For Mercury the outer layer would necessarily be $-209°$ C., 894° C., and 1371° C. for hydrogen, air, and carbon dioxide respectively. For Venus the outer limit of the respective atmosphere would be 20.5° C., 5051° C., and 7403° C. The temperature of the outer limit of the Earth's atmosphere, in order that the respective atmospheres would escape, would be, for hydrogen 291° C., for air 9937° C., and for carbon dioxide 14447° C. For Mars the outer limit of the respective atmospheres would be $-195°$ C., 1139° C., and 1807° C.

In order to draw any more definite conclusions in regard to the atmosphere of the Moon and planets, it will be necessary to know the temperature of the planets and the temperature gradient of their atmosphere. It appears, however, from the above figures, that an atmosphere like that of the Earth would not remain on the Moon, but would remain on any of the planets

mentioned. The Earth and the major planets would not only retain an atmosphere of nitrogen and oxygen, but also hydrogen and helium.

My thanks are due to Professor D. B. Brace for many valuable suggestions, and also to Professor Cleveland Abbe for valuable data and references.

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ON A BALANCING RHEOSTAT FOR BOLOMETERS.

By P. G. Nutting.

Professor F. L. O. Wadsworth some time ago described in this Journal[1] a device for making fine bridge adjustments by means of a double contact arrangement for one of the battery terminals. With the best contacts we could construct, however, we found the error due to varying contact too great to make the arrangement available for use with our differential bolometer in the determination of absorptive indices. We finally hit upon the device of connecting *both* galvanometer terminals by variable contact to the bridge angles instead of only one as is usually done. The bridge junction at one galvanometer terminal is made of rather fine wire, of just sufficient resistance to overcome the inequalities of the bolometer arms. The other bridge junction at the remaining galvanometer terminal is made of large wire of very low resistance, and serves for the delicate final setting. The resistance of this whole wire may be made equal within about 1 mm, or less, to that at the opposite terminal, but for rapid work it is conveniently of a resistance sufficient to balance the greatest bolometric variations expected. It is further convenient to choose the balancing arms of such diameter that their lengths may be made equal, and not more than 20 cm. These are stretched side by side on an ebonite plate, and a millimeter scale inlaid under the one where the final adjustment is made. The galvanometer terminal makes contact with this wire by means of a sharpened piece of No. 8 wire, which is frequently repointed. The rough adjustment is made first, and that terminal fastened with wax. If a null method is not used, it is packed and enclosed to guard against temperature waves, leaving only the large, low resistance wire exposed for adjustments.

For the most delicate bolometric work there remain only

[1] This Journal, 5, 268–276, April 1897.

thermal currents and variations in resistance with room temperature to be eliminated in the balancing rheostat. Working with a null method, we make all external connections of copper wire, rheostat included. Junctions are made close fitting and soldered. The only thermal junctions are at the ends of the platinum receiving strips, and these are enclosed within the bolometers themselves. Changes in resistance with room temperature are of no consequence when a null method is used. For other work, nickeline or manganin is used for one rheostat wire, or else a temperature correction applied. This may easily be done to within the probable error due to other causes.

The balancing rheostat used here in connection with the double bolometer is constructed of copper wires of about 0.005 and 0.0003 ohms, each 10 cm long. The adjustment of the galvanometer terminal can be made to within $\frac{1}{10}$ mm; readings of its position are unnecessary using the null method. $\frac{1}{10}$ mm on the larger wire corresponds to 0.0000003 ohm. This is but 0.000000015 of the resistance of the bolometer arms. Calling the temperature coefficient of platinum 0.005 per degree, a temperature change of 0.000003° in the bolometer strips can be detected. For these tests a commercial d'Arsonval galvanometer and an old 1.5 volt, 20 ohm Leclanche cell were used. Our new, specially constructed, light suspension, Paschen galvanometer is sensitive to a current of less than 1×10^{-11} amperes. Using only $\frac{1}{5}$ the former current, and a larger and longer rheostat wire, the sensitiveness is increased about one hundred times.

Excellent quantitative work may be done with the same arrangement. A simple computation will show that in the determination of a small percentage variation in resistance, the coefficient is certain to as many significant figures as is the resistance itself. For example, when copper wires of 2.002 and 2.006 ohms resistance were surrounded with gasoline, a shift of 22.4 mm in the rheostat wire was necessary to restore equilibrium. This shift corresponded to 0.00000683 ohm, or 0.000167 of 1 per cent. change in resistance for each wire. Changes due to temperature were eliminated by stretching the

four very nearly equal wires forming the bridge arms side by side in a heavy brass tube, and only exposing opposite arms to the dielectric.

By a somewhat similar method, the temperature coefficient of some new resistance wire sent us from Germany was found to be — 0.00000271 per degree.

It is hoped that the above may be of use to a few of the many investigators now using bolometers in research.

University of California,
November 1899.

SOLAR ECLIPSE PROBLEMS.

By George E. Hale.

The writer, as secretary of the Eclipse Committee of the Astronomical and Astrophysical Society of America, has received so many requests for suggestions from intending observers of the coming total eclipse that it has seemed advisable to bring together, for the convenience of those who may not have ready access to astrophysical literature, various data bearing upon some of the more important problems which this eclipse may help to solve. As no total eclipse of the Sun has been visible in the United States since 1889 it may be expected that many individuals and institutions will be anxious to take advantage of the present opportunity. Considered merely as a spectacle, the eclipse will amply repay those who may journey far to see it. But if the maximum scientific return is to be derived from this rare phenomenon, every intending observer must familiarize himself with the results obtained at previous eclipses, devote much time to the consideration of the questions still demanding solution, and give particular attention to the design of the apparatus required for the work selected. It fortunately happens that valuable scientific results can be obtained with simple and inexpensive instruments. The unsolved problems, too, are numerous and varied, offering a wide field for choice, and appealing to the individual tastes of many observers. It is only necessary to chose intelligently, basing the decision upon one's previous experience in observational work and a consideration of available instruments. If possible, there should also be some measure of coöperation between different parties, and here the Eclipse Committee stands ready to lend its aid.

The eclipse will occur on May 28, 1900, between 8 A. M. and 9 A. M. The central line of the shadow will cross the states of Virginia, North Carolina, South Carolina, Georgia, Alabama,

Mississippi, and Louisiana, the duration of totality decreasing from $1^m 46^s$ on the Atlantic seaboard to $1^m 18^s$ near New Orleans. The advantage due to a longer duration of the total phase is more than offset, however, by the much greater probability of cloudiness at points near the ocean, as shown by the survey of cloud conditions recently made under the supervision of the United States Weather Bureau. From these observations it appears that the chances of fair weather are best on the highland of the southern portions of the Appalachian Mountains.[1] It is to be hoped, however, that all of the stations will not be confined to this region, as there are many reasons why they should be distributed all along the line of totality.

In preparing a plan of operations it is first necessary to remember that the eclipse will be a short one, even for those who may view it from the Atlantic coast. Other things being equal, it therefore will not be desirable to undertake observations which cannot be successfully completed within a space of little more than a minute. Thus it would hardly be advisable to attempt to photograph the fainter portions of the spectrum of the corona on this occasion, and it is exceedingly doubtful whether it would be worth while to repeat the hitherto unsuccessful attempts to measure spectroscopically the period of rotation of the corona. Although some reference to such experiments may be found in this paper, it will probably be best to reserve these observations for the Sumatra eclipse, when totality will last over six minutes. On the other hand, the spectrum of the reversing layer can be photographed next May to good advantage, especially, as Mr. Evershed has pointed out, at stations not far removed from the edges of the shadow path. Many other phenomena can be advantageously studied during the total phase, provided the observer will limit himself to some well-defined piece of work, which must not be too comprehensive in its scope.

[1] See *Weather Bureau Bulletin*, No. 27. In addition to statistics of cloudiness, this bulletin contains much valuable information regarding suitable sites for stations, hotel accommodations, etc.

The suggestions offered in the following paragraphs make no pretensions to fullness of scope or adequacy of treatment. They are in no sense intended to take the place of "Instructions" to eclipse observers, and it is not supposed that they will be of service to experienced astrophysicists, except possibly in recalling certain matters which are sometimes overlooked. To amateurs possessing small instruments, and to others who have had little practical experience in solar work, it is hoped that they may be of some value.

I. NAKED-EYE DRAWINGS OF THE CORONA.

The experience of previous eclipses has shown that drawings of the corona for the most part serve no useful purpose, unless it be to illustrate the personal peculiarities of the draftsman. At the present time, when everyone who possesses a suitable camera may secure a photograph which can be relied upon, it seems quite superfluous to devote the brief time of totality to sketching. Naked-eye observations are specially referred to here. If drawings are made it should be only after long and faithful drill, and the more limited the region drawn the better.

2. DRAWINGS AT THE TELESCOPE.

What has been said regarding naked-eye drawings applies with much less force to drawings made with the aid of a telescope. According to the testimony of the best observers the most successful large-scale photographs hitherto made fail to reveal the most interesting details of the coronal structure. These details can be seen with telescopes of from three to six inches aperture, and an observer practiced in rapid sketching should be able to secure a valuable record if he confines his attention to a limited region. Concentration of this kind, especially if the field of view is small, should eliminate many of the distractions which so seriously interfere with naked-eye observations. It is particularly to be desired that some observers should devote their attention to the lower corona in the immediate

vicinity of large prominences. Those who would be likely to distinguish an active prominence by a glance at its form would do well to record the coronal details near it. Others should sketch the polar streamers close to the Moon's limb, and attention may also be given to the structure of the outer corona, both in the polar and equatorial streamers. It is evident that coöperation of some sort is desirable in this work, in order that the various parts of the corona may receive proper attention.

3. COLOR OF THE CORONA AND PROMINENCES.

Reliable information regarding the color of the prominences and various parts of the corona is much to be desired. But if the record of observations is to be of real service, it must consist of something more definite than fanciful similes, such as are likely to be called forth by the excitement of the moment. The colors must be well and clearly defined, and if possible referred to some standards of color, prepared for the purpose. If a telescope is not employed the observations will be much facilitated by the use of a field or opera-glass, provided instrumental color is not introduced in this way. The presence of the prominences greatly interferes with determinations of the color of the corona, and in such observations it would be advantageous to occult them with a small disk in the focal plane of the telescope.

The importance of noting the color of the prominences rests upon the fact that Tacchini and others have observed the so-called "white prominences," of which an adequate explanation has yet to be offered. The subject has been discussed by the present writer in an article entitled "The Effect of a Total Eclipse of the Sun on the Visibility of the Solar Prominences,"[1] where Tacchini's observations are fully described. To mention only a single case, it may be said that at the eclipse of 1886 Tacchini distinctly saw an enormous white prominence, which appeared on the photographs of the inner corona, but could not be seen with the spectroscope in the $H\alpha$ line either before or

[1] This JOURNAL. 3, 374, 1896.

after totality. In the article referred to I have suggested that the white color of such a prominence may be due to the comparative faintness of the less refrangible lines in its spectrum, whereas the calcium lines H and K were of their normal intensity. Objective prism photographs of the chromosphere and prominences have indeed shown marked variations in the relative intensity of prominence lines, but it is not yet certain that a "white prominence" could be caused in this way. From this point of view it would be well to compare together the colors of a small number of prominences, as seen in a field-glass or telescope, noting the degree of redness in each case. Such observations should of course be supplemented by work with the spectroscope whenever possible. Photographs of the "flash" spectrum made with an objective prism will be very useful in this connection. If circumstances permit, the chromosphere and prominences will be photographed in the K line with the large spectroheliograph of the Yerkes Observatory at the time of totality for purposes of comparison.

4. SMALL SCALE PHOTOGRAPHS OF THE CORONA.

Every possessor of a suitable camera is equipped with apparatus sufficient to give valuable results in photographing the corona. As Mr. Maunder has pointed out in his interesting discussion of the results obtained at the recent Indian eclipse, with lenses of focal length not greater than five feet and ratio of aperture to focal length not less than $\frac{1}{15}$ it is not necessary to have an equatorial mounting for the camera when the exposure does not exceed one half second. It must not be supposed, however, that it is only necessary to point any camera at the Sun and make a snap exposure. Such a procedure might possibly produce a good picture of the corona, but the most valuable results will be obtained by those who observe a few simple precautions:

a. It is advantageous to use lenses of considerable focal length (from 30 to 60 inches) provided the angular aperture is not too small. Such lenses should show the corona on a scale

large enough to bring out many important details. Very short focus lenses will be useful for long-exposure photographs of the coronal streamers, if mounted equatorially and moved by clock-work.

b. Light and unsteady camera boxes should not be employed. A strong wooden box of the proper length is much more suitable than a camera having leather bellows.

c. It is of great importance that the camera box should be very solidly mounted, so that it will not be affected by the wind and not easily set into vibration by any cause. Unless an equatorial mounting is to be used it will be advantageous to screw the wooden camera box to the top of a post firmly planted in the ground. The optical axis of the lens should point toward the Sun at the middle of totality. Unless the focal length of the lens exceeds 5 feet, or the diameter of the field is very small, it will be unnecessary to change the position of the camera during totality.

d. The lens can be most accurately focused by photographing star trails, a series of exposures being made for different positions of the lens. After the best focus has been ascertained the lens should be firmly clamped in place.

e. The results obtained at the Indian eclipse demonstrate the great advantage of using multiple coated plates. It will be best to employ triple or quadruple coated plates, and as a further safeguard the plates may also be given a suitable backing in order to still further reduce the danger of solarization. With such plates it is possible with an equatorially mounted camera to obtain the faint outer extensions of the corona without serious interference from the bright inner corona. However, too much should not be left to the plates, for the care exercised in development is of the greatest importance.

f. With the most rapid plates it would appear from the results discussed by Mr. Maunder that for an aperture of $\frac{f}{15}$,[1]

[1] For other angular apertures the exposure may be considered to vary inversely as $\left(\frac{f}{a}\right)^2$.

exposures of from $\frac{1}{60}$ second to $\frac{1}{45}$ second will suffice for the prominences and $\frac{1}{10}$ second to $\frac{1}{5}$ second for the inner corona. Such short exposures can of course be given only with the aid of a suitable shutter. A greater extent of the corona may be obtained with exposures from $\frac{1}{2}$ to $\frac{3}{4}$ second. Mr. Maunder believes that in order to obtain a photograph of the corona as a whole the exposures should not exceed one second. It is evident that most of the exposures so far referred to are easily within the reach of those who employ fixed cameras. If it is desired to secure the great extensions of the corona, which at the Indian eclipse were traced to a distance of nearly fourteen lunar radii from the Sun, the camera must be mounted on an equatorial telescope provided with a driving-clock. In this case the exposure might well continue throughout the entire time of totality, but great care should be taken to cap the lens several seconds before the expected reappearance of sunlight. It is desirable to have several lenses of different focal lengths carried by the same equatorial mounting. The long focus lenses, giving large solar images, should be used in photographing the prominences and the structure of the inner corona, while the short focus lenses may be employed to record the long streamers.

g. Care must be taken to provide for the accurate orientation of all photographs, as it is highly important that the north and south points of the image shall be known. If the plate stands in a vertical position, as it may when a heliostat is used, a fine plumb line hanging immediately in front of it will give satisfactory results. In the more common case, where a camera pointing at the Sun is employed, a series of rapid exposures made with reduced aperture during the partial phase, while the Sun moves across the stationary plate, will serve the purpose.

It is to be hoped that during the coming eclipse cameras in the hands of amateur observers will be distributed all along the line of totality in the United States, in Spain, and in Algeria.

A comparison of such a series of photographs might lead to very valuable results.[1]

5. PHOTOGRAPHS OF THE CORONA DURING THE PARTIAL PHASE.

During the recent Indian eclipse Mrs. Walter Maunder, using a triple coated plate and a Dallmeyer stigmatic lens of 1½ inches aperture and 9 inches focal length, succeeded in securing a photograph of the inner corona 39 seconds after the end of totality. As Mr. and Mrs. Maunder point out in the report to which such frequent reference has been made, it will be advisable to repeat and extend these experiments at the coming eclipse. The problem of photographing the corona in full sunlight is one which has received a great deal of attention, but hitherto without tangible result. It may be doubted whether the use of multiple coated plates will be sufficient of itself to overcome the great obstacle of atmospheric glare, but it is not impossible that the corona can be photographed in this way during partial eclipses. A series of exposures made before and after totality on multiple coated and heavily backed plates should be of value, especially if the image of the photosphere is prevented from falling on the sensitive film. An occulting disk before the plate should therefore be employed if possible. Mr. Maunder believes that with an aperture of $\frac{f}{15}$ the exposures out of totality should range from $\frac{1}{5}$ second down to $\frac{1}{50}$ second.

6. PHOTOGRAPHIC SEARCH FOR POSSIBLE INTRA-MERCURIAL PLANETS.

The possibility that planets may exist within the orbit of Mercury has led to many attempts to detect such objects under the favorable conditions presented by a total solar eclipse. As the unsuccessful visual observations made at previous eclipses tend to indicate that no very bright planet is likely to be found, preparations should be made to record all objects, down to the

[1] In this connection one cannot do better than consult the *Report on the Indian Eclipse of 1898*, recently issued by the British Astronomical Association. Many of the above suggestions are derived from this source.

lowest attainable magnitude, in the region about the Sun. Except by a most fortunate chance nothing can be expected to result from visual observations. Even if the region were divided among a great number of observers, little could be done in the brief space of totality, and disputes would be likely to arise on account of the uncertainties inseparable from hurried work. It would therefore seem inadvisable to waste effort on visual observations, but a thorough photographic search should undoubtedly be made. In undertaking such a search the following considerations are among those which should be borne in mind:

a. The lens should combine the greatest possible rapidity of action with the largest possible field. Good portrait lenses having an aperture of about $\frac{f}{5}$ should give well-defined star images over a field about nine degrees in diameter on a flat plate. Experiments recently made by Professor Wadsworth and Mr. Brashear show that the diameter of the field can be greatly increased by the use of concave plates of suitable curvature. It is advisable to employ lenses in pairs, in order that suspected images may be verified on an exactly similar plate made with another lens. In addition to portrait lenses it will probably be advantageous to use rapid rectilinear lenses giving very large fields.

b. The suitability of a lens, the best exposure time, and the proper development can all be ascertained by photographing stars under conditions as nearly as possible like those prevailing during totality, when the general illumination will probably be not very different from that of a clear sky half an hour after sunset. The maximum exposure given should be a few seconds less than the computed duration of totality, for in order to avoid the possibility of complete loss it will be advisable at the eclipse to cap the lens an appreciable time before the Sun is expected to reappear. Such experiments will enable one to select the lens best adapted for the purpose, to determine approximately the exposure time giving the greatest number of stars, and to

form an idea of the number of stars likely to be recorded during the eclipse. Professor W. H. Pickering, who has great confidence in the photographic method, finds from experiment that it should be possible, under eclipse conditions, to photograph stars as faint as the sixth magnitude.

c. The camera should be carried by a good equatorial mounting, provided with driving-clock. Care should be taken in adjusting the instrument, for without perfect following the maximum photographic effect of course cannot be obtained.

d. Those who possess large portrait lenses of great angular aperture, but lack a suitable mounting, will do well to try photographing star trails under the conditions named above. It is probable that even with a fixed camera the chances of success would be decidedly greater than in the case of visual observations.

e. It goes without saying that the most rapid plates obtainable should be used. How far it will be safe to push the development can be roughly determined from the preliminary experiments.

f. In order to cover the largest possible area of sky it is very desirable that parties planning to undertake this work should coöperate, each devoting its attention to a particular region. All persons provided with suitable instruments, who are willing to participate in a general plan of operations, are invited to address the Eclipse Committee.

It may be added that Professor Newcomb, whose suggestions are embodied in the above paragraphs, considers a photographic search for possible intra-Mercurial planets as one of the most important pieces of work that can now be attempted during total eclipses of the Sun.

7. LARGE SCALE PHOTOGRAPHS OF THE CORONA AND PROMINENCES.

Portrait lens photographs of the corona, while of great value for certain purposes, are on too small a scale to permit the finer structure of the corona and prominences to be recorded. For this purpose the largest possible images are required. The

results obtained at recent eclipses with objectives of great focal length indicate the value of this method. The lens may be pointed directly at the Sun and used in a fixed position with a moving photographic plate, or it may be mounted with its axis horizontal, and supplied with light by a heliostat or coelostat. A double mirror heliostat or a coelostat should be used in preference to a single mirror heliostat (unless this be of the polar form), as no large instrument of the latter type, so far as the writer is informed, has ever been made to drive accurately. If the mirrors show any tendency to distortion by the Sun's heat they should be screened until a few seconds before totality. Of the lens it may be said that it is doubtful whether any advantage will result from the use of focal lengths much greater than 75 or 100 times the aperture. A consideration of the size of the silver grains would indicate that with perfect seeing the full visual resolving power of the objective should be realized when the focal length is about 50 times the aperture. Excellent results have, however, been obtained with objectives whose focal lengths are as much as 120 times the aperture. But it must not be forgotten that for an objective of given aperture the exposure time and the effects due to poor seeing and inaccuracies of the coelostat or heliostat increase rapidly with the focal length. Hence one should not go too far in his desire to secure a large focal image of the corona.

It is probable that a photographic objective of comparatively short focus (say $\frac{f}{16}$), if used in conjunction with a suitable enlarging lens, would give results but little inferior to those obtained with the apparatus described above. This would have the advantage of requiring only a small equatorial mounting, but it might be difficult to avoid setting the instrument into vibration when changing plate-holders. All apparatus used in large scale photography must of course have stable foundations and adequate protection from the wind.

It will probably be advantageous to use multiple coated and backed plates, particularly for the longer exposures. The

approximate lengths of exposures may be calculated from the data given in section 4.

The device employed at the Indian eclipse by Mr. Burckhalter, for the purpose of giving to each part of the corona an exposure inversely proportional to its brightness, merits more general use. It consists of a revolving disk, mounted concentrically with the Sun's image, and admitting light to the plate through an aperture of suitable form. The width of the opening at a given point cannot be exactly known until an expression giving the (photographic) brightness of the corona as a function of the distance from the Sun's limb has been found. Nevertheless, data sufficient for practical purposes are available, and there should be no difficulty in repeating Mr. Burckhalter's experiments.

8. DISTRIBUTION OF "CORONIUM."

It is a matter of considerable importance to determine whether "coronium"—the gas whose spectrum is characterized by the bright green line formerly known as "1474 *K*"—forms a structureless envelope about the Sun, or conforms in appearance to the corona. In other words, does the bright green line pass without marked change of intensity over coronal rifts and streamers, as Tennant long ago found it to do? It is evident that this question may be tested in several ways. One of the simplest is that employed by Maunder at the Indian eclipse of 1898. A small direct-vision prism was inserted in one of the eyepieces of a binocular of about two inches aperture. It was thus possible to compare the corona, as seen directly with one eye, with its monochromatic green image, as seen with the other. Thus the distribution of coronium could be ascertained with rapidity and certainty. Unless adequate shade glasses are used the Sun should not be observed with such an instrument before the beginning of totality, as the eye is likely to be temporarily injured by the brilliancy of the image. This was Mr. Maunder's experience, and as a result he was able to see the faint green image only after mid-totality had passed. No trace of rifts or rays could be distinguished by this observer. On account of its

small size and convenience, a prismatic binocular will be very suitable for persons who wish to make useful observations, but are not in a position to provide elaborate and expensive instruments. Care must be taken, however, to secure an efficient binocular and direct-vision prism. The magnifying power of the binocular should be sufficient to show the coronal structure clearly and distinctly. The instrument, with prism in place, can be tested by the aid of an alcohol or Bunsen flame, colored with lithium or strontium chloride, and partially screened by a piece of cardboard in which openings are cut having the same angular magnitude as the coronal rays.

It would be interesting to examine with such an instrument the green coronal image in the immediate vicinity of a prominence. An objective prism or grating, or a small direct-vision prism, used in conjunction with a telescope of three or four inches aperture, would be more suitable for this purpose, however, on account of the larger scale of the image.

9. POSITION OF THE GREEN CORONAL LINE.

It has been recently shown by Lockyer, Campbell, and others that the green coronal line photographed at the Indian eclipse of 1898 did not correspond in position, as it had previously been supposed to do, with the line falling at 1474 on Kirchhoff's map of the solar spectrum, Undoubtedly the best way to accurately determine the wave-length of the green coronal line at the coming eclipse is by photography, but direct visual comparisons of the chromospheric and coronal lines will also be of value. Any spectroscope that easily separates the D lines will serve for the purpose, but a dispersion of two or three prisms would be advantageous. The spectroscope should be attached to a telescope of from four to six inches aperture, provided with good slow motions and driving-clock. The slit should be made radial at the computed point of second contact, and set in the focus of the telescope corresponding to λ 5317. The micrometer of the spectroscope should be provided with a rather coarse wire, extending only half way across the field, and fined down to a

sharp point. At the beginning of totality this wire may be set so as to occult the bright chromospheric line at λ 5317 (1474 *K*). A few seconds later the green coronal line should become visible, and it may be seen at once whether it coincides in position with the wire. If time permits, micrometer settings may be made to determine the wave-length of the coronal line. Or the same instrument may be employed in an attempt to trace the green line across the coronal rifts.

10. PHOTOGRAPHS OF THE SPECTRUM OF THE CHROMOSPHERE.

The importance of securing a complete record of the spectrum of the base of the chromosphere is so great that no pains should be spared to surpass the admirable results recently obtained in India. So far as the writer knows, the numerous lines of the green and yellow flutings of carbon vapor, which can be seen at the Sun's limb on any good day with the spectroscope attached to the 40-inch Yerkes telescope, have not yet been observed or photographed during eclipses. This indicates that the spectrum of the "flash" is probably far more complicated than even the best photographs show it to be, and thus emphasizes the fact that too much attention cannot be devoted to this field of eclipse research. The brief duration of the flash is the principal obstacle to success. For this reason Mr. Evershed's suggestion that stations lying only a few miles within the north and south boundaries of the shadow path be selected for this work is an excellent one. At such points the duration of totality may be only one third that on the central line, but during this entire time the spectrum of the chromosphere would be visible. Thus photographs taken at second and third contacts would give the combined spectra of the lower and upper strata, while exposures made at mid-eclipse would record only the higher layers. Observers who wish to give special attention to the smaller prominences and the lower corona could also work to advantage at such stations, but all other observations should be made on or near the central line.

There can be little doubt that of all forms of spectroscopes suitable for photographing the spectrum of the flash the objective prism is the one most likely to give good results. As compared with a slit spectrograph it has certain disadvantages, notably on account of the absence of a slit, which renders the use of a comparison spectrum practically impossible. This defect is rendered less serious by the fact that a sufficient number of well-known lines to serve as standards can be found in the chromospheric spectrum itself. Thus the wave-lengths of the unknown lines can be determined with a considerable degree of accuracy, especially if Hartmann's valuable interpolation formula is used, and the reductions made by the method of least squares. Again, the necessary exposure times are less than with a slit spectrograph of equal resolving power, and the danger of complete failure is very small.

A slit spectrograph, if it is to be comparable in working efficiency with an objective prism, should be of large aperture, while the camera objective should be a triple lens or some other combination capable of giving good definition over a fairly large field. If a tangential slit is to be used the solar image should be as large as possible, and special care should be taken to provide reliable slow motions for use in placing the slit tangent to the Sun's limb. A train of prisms is undoubtedly superior to a grating in an instrument of this kind, unless one is fortunate enough to possess a grating which concentrates most of the light in a single spectrum. The time of exposure may be reduced to a minimum by using a long collimator (of correspondingly large aperture) and a comparatively short camera. The focal length of the camera objective will of course depend on the angular dispersion of the prism train and the linear dispersion desired on the photograph. In general, when it is necessary to reduce the time of exposure to a minimum, it is not advisable to attempt to realize the full theoretical resolving power of the prism train. This could be done only by the use of a very long camera, which would require a great increase in the exposure time.

Summing up, it may be said that the objective prism has a decided advantage in requiring less care in manipulation, and in

showing a long arc of the chromosphere with a comparatively short exposure. It is therefore likely to be used in the majority of cases. Slit spectrographs should be employed only by experienced observers, who must take pains to secure the highest possible degree of efficiency in the design. The chances of photographing the spectrum of an active region in the chromosphere will be much increased if a slit having the curvature of the image of the Moon's limb is substituted for a straight slit. This presupposes that the slit is to be used tangentially, the position which demands the greatest skill on the part of the observer, but the one which may ultimately give the best spectrum of the lowest stratum of the chromosphere. Although a large solar image is needed, brightness should not be sacrificed for size through the use of an objective of very small angular aperture.

Reflecting jaws would facilitate setting the image on the slit. If a radial slit is used much less care is necessary in adjusting the position of the solar image, which may also be smaller in diameter. In this case the plan used by Campbell at the Indian eclipse, of moving the photographic plate by clockwork during the exposure parallel to the spectral lines, should prove effective in giving the spectrum of the various levels of the chromosphere on a single plate. The same method may be adapted to the requirements of a tangential slit or to those of an objective prism by limiting the length of the spectral lines by means of a slit parallel to the length of the spectrum, placed immediately in front of the photographic plate.

A third method of using the spectrograph is to set the slit so that it makes a small angle with the tangent to the Moon's limb at the point of contact. With a fairly large image of the Sun the spectrum obtained in this way on a fixed plate is quite wide enough to differentiate the spectra of the various strata. In observations of the faintest lines in the chromospheric spectrum with the 40-inch telescope this manner of placing the slit is found to give excellent results.

It is to be hoped that someone will use a quartz spectrograph for work in the ultra-violet. For this and other purposes the

valuable combination of polar heliostat and spectrograph used by Newall in India should prove useful.[1]

Little need be said regarding the design of the objective prism spectroscope, unless it be to emphasize the importance of employing a lens corrected for the wave-lengths comprised by the photograph and the advantage of using several prisms with a short focus objective rather than a single prism with an objective of long focus. The dark line spectrum of the cusps should be photographed for use in connection with the reductions.

A direct concave grating spectroscope, similar to that used in 1899 by Mitchell at the Yerkes Observatory and described by him in the June number of this JOURNAL, should give good results in eclipse work. Newall describes in the paper referred to above an objective grating with which he successfully observed the distribution of "coronium" about the Sun.

In all cases it will be advantageous to employ multiple coated and backed plates, and to determine the proper moment to expose for the spectrum of the flash by the aid of a solar spectroscope used visually. A suitable photographic shutter should be provided, and a device for rapidly bringing fresh plates into position would prove of great service.

II. PHOTOGRAPHS OF THE SPECTRUM OF THE CORONA.

As already remarked, the eclipse will be of so short duration that it will be inadvisable to attempt any exhaustive study of the spectrum of the corona. Nevertheless, in view of the importance of definitely determining its wave-length, the green coronal line should be photographed with the highest dispersion it is safe to employ — probably three prisms, with a long collimator and a comparatively short camera. If possible, the chromospheric line at λ 5317 and the solar spectrum should be photographed on the same plate for purposes of comparison. From such a plate an accurate determination of the wave-length of the green coronal line may easily be obtained.

[1] See *Proc. Roy. Soc.*, 64, 55, 1898.

Further attempts to measure the rotation of the corona will probably be put off until the Sumatra eclipse, but if any work is done it would appear from Campbell's results that the green line should be chosen in preference to all others. In any event the H and K lines should not be used, as the objective prism has shown that they do not belong to the spectrum of the corona.

12. HEAT RADIATION OF THE CORONA.

Direct measurements of the heat radiation of the corona, although attempted at two previous eclipses, have hitherto yielded no satisfactory results. As Professor Very has pointed out in the October 1899 number of this JOURNAL, the question is one of great intrinsic importance, and it has an additional interest because of its bearing on the possibility of mapping the corona without an eclipse. If the corona radiates sufficient heat to produce easily measurable deflections with a bolometer, radiometer, or other sensitive heat-measuring instrument, and if the intensity of heat radiation from the rifts and streamers is sensibly different, there is a chance that it may ultimately become feasible to map the outline of the corona in full sunlight. Discussion of the method would be out of place here;[1] suffice it to say that the principle on which it is based has been proved by laboratory experiments to be sound. Two sets of measures are particularly desirable just at present:

a. Measures of the heat radiation of a streamer at various distances from the limb, to determine the law of decrease with distance.

b. Measures of the relative heat radiation of rifts and streamers.

Of the four heat-measuring instruments available for this work the observer must endeavor to select that which is best adapted for his purpose. The radiometer and radiomicrometer are not affected by magnetic disturbances, and have other important advantages. The bolometer and thermopile require the

[1] "On a New Method of Mapping the Solar Corona without an Eclipse." This JOURNAL, 1, 318.

coöperation of an exceedingly sensitive galvanometer, but they are small and light, and can be used in any position. The thermopile needs no battery, and if made in Ruben's new form, as described in the last volume of the *Zeitschrift für Instrumentenkunde*, it ought to be well adapted for eclipse work. Each observer must, however, base his selection upon experience in the use of the various instruments. Whichever is chosen, means must be provided for instantly varying the sensitiveness through a large range, as with our present ignorance of its intensity it would be unsafe to expose a very delicate instrument to the full radiation of the corona. With a bolometer or thermopile a good balancing bridge is essential, and for the former it is necessary to have a battery giving a very uniform electromotive force. Every precaution must be taken to shield all junctions (which should be of copper throughout) from convection or radiation, and to protect all parts of the apparatus from the wind and other sources of disturbance. If possible, the measures should be differential, one strip, vane, or junction being exposed to the radiation from the Moon, the other to that of the corona. An observer with a single instrument should not attempt to measure the heat radiation at more than one or two points, using the lowest sensitiveness of the instrument that will give reliable readings. On account of the difficulty of the work, and the danger of a mishap at the critical moment, it is hoped that several observers with independent instruments may take part in it. The writer will be glad to discuss details of the apparatus (including heliostat, device for rotating coronal image, differential radiometer, radiomicrometer, bolometer or thermopile, galvanometer, balancing bridge, etc.) with anyone who is prepared to undertake the measurements.

It might be profitable to consider many other questions relating to the eclipse, particularly those connected with photometric and polariscopic observations. But, as stated at the outset, this article is intended to be merely suggestive in character, and no attempt has been made to cover the whole field. It will be

well worth while to undertake photometric observations of the corona, if photographic methods are employed in conjunction with some such instrument as Hartmann's new photometer. Work with the polariscope is less important at this time. Arrangements should always be made, if possible, to accurately determine the latitude and longitude of eclipse stations. The contacts should of course be recorded with all possible accuracy.[1]

It should perhaps be said in conclusion that the above suggestions do not necessarily represent the views of the Eclipse Committee, as the writer, except in a single instance, is alone responsible for them. The committee desires to secure for publication all possible information regarding the plans of eclipse parties, and communications may be addressed to the secretary at the Yerkes Observatory.

[1] Special attention is called to Professor Campbell's article on the observation of contacts in the *Astronomical Journal*, No. 474, which has appeared since the present paper was put in type.

RESEARCHES ON THE ARC-SPECTRA OF THE METALS.

V. SPECTRUM OF VANADIUM.[1]

By B. Hasselberg.

VANADIUM AND THE OTHER METALS.

After this comparison of vanadium with the metals I had previously investigated, I proceeded to a similar comparison of my observations with the spectroscopic investigations of other metals, made by Kayser and Runge. It appeared as a result of this comparison that several of these metals contained in their spectra no line in such position or of such character that any connection whatever with vanadium could be inferred. The metals are these :

Sodium,	Strontium,	Indium,
Potassium,	Magnesium,	Tin,
Lithium,	Zinc,	Lead,
Rubidium,	Aluminium,	Arsenic,
Caesium,	Gold,	Bismuth.

The metals

Calcium,	Mercury,	Thallium,
Barium,	Copper,	Antimony,
Cadmium,	Silver,	

show, however, occasional approximate coincidences with lines of vanadium, but in view of their diffuse character and the consequent considerable uncertainty of the wave-lengths given by Kayser and Runge, most of these are accidental and certainly not suitable as a basis for any inference as to a connection. These lines are given in the following tables along with the limits of error of the wave-length assigned by Kayser and Runge, with other remarks.

[1] Continued from Vol. X, p. 361.

λ	i	λ	i	Δλ	Remarks
V		Sb			
3637.05	1.2	3637.04	2.3	0.03	
		Th			
3775.85	1.2	3775.87	6	+0.03	Reversed
		Ag			
4379.38	4.5	4379.45	2.3	±0.15	Broadened
		Cu			
3524.38	2+	3524.31	1.2	±0.10	Broadened
3533.85	3	3533.84	2.3	0.05	Broadened
4531.01	2+	4531.04	4.5	0.10	Reversed, broadened toward red
		Hg			
3543.68	1.2	3543.65	3.4	±0.10	Hazy
4916.48	1.2	4916.41	3.4	0.10	Hazy toward red
		Ba			
3910.01	3	3910.04	4.5	±0.05	Reversed
3975.51	1	3975.55	1.2	0.10	Hazy toward red
4179.57	2.3	4179.57	1.2	0.20	Hazy toward red
4332.98	3	4333.04	3	0.05	Hazy
		Cd			
3729.22	1.2	3729.21	3	+0.20	Broadened toward red
		Ca			
4240.53	2+	4240.58	3	±0.10	
4586.15	1.2	4586.12	6	0.10	
5260.56	1	5260.58	3	0.05	

Among these lines those of calcium in the last group are the only ones whose assignment to vanadium can be doubted. I have, nevertheless, retained them here because two of them have only a moderate intensity in calcium, and the third certainly corresponds to the solar line 4586.05, but not to the vanadium line. The observation made during the measurement that the last occupied a position somewhat to one side of the solar line also favors this view. The first two lines of the table indeed show a very accurate agreement with the corresponding lines of antimony and thallium; since nothing was seen, however, of the rest of the strongest lines of these metals in the vanadium

spectrum, we can hardly conclude that they occur as an impurity in the vanadium.

In addition to the investigations of metallic spectra by Kayser and Runge which have so far been utilized, a similar research on the spectra of the platinum group by Kayser[1] alone has recently appeared, which seems to give an accuracy in the positions of the lines not hitherto attained. The precision in fact appears to be of the same degree as that obtained by Rowland in the solar spectrum, so that it is justifiable to carry the third decimal of the tenth-meter. The accordance reaches this high degree not only in the measures among themselves, but also in comparison with those of Rowland; while in similar series of measures up to this time the individual values of an individual observer may present a similar accuracy, but the final results differ by several units of the second decimal place from those of other observers. It is evidently entirely illusory to carry the third place under such circumstances. That so sharp measurements could be obtained in this case must nevertheless be attributed not only to the superior dispersion of the large concave grating, but also, and perhaps chiefly, to the excellent character of the lines of the metals here investigated, which exceed in sharpness the lines of most of the other metals.

Since in my measurements of the vanadium lines the probable error of the wave-lengths must be set at about $\pm$ 0.02 tenth-meters, while in Kayser's determinations of the spectra of the platinum group the uncertainty affects only the third place, I have limited more closely the range of approximate coincidences in the comparison of my observations with his, and have only included those lines whose wave-lengths differ from each other at most by $\pm$ 0.05 tenth-meters; for it would seem quite certain the lines with a larger difference of wave-length are actually separated, and hence independent of each other. In this way I arrive at the following summary:

[1] *Abhandlungen der Berliner Akademie der Wiss., 1897. Anhang.*

V λ	V i	Pt λ	Pt i
3505.83	1	3505.85	1
3687.61	2.3	3687.58	2+
		Pd	
4406.80	4.5	4406.76	3
5668.61	2.3	5668.60	1+
		Ru	
3493.34	1	3493.38	1+
3605.83	1.2	3605.79	1.2
71.37	2—	71.36	1+
72.54	2	72.53	1+
3676.86	3	3676.82	2—
3755.85	1+	3755.86	1+
3790.62	1.2	3790.65	3
3890.34	3	3890.35	2.3
3942.16	2	3942.21	2.3
3950.37	2—	3950.37	2.3
4118.73	2	4118.68	1+
21.13	1	21.15	1+
82.77	2.3	82.81	1—
4197.77	2—	4197.75	2.3
4226.78	2	4226.82	1—
4297.86	2.3	4297.89	4.5
4341.15	3	4341.20	1+
4426.17	3	4426.18	1—
4428.67	3	4428.62	2+
4531.01	2+	4531.03	2+
4814.28	2.3	4814.33	1—
4833.17	2	4833.16	1+
5657.11	1+	5657.13	1+
5725.90	2	5725.90	2.3

V λ	V i	Rh λ	Rh i
3775.85	1.2	3775.86	1.2
3806.93	2	3806.92	2.3
17.98	2	17.99	1—
18.37	3	18.35	3
28.66	3.4	28.62	1+
3888.50	2	3888.47	1+
4506.77	2—	4506.82	1—
4922.60	1.2	4522.63	1+
5605.20	2.3	5605.21	1—
5626.27	2.3	5626.25	1+
		Os	
3930.19	2.3	3930.15	1—
3963.77	2	3963.77	2.3
4003.70	1.2	4003.65	1+
15.20	1+	15.20	1—
4097.09	1.2	4097.09	1—
4400.74	4	4400.75	1—
4529.80	2.3	4529.85	1—
4738.51	1.2	4738.51	1.2
		Ir	
3952.10	2	3952.10	1+
4048.77	1.2	4048.78	1—
4051.11	2.3	4051.07	1+
4257.54	2	4257.53	1+
4261.37	2	4261.41	1+

The number of approximate coincidences, particularly in the case of ruthenium, is greater than would have been expected in view of the narrow limits taken for the difference of wave-length. A direct comparison of the two spectra will alone decide whether or not these are actual coincidences and in certain cases are impurities in the vanadium spectrum due to this metal. Judging by the estimates of intensity in perhaps the majority of the cases the reverse is indeed equally probable, or that the vanadium occurs as an impurity in the platinum metals.

RESULTS OF OBSERVATIONS.

On the basis of what has preceded I have derived from my measures the following catalogue of the lines to be assigned to the spectrum of vanadium. The last column contains the corresponding determinations by Rowland, but those lines of his catalogue which do not occur in mine are given in the column of remarks with the designation R.

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
3486.05		1	2			
89.64		1			89.65	1
———	3491.46					
93.34		1				
97.13		1			97.08	1
3498.23		1				
3501.65		1			01.61	1
04.57		1.2	1.2			
05.83		1+	1.2			
17.44		2+	2		17.44	1
———	3518.49					
20.18		2	2.3	Faint Ti Line		
24.38		2+	1.2	Also Fe		
24.89		1.2	?			
29.90		2—	3	At edge of ⊙ line	29.88	1
30.91		2	2			
33.85		3	1		33.82	1
———	3540.27					
43.68		1.2	—		43.63	1
45.34		2	2		45.33	1
45.52		1.2	—	R. 51.67 1	45.42	1
53.43		3	—		53.41	1
55.32		1.2	?			
56.42		1.2	—			
56.97		2.3	—	Very sharp; at edge of ⊙ line 57.03 (Fe)		
60.75		1+	1.2			
62.32		1	?			
63.59		1	?			
———	3564.68					
66.33		1.2	2	V? Also an inconspicuous Ti line		
69.11		1.2	2	V?		
71.18		1.2	—			

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
3571.82		1.2	1.2			
73.69		1.2	—		73.65	1
74.92		1	1		74.92	1
75.26		1	1.2			
78.01		1+	2		78.01	1
81.00		1	—			
83.00		1+	—		82.95	1
———	3583.48					
83.84		1+	2		83.84	1
89.91		2+	2		89.89	1
92.15		2+	2		92 16	1
92.71		1+	—			
3593.48		2	2		93.52	1
3600.20		1+	—		00.17	1
05.75		1.2	—			
———	3609.02					
09.45		1.2	2	Very sharp; V? Also faint Ti line		
16.91		1+	—			
19.10		1	—	Very sharp		
22.82		1	1	V?		
———	3623.33					
36.09		2	—			
37.95		1+	1			
38.57		1	—			
39.21		1.2	1	R. 39.72 1 Pb	39.16	1
40.25		1+	—			
41.28		1.2	—			
44.05		1.2	—		44.04	1
44.88		1.2	1.2	Very sharp; ⊙ line fine Dpl. viol. Comp.		
45.77		1.2	—			
49.13		2—	1.2		49.06	1
———	3653.64					

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
3663.73		2.3	—		63.69	1
65.30		2	—		65.26	1
67.87		2.3	—	Diffuse	67.84	1
69.57		1.2	1	⊙ has a triple; the strong one in the middle = Fe		
71.37		2—	1	Coin. ?		
				R. 71.84 1		
72.53		2	—	Diffuse; Ru 72.53	72.52	1
73.55		3	1	Diffuse		
75.85		2.3	1.2	Very sharp	75.84	2
76.86		3	—	Diffuse; Ru 76.82 2	76.81	1
—	3680.06			R. 80.05 1		
80.26		3	?	⊙ line ?	80.21	1
83.26		3	—	Fe 83.23	83.24	3
				R. 83.60. 1 Pb		
84.83		1.2	—			
86.40		2	2		86.39	3
87.61		2.3	3	Also a strong Fe line; other Fe lines are hardly visible in the region		
88.22		2.3	1.2		88.21	3
90.41		2.3	1		90.41	3
92.36		3	1		92.36	3
—	3695.19					
95.48		2.3	1		95.45	1
3696.00		3	1		95.99	4
3703.71		3.4	2	Also a faint Fe line		
				R. 04.66 1		
04.85		3	1.2		04.83	7
05.19		2.3	1		05.17	5
				R. 06.17 Ca, Mn		
08.88		1.2	—	Very sharp	08.85	1
14.12		1	—			
15.62		2	2	Very sharp; Ni 15.61 1.2		

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
—	3720.09					
3722.[illegible]5		1+	—		22.14	1
22.27		1	1+		22.33	1
22.70		2	?			
23.52		1.2	—			
27.49		2	2			
29.22		1.2	—			
32.88		2	1.2	Very sharp		
34.59		1.2				
38.15		1.2	—		38.13	1
38.93		2—	—		38.90	1
40.38		1.2	2		40.37	1
41.65		1.2	1.2		41.63	1
46.02		2—	—	Very sharp. At edge of the ⊙ line 46.06 (Fe)		
—	3747.10					
48.11		1+	—			
51.02		2—	1.2	Very sharp		
51.94		1+	1.2			
53.44		1+	—			
55.85		1+	—			
56.18		1	—	At edge of ⊙ line 56.21 (Fe)		
60.40		1	1	V?		
60.96		1	—			
63.30		2—	—			
64.96		1	—			
69.23		1+	?	Perhaps ⊙ line		
—	3770.13					
70.68		1	—			
71.11		2	1			
71.31		1	1			
71.87		1.2	—	Diffuse or dpl.		

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
3774.27		1+	—			
75.34		1.2	1	Weak. Coin.?		
75.85		1.2	—	Weak		
76.31		1.2	—	Weak		
77.31		1	—			
77.63		1	2	V?		
78.48		1	2	V?		
78.83		2.3	2	Very sharp	78.81	3
79.80		1.2	—			
81.54		1.2	—			
82.70		1	—			
84.84		1	—			
87.68		1	—			
90.46		2.3	—		90.45	2
90.62		1.2	2	{ Ru has 90.65 3 { Cr has .61 2	90.59	1
93.76		2—	2			
———	3794.01					
3795.12		3.4	?	At edge of a strong ⊙ line		
3800.05		2.3	?		00.05	3
03.62		2.3	1		03.61	3
03.92		1.2	—			
04.05		1.2	—			
06.93		2	—	Beside the ⊙ line 06.86 R. 07.42 2		
07.64		2+	?	Beside the ⊙ line 07.68 R. 08.14 4	07.63	3
08.64		2.3	1	Very sharp		
13.63		3	?		13.61	4
15.65		2	—			
———	3815.98					
17.98		2—	1			

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
3818.12		1.2	—			
18.37		3	2		18.37	5
20.10		2+	—		20.09	4
20.41		1+	—	In the shade of a strong ⊙ line R. 20.59 2		
21.63		2+	1		21.61	4
22.14		2.3	1.2			
23.00		2+	1.2		23.01	2
23.35		2+	1			
24.12		2—	—			
28.67		3.4	2		28.68	7
32.97		1+	—			
33.36		1+	1			
35.70		2—	—			
36.20		2—	—			
———	3836.65					
39.12		2	1	Coin.?		
39.53		2	—			
40.27		2—	?			
40.56		2.3	—	At edge of ⊙ line 40.60(Fe)		
40.88		3	1		40.87	6
42.03		2	?	Coin.?		
44.58		2.3	1	Sharp	44.57	4
45.03		1	—			
47.46		2.3	?	Sharp. ⊙ has a group of fine lines; coin.?	47.45	3
49.48		2	—		49.43	2
51.32		1.2	—			
52.27		1	—			
55.50		3	2	⊙ line broad, dpl.? tripl.?	55.49	4
56.00		4	2		55.96	7
———	3856.52					
58.83		1.2	1.2	Coin.?		

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
3859.51		1.2	—			
62.37		2	—	Sharp		
64.02		2—	—	Sharp		
65.02		3.4	2	At edge of ⊙ line	64.98	5
67.50		1	—			
67.77		2.3	2			
70.72		1+	—			
71.23		2	?	Falls in a broad band in ⊙		
73.80		1+	—			
75.22		3	2		75.20	5
——	3875.22					
76.05		2+	1.2			
76.21		2.3	2	Also an inconspicuous Fe line		
79.82		1.2	—			
84.04		1.2	—			
84.60		1.2	—			
85.00		1	1			
85.91		1+	1.2			
86.72		2	—	Sharp	86.69	2
88.23		1+	—			
88.50		2	—	Diffuse; weak		
90.33		3	1.2	Very sharp	90.30	4
91.27		2+	—	Diffuse; broad; weak R. 92.47 4		
93.03		3	—	Very sharp beside the ⊙ line 93.00		
94.19		2	2	Sharp; ⊙ line a close dpl.		
96.29		2	—		96.26	2
97.22		2	—			
——	3897.60					
98.15		3	3	Diffuse; weak. ⊙ line strong dpl. Fe	98.08	1

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
3899.30		1+	2	⊙ line dpl. $\left\{ \begin{array}{l} 99.30 \text{ Fe} \\ .17 \text{ V} \end{array} \right.$		
3900.33		2.3	—	Diffuse		
01.30		2.3	—	Diffuse		
02.40		3.4	2		02.37	7
02.71		1+	—			
03.42		1.2	1			
04.63		1	—			
06.89		2	2	Sharp. Also a faint Fe line		
10.01		3	2.3	V dpl.; also ⊙ line	09.99	5
10.95		2	—	Sharp		
12.36		2.3	—	Very sharp		
13.03		2—	—			
16.55		1.2	2	V? R. 14.44 1 R. 19.60 1		
20.15		1	—			
20.65		1.2	—			
22.05		2—	—		22.02	1
22.58		2.3	1		22.55	3
——	3924.67					
24.84		2.3	—	Companion toward red	24.77	3
25.36		2+	2		25.35	3
30.19		2.3	—			
31.40		1+	—			
31.50		2	—	R. 33.77 3 Ca		
34.16		3.4	—			
35.28		2.3	—			
36.42		2	—			
37.68		2—	—			
38.35		2—	—			
39.48		2—	—			
40.75		1	—			
41.40		1.2	2	Coin.?		

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	Rowland i
3942.16		2	—			
43.77		2.3	—			
———	3950.10			R. 44.13 3 Al		
50.37		2—	—			
52.09		2	2	Sharp. Ir has 52.10 1+ R. 61.65 5 Al	52.07	1
63.77		2	2	V line somewhat displaced toward violet		
68.24		2—	—			
72.10		1+	—	R. 68.59 1 Ca		
73.49		1	1			
73.79		2—	?			
———	3977.89					
79.30		2—	—			
79.59		2—	—		79.54	1
80.66		2—	—			
84.75		2	2			
88.97		2	—			
90.71		3	1		90.69	5
92.95		3	2		92.92	3
97.30		1.2	1.2			
3998.87		3	—	Near 98.80 (Ti)	98.85	3
4000.24		1	—			
03.10		1.2	1.2			
03.70		1.2	—			
———	4003.92					
05.86		2+	2		05.84	1
09.94		1	—			
11.45		1+	—			
15.20		1+	—	{ Several fine lines here R. 22.04 1	23.51	1
23.50		2	2			
25.46		1+	—			
30.04		1.2	1—	⊙ line?		
31.37		1.2	1			

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	Rowland i
4031.98		2	—	Between { 32.12 Fe, 31.94 Mn	31.96	1
32.62		1.2	1.2	Coin.?		
33.01		1+	—			
———	4034.64			R. { 34.62 2 Mn, 33.19 3 Mn		
35.77		2	2.3	{ 35.88 Mn, .77 V, .73 Co		
36.93		1	1			
40.46		1+	—			
41.72		2	—	Beside ⊙ line 41.80		
42.78		2+	?	Perhaps ⊙ line	42.76	1
47.05		1+	—			
48.77		2—	—			
51.11		2.3	?	Perhaps ⊙ line		
51.48		2.3	1		51.48	4
52.60		1	—			
57.21		3	—		57.21	2
60.97		1+	?	Coin.? R. 57.96 1 Pb		
———	4062.60					
64.09		2.3	—		64.06	2
67.90		1.2	—			
71.67		2.3	2		71.66	2
72.30		2+	—			
———	4083.77			R. 77.85 1 Sr		
90.70		3	1.2		90.70	5
92.09		1.2	—			
92.54		2	2	Also in Mn	92.53	2
92.83		3	2			
93.65		2+	—			
94.42		2	—			
95.64		3	1		95.61	5
97.09		1.2	1			

Vanadium λ	R[1]	i V	i ⊙	Remarks	Rowland	
4098.54		2	1—	⊙ line ?	98.51	1
4099.93		3.4	2		99.92	7
4102.32		3	1	⊙ line ?	02.29	3
—	4103.10					
04.55		2	—		04.52	2
04.92		2	1			
05.32		3	1.2			
07.64		1.2	2.3	Close by the Fe line 07.65	07.60	1
08.36		2	?			
09.94		3.4	2.3	Close by the Fe line 09.95	09.91	7
11.92		4	2.3		11.92	5
12.47		1.2	1.2			
13.65		2.3	—		13.64	3
14.69		1.2	—			
15.32		3.4	2		15.31	7
16.64		3	1.2		16.63	9
16.85		1.2	1.2			
18.34		2.3	?	Coin.?	18.32	1
18.73		2	—	Beside 1871 (Fe)		
19.58		2	1		19.58	3
20.69		2	?		20.66	2
21.13		1+	?			
—	4121.97					
23.65		3	2			
24.23		2—	—		24.20	1
28.25		3.4	2.3		[1] 28.15	7
29.00		2	?			
31.32		1	—	Mn has 31.36; separated	31.30	1
32.13		3.4	2.3	⊙ line dpl. { 32.22 Fe / .13 V	32.12	6
33.92		2	—			
34.61		3.4	2.3	⊙ line dpl. { 34.60 / .50	34.62	7
36.25		2	—			

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
4136.52		2	—			
39.39		2	—			
41.96		1.2	—			
42.75		1.2	1.2			
43.02		1+	?			
49.02		1.2	—			
50.84		2—	—			
51.52		1	—			
52.81		2—	—			
53.49		1.2	?			
55.39		1	—			
56.00		1.2	?			
—	4157.95					
58.14		1+	—			
59.84		2.3	1		59.82	2
60.57		1+	—			
62.51		1	—			
69.40		1.2	1	Ti 69.46 near by		
71.45		2	—			
74.18		2	1		74.15	1
75.30		1	1.2			
76.83		1—	—			
77.02		1	—			
77.25		2—	1			
79.53		2.3	2			
80.99		1	2	V ?		
82.23		1.2	—			
82.74		2.3	—	R. 83.07 4	82.73	1
83.43		1+	—			
83.59		1+	1.2			
—	4185.06					
86.95		1	—			
87.82		1	—			

[1] Misprint; should be 28.25

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4189.99		2.3	—	In a group of faint ⊙ lines Coin. ?	90.01	—
91.70		2.3	—			
94.17		1+	—			
97.45		1	1			
97.77		2—	1	Sharp		
4198.78		2—	2.3	Sharp		
4200.35		2—	—			
—	4202.19					
02.52		1+	1		02.51	2
04.67		1	?			
05.23		1+	1.2		05.20	2
09.98		2.3	1.2		10.00	5
10.55		1	—			
11.02		1	—			
16.52		1	?			
18.86		2	2	Very Sharp		
19.65		1.2	—			
21.17		1+	—	Diffuse		
22.49		1+	—			
24.30		2	—			
25.40		1	1		25.37	1
26.78		2	—	In shade of Ca line 26.90. R. 26.87 4 Ca		
—	4226.89					
27.90		2	1			
29.87		2	—			
32.62		3	1—		32.60	7
33.09		3	1.2		33.01	7
34.12		3	—		34.15	7
34.70		2.3	1		34.67	7
35.90		2.3	1		35.91	4
39.12		1.2	—			
40.25		2	1	Coin. ?		
40.53		2+	2	Fe 40.56. Separated		

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4241.48		2	—			
47.46		1+	?	Coin. ?		
—	4250.96					
51.45		1	?	⊙ 51.49		
53.02		1.2	—			
55.60		1.2	—			
57.53		2	1	Very sharp ⊙ line, trace	57.52	4
59.46		2	1+	Very sharp	59.45	4
61.37		2—	2			
62.32		2	—		62.31	4
65.28		2	—	Broad, dpl. ?		
67.50		1.2	1.2			
68.78		3	1		68.79	0
69.92		2	1	⊙ line dpl. { 70.02. 69.90.		
70.49		2	—			
71.71		3	—		71.71	17
—	4271.92					
77.12		3	1		77.10	7
79.12		1.2	—			
83.06		2	—			
84.19		3	—		84.21	5
86.57		2	—	⊙ has 86.63 diffuse		
87.97		2	—			
91.46		2—	—			
91.97		3	—		91.98	1
—	4293.25					
96.28		2.3	—		96.27	7
97.86		2.3	—		97.84	7
98.17		2.3	2	V? Also inconspicuous Fe line. A dense group between 03.7 and 98.2; partly Ti		

Vanadium λ	R	i V	i ☉	Remarks	Rowland	
4303.70		2	?	R. 09.24 1	03.70	2
06.35		2.3	—			
07.33		2.3	—			
09.69		1.2	—			
09.95		3	?		09.95	7
12.56		1+	—			
14.06		1.2	—			
16.02		1				
———	4318.82			R. 18.80 2 Ca		
20.46		1+	—			
22.51		1+	—			
30.18		3	1			
32.56		1.2	—			
32.98		3	1		32.98	10
34.23		1.2	—			
36.29		1.2	—			
41.15		3	1		41.16	10
42.36		1.2	—			
43.00		2	—			
———	4343.39					
53.02		3.4	1		53.04	18
55.09		2	?	☉ line ?	55.14	4
56.10		2.3	1.2		56.10	4
57.60		1.2	—			
57.82		1	—			
60.75		1.2	—			
61.18		1+	—			
61.57		1.2				
63.48		1	1.2			
63.69		2	?		63.69	4
64.37		2	1+		64.38	4
65.92		1.2	—			

Vanadium λ	R	i V	i ☉	Remarks	Rowland	
4367.24		1+				
68.25		2+	1			
68.76		1.2	1		68.76	4
69.25		1+	—			
———	4369.94					
73.40		2	1		73.38	6
73.99		2	1	☉ dpl. { 73.95. 74.05.	73.98	3
75.47		2	1			
76.25		1	—			
78.06		2	—	Diffuse		
79.38		4.5	2.3	Reversed	79.39	1
80.69		2	—		80.72	4
84.07		1	—	R. 81.19 1		
84.37		1	—			
84.87		4.5	2.3		84.87	1
87.40		1.2	—			
90.13		4.5	2	Reversed	90.14	7
90.79		1	—			
———	4391.15					
91.84		1.2				
92.24		2	1		92.23	4
93.26		2—	?		93.26	3
94.01		2—	1		94.00	4
94.98		1.2	1			
4395.40		4.5	2	R. 97.39 1	95.38	10
4400.74		4	1.2		00.74	10
03.86		1.2	—		03.83	4
05.20		2.3	1	Very sharp		
06.80		4.5	2	R. 06.28 8	06.80	8
07.85		4.5	3	☉ dpl. { Comp. to V. = V / Comp. to R. Fe, Ti	07.80	8
———	4407.85					
08.36		4	2		08.37	5

Vanadium λ	R	T	⊙	Remarks	Rowland	
4408.67		4.5	3	⊙ line dpl. { 08.58—Fe; .67—V }	08.66	5
12.30		2	—	Very sharp	12.30	4
16.63		3	1+		16.63	5
20.08		2.3	—			
21.73		3	1	⊙ line extremely faint	21.74	10
22.40		1.2	1			
23.32		1.2	—	}	23.37	8
23.41		1.2	—	}		
24.10		1.2	—		24.08	2
24.74		1.2	1	R. 25.59 1 Ca	24.74	4
25.86		2	1			
26.17		3	1			
28.68		3	1		28.68	5
29.95		3	—			
30.68		2	—			
34.80		2+	—			
———	4435.85					
36.31		3.4	1		36.31	7
38.02		3.4	1		38.00	7
41.88		3.4	1.2	Coin.? Also faint Ti line	41.85	2
43.52		2	—		43.51	4
44.40		3.4	1	Also faint Ti line	44.38	3
49.77		2.3	—		49.74	4
51.09		2+	1.2		51.07	4
52.19		4	1		52.18	8
52.91		2	?	⊙ line ?		
———	4456.05			R. { 54.94 Ca; 56.07 1 Ca }		
56.68		2	—		56.67	3
57.65		3.4	3	⊙ line dpl. { 57.60 Ti, V; .71 Mn }	57.63	3
57.97		2.3	1	R. 58.91 1		
59.93		4	2		59.92	8

Vanadium λ	R	V	⊙	Remarks	Rowland	
4460.46		4.5	2.3	⊙ line broad. Group ? R. 60.85 4	60.46	10
62.56		3.4	—	Beside ⊙ line 62.62 (Ni) R. 65.67 3	62.53	10
67.04		2	2	Coin. uncertain		
68.19		2.3	?		68.17	3
68.94		2	—		68.93	3
69.88		3.4	—	R. 70.87 1	69.87	7
74.21		3	—		74.21	7
74.89		3.4	—		74.90	7
76.06		2	—			
———	4476.19					
80.20		2.3	—	Very sharp	80.21	3
86.44		1	—			
89.06		3.4	2		89.10	7
90.95		2.3	2	Very sharp	90.98	4
91.35		1.2	—	Sharp	91.34	2
91.66		1	—		91.65	1
———	4494.73					
95.16		1.2	?	⊙ line ?		
96.26		3	—	Beside ⊙ line 96.32 (Ti)		
97.03		2+	3		96.23	5
4497.57		2	—			
4501.01		2	—	Diffuse R. 01.41 1 (Ti)	97.57	5
02.12		3	—		01.00	2
06.30		2	—		02.12	4
06.41		1.2	—			
06.77		2—	1		06.74	
08.11		1	—			1
———	4508.46					
09.49		2	1	⊙ line extremely faint	09.46	2
11.64		2	—		11.61	2
13.79		2	—		13.79	2
14.36		2.3	2	Also Fe, Co	14.36	4

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4515.74		1.2	—		15.73	1
17.77		2	2	Fe 17.70	17.74	3
20.31		2 —	—	Diffuse	20.33	2
20.67		1.2	—	Diffuse	20.68	2
24.38		3	—		24.38	5
25.31		2+	2.3	Sharp. Also Fe	25.34	2
28.16		2.3	—		28.17	3
28.66		2—	—			
29.47		2+	—		29.48	2
29.76		2.3	—			
30.97		2+	—	Beside ⊙ line 30.91 (Cr)	30.97	3
——	4534.95			R. 34.11 3		
37.84		2	1		37.83	4
40.18		2	—	Perhaps ⊙ line	40.18	4
41.57		1	—			
45.57		3.4	?	⊙ line ?	45.57	10
49.81		3	2.3		49.82	8
52.05		2	—		52.02	2
53.25		2.3	—	R. 52.73 5		
——	4554.21					
60.90		3	?	⊙ line ?	60.89	7
64.76		1+	?	⊙ line ?	64.76	1
70.60		2	—			
71.96		3	—		71.96	5
77.36		4	1		77.35	7
——	4578.73					
78.92		3	?	⊙ line	78.91	5
79.38		2.3	—		79.37	2
80.57		4	2		80.56	8
				R. 81.41 1		2
83.96		2	—	Beside ⊙ line 84.02 (Fe)	83.97	
86.15		1.2	—	Beside ⊙ line 86.05 (Ca)		
86.54		4.5	1		86.55	8
88.94		1+	—			

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4591.30		2.3	—		91.41	5
4594.27		4.5	2		94.22	10
4600.34		1.2	—			
——	4602.18					
06.33		2.3	—		06.32	4
07.40		1.2	—	R. 08.63 1	07.39	1
09.84		2	—		09.82	4
11.10		1.2	—		11.10	1
11.92		2—	—	R. 13.98 1		
14.08		1	1.2	V ?	14.09	1
16.18		1	—		16.19	11
17.03		1	—			
18.00		1	—			
19.85		2	?	⊙ line ?	19.90	0
19.97		2.3	?	⊙ line ?		
21.43		1	—		21.43	1
24.62		2+	?		24.58	4
26.67		2+	1.2		26.67	4
——	4629.51					
30.24		1	—		30.24	1
35.35		2.3	?	⊙ line ?	35.35	7
36.34		1.2	—		36.34	1
40.25		2	—		40.24	5
40.92		2	—		40.91	6
44.64		2—	?	⊙ line ?	44.62	2
46.17		1+	—		46.16	1
46.59		2.3	?	⊙ line ?	46.57	8
48.08		1	1.2	Coin. uncertain	48.05	1
——	4648.83					
49.08		1.2	1		49.07	2
53.15		1	—		53.11	1
54.84		1.2	—	⊙ has { 54.80 Fe, .91 Cr }		
55.47		1	?	Perhaps a fine ⊙ line	55.41	1

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
4657.17		1	1.2		57.14	1
61.01		1	—			
62.02		1	--			
63.33		2+	1.2	⊙ line dpl.		
				Viol. comp. R. 62.61 1		
				Coin. with R. 63.31 3		
———	4668.30					
69.50		1+	1		69.49	1
70.66		4	—		70.67	8
72.48		1	2	V?		
73.83		1	—		73.84	1
79.65		1	—			
79.95		1.2	—		79.96	1
81.07		1.2	1	Coin. uncertain	81.07	1
82.09		1+	1.2	V?		
———	4683.74					
84.64		2	—		84.63	3
87.10		2.3	—		87.10	5
88.24		1	—			
90.45		1	—		90.44	1
4699.52		2—	2		99.51	2
———	4703.18			R. 02.69 1		
4705.26		2	—		05.28	3
06.34		2.3	—		06.36	5
06.75		2.3	1.2		06.76	5
07.62		2	2	R. 08.40 1	07.63	3
10.74		2.3	—	R. 09.13 1	10.75	5
13.61		1.2	—		13.64	1
14.28		2.3	1.2	R. 15.49 1 (Ti)		
15.61		1.2	—		15.65	1
16.08		2+	—		16.08	4
16.36		1.2	—		16.38	1
17.85		2.3	—		17.87	5
21.42		1.2	—		21.44	1

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
4721.70		2.3	—		21.70	4
23.06		2.3	—		23.06	4
23.65		1	—		23.63	1
———	4727.63			R. 24.07 1		
28.85		1+	—		28.84	1
29.73		2+	—		29.72	5
30.57		2	—		30.57	2
31.42		1.2	—		31.44	1
31.74		1.2	—		31.74	1
32.12		1.2	—		32.11	1
37.91		1+	—		37.92	1
38.51		1.2	—	Os has 38.51 1+	38.51	1
39.79		1	–		39.85	1
42.79		2	1		42.82	5
46.81		2	—		46.83	5
47.30		1.2	—		47.31	1
48.70		2+	—		48.72	5
51.16		2+	—		51.21	5
51.45		1	—		51.46	1
51.75		2	—		51.76	5
				R. 52.04 1		
54.13		2.3	—	⊙ has 54.23 (Mn)		
———	4754.23					
57.55		2	—			
57.68		2.3	—	⊙ has 57.77 (Fe)	57.69	4
58.92		1.2	—	R. 59.21 1	58.94	1
65.84		1.2	—	R. 64.22 1	65.86	1
66.80		2.3	1	R. 69.21 1	66.84	7
72.74		1+	—		72.78	1
73.25		1.2	—		73.26	1
76.54		3	1.2			
76.70		2+	1.2		76.64	5
———	4783.60			R. 81.51 1		
84.65		2	—		84.66	5

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
4786.70		3	3	R. 89.10 1	86.71	7
93.10		2	—	R. 94.73 1	93.13	2
95.27		2	—		95.29	2
97.07		3	1		97.12	8
98.12		1.2	—		98.15	1
99.20		1+	—		99.21	1
4799.94		2+	1.2		99.97	4
———	4805.25			R. 03.24 1; 02.37 1		
4807.70		3.4	1	R. 08.84 1	07.74	10
19.22		1.2	—		19.22	2
———	4824.32			R. 23.03 1		
27.62		3.4	1		27.64	10
29.00		1.2	—	R. 29.43 1	29.01	1
30.86		1.2	—		30.88	1
31.80		3.4	1+		31.84	8
32.59		3	1		32.62	8
33.17		2	—		33.21	3
				R. 34.00 1; 34.26 1; 35.04 1		
43.16		1.2	—		43.19	2
				R. 46.80 1		
48.98		1.2		R. 49.26 1; 49.46 1	49.00	1
51.65		4	1.2		51.69	10
				52.15 1; 54.11 1; R. 57.24 1; 58.81 2		
59.34		2	1	⊙ has 59.32		
———	4859.93			Coin. ?		
62.83		2	1	Coin. ?	62.80	4
64.93		4	?	Coin. ?	64.94	10

Vanadium λ	R	V i	⊙ i	Remarks	Rowlan	
				R. 70.33 1		
4871.46		2	—	⊙ has 71.51 (Fe) No coin. with V	71.45	3
				R. 73.17 1		
75.66		4	1+		75.67	10
80.77		2.3	—		80.75	6
81.75		4	1.2		81.75	10
85.86		2	—	R. 82.36 2	85.83	2
87.02		2	—		86.99	2
90.32		1.2	—		90.26	1
———	4890.94					
91.43		1.2	—		91.41	2
91.81		2	—	⊙ has 91.68 (Fe)	91.77	3
4891.43		2	—		94.40	3
4900.84		2.3	—		00.82	3
———	4903.49					
04.59		3	2	Dpl. Coin. exact ?	04.58	5
05.10		1.2	—		05.05	3
06.06		1	—			
				R. 07.05 1		
08.92		1	—	R. 13.28 1	08.88	1
16.48		1.2	—	R. 19.17 1	16.44	1
22.60		1.2	—		22.54	1
———	4924.05					
25.83		2.3	—	⊙ has 25.75 (Ni)	25.84	7
32.24		2	1		32.21	3
33.82		1+	—		33.79	1
———	4942.66					
4943.04*		1.2	—			
5002.54		2+	—		02.51	2
———	5014.42					
14.83*		—	—	R. 47.84 1	14.81	4
60.91		1	—	R. 51.78 1	60.83	1
5064.32		1.2	—		64.30	1
———	5064.83					
5105.37		1.2	—		05.32	2

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
——	5109.82					
5128.71		2.3	—	R. 37.77 1	28.71	7
38.58		2+	—		38.60	4
39.74		2—	?	⊙ line ?	39.70	2
——	5141.92					
48.95		2	?	⊙ line ?	48.89	
57.27		1—	—	R. 59.44 2		
59.56		2	—		59.52	2
65.14		1	—		65.07	1
——	5165.59					
67.04		1	—	R. 69.13 1	66.96	1
70.15		1+	—		70.11	1
72.35		1—	—	R. 74.71 1	72.28	1
77.03		2-	—	76.68 1	76.96	1
78.75		1	—		78.73	1
79.35		1	—		79.28	1
81.01		1	—		80.93	1
83.07		1—	—		83.03	1
——	5183.79					
92.22		1	—		92.19	1
93.18		2	2	V?	93.18	5
93.82		1+	—		93.80	1
95.01		2+	—		95.02	6
5195.58		2	2		95.56	2
——	5202.48			R. 97.22 1		
				00.52 1		
5206.82		1+	—		06.79	1
07.89		1	—		07.84	1
12.47		1-	—		12.40	1
13.87		1+	—		13.84	1
16.80		1.2	—		16.77	1
——	5217.56					
25.97		1.2	—		25.92	3
33.91		1	—		33.90	2

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
5234.31		2	—	Very sharp	34.25	7
40.40		1	—		40.36	2
41.06		2	—	Very sharp	41.06	3
——	5242.66			R. 58.31 1		
60.56		1	—		60.53	1
61.20		1	—		61.15	1
66.33		1	—			
——	5270.50					
71.28		1	—		71.12	1
72.92		1	—			
82.75		1	—			
5287.88		1	—	Several fine lines		
——	5288.71					
5302.40		1	—	Several fine lines here.		
——	5307.55			V?		
29.05		1	—			
30.65		1	—		30.62	1
——	5333.09			R. 38.81 1		
				58.62 3		
83.68		2-	—	⊙ has 83.58 (Fe)	83.65	1
85.39		2	—			
88.56		1.2	1	Coin. with a fine ⊙ line	88.53	1
——	5397.35					
5398.13		1.2	—			
5402.17		2.3	—		02.15	5
——	5415.42					
15.51		2.3	?	Fe has 15.42	15.48	5
18.33		1.2	—		18.32	1
20.32		1	—			
21.96		1	—	R. 24.28 2 (Fe)		
34.43		2	—		34.41	4
——	5434.74					
37.93		1.2	—		37.89	1
43.50		1+	—		43.47	1

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
—	5447.13					
5458.39		2	—	R. 55.03 1 Ru has 55.02 3.4		
—	5463.17					
64.30		1	—			
68.05		1	—		68.03	1
71.56		1+	—		71.56	1
—	5477.13					
87.48		1.2	—		87.45	1
88.18		2+	—		88.31[1]	4
5490.22		1.2	—		90.18	1
—	5497.73					
5505.13		1.2	—	06.10 1 07.74 1 08.86 1	05.10	1
07.97		2.3	—			
11.41		1.2	—	15.30 1 17.44 1 34.06 1 R. 35.08 1 35.66 1 42.95 1 45.10 1	11.41	1
—	5513.21					
46.18		2	—		46.16	1
47.31		2.3	—		47.31	4
48.41		1+	—		48.40	1
57.71		1+	—			
59.00		2	—		58.99	1
61.92		2	—	66.16 1 R. 67.70 1 76.75 1 84.60 1	61.90	1
—	5569.85					
84.75		2.3	—		84.75	5
85.00		1.2	1+	V ?	84.98	1
86.26		2	—		86.23	1
88.71		1.2	—		88.71	1
92.67		3	—		92.67	4
5593.22		1.2	—	R. 94.73 1 98.05 1	93.21	1
5601.63		2	—		01.63	1

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
—	5603.10					
5604.44*		1.2	—		04.44	1
04.91*		1	—		04.88	1
05.20*		2.3	—		05.19	4
22.34		1.2	—	R. also has 24.45 1	22.32	1
24.80		2.3	—		24.85	7
25.16		2	—		25.12	4
26.27		2.3	—		26.27	7
27.86		3.4	1		27.89	7
32.73		1+	—		32.70	1
—	5634.17					
—		—	—		34.14	1
35.76		1.2	—		35.74	1
46.36		2.3	—		46.35	5
—	5655.71					
57.11		1+	—		57.12	1
57.67		2.3	—		57.69	5
68.61		2.3	—		68.61	5
71.10		3.4	?	Perhaps a fine ⊙ line	71.09	10
—	5675.65					
83.47		1.2	—		83.45	1
88.02		1+	—		88.00	1
—	5688.43					
5698.74		4	?		98.76	1
5703.83		3.4	1		03.83	10
07.26		3.4	1.2		07.24	10
09.25		1.2	—		09.20	1
—	5709.69					
16.49		1.2	—		16.46	1
25.90		2	—	Sharp	25.88	3
27.25		4	1.2		27.30	10
27.90		2.3	—		27.90	5
31.48		3.4	—			
33.34		1.2	—		33.34	1

[1] Typographical error for 88.21 ?

* The asterisk indicates that the particular wave length is deduced from the following ⊙ line.

Vanadium λ	R	V	i	Remarks	Rowland	
5733.63		1+	—			
34.26		2	—		34.25	2
37.28		3	—		37.31	5
——	5742.07					
43.67		2.3	—		43.67	5
47.98		1	—	Sharp		
49.13		2	—	Sharp		
50.90		1.2	—			
52.99		1.2	—		52.99	1
61.70		1.2	—		61.67	1—
——	5763.21					
72.66		2+	—	Sharp	72.66	2
76.95		2	—	Diffuse	76.93	1.2
82.85		1+	—		82.85	1
83.14		1	—			
83.76		1+	—		83.76	1

Vanadium λ	R	V	i ⊙	Remarks	Rowland	
5784.64		2	—		84.65	1
86.42		2	—		86.41	1
5788.85		1.2	—			
——	5791.21					
5800.17		1.2	—			
07.40		2—	—			
——	5809.44					
17.33		1.2	—			
17.80		2—	—			
30.97		2	—	Diffuse		
——	5831.83					
39.34		1	—			
46.56		2	—	Diffuse		
5850.60		1	—			
——	5853.90					

I am well aware that among these lines several can be found which are due to other metals hitherto not recognized. In this category of suspicious objects probably belong those lines which are inconspicuous in the vanadium spectrum, but correspond to strong absorption lines in the solar spectrum. This seems to be a reasonable inference from the fact that vanadium is represented in the general solar spectrum by only faint and inconspicuous lines corresponding to the most intense lines of the metal, while the metallic lines of medium and slight intensities are entirely absent in the solar spectrum. Provisionally, however, and as long as their origin cannot be established, the exclusion of these seems to me not quite justifiable, as slightly so as in cases where the lines arouse a suspicion of foreign origin on account of their extreme faintness.

The probable error of the above wave-length may be said to be about ± 0.02 tenth-meters, as already remarked. Rowland's values are doubtless more accurate, and in general are probably certain to two places of decimals. A comparison of our determinations nevertheless yields an entirely satisfactory result, the differences exceeding 0.03 tenth-meters only in a few cases. Among 431 lines compared the following differences occur the number of times given under N.

Δ	N	Δ	N
0.00 t.-m.	80	0.05 t.-m.	23
0.01	120	.06	7
.02	85	.07	8
.03	75	.08	4
.04	29		

The number of cases where the deviation exceeds 0.03 tenth-meters constitutes about 16 per cent. of the whole.

In Rowland's catalogue of the vanadium lines several occur which may be easily recognized as due to foreign impurities. In the following table I have collected these with the corresponding values of the wave-length according to Kayser and Runge, or myself, the estimates of intensity by Kayser and Runge, being transformed to my scale:

Rowland		Kayser and Runge		Hasselberg		Element
λ	i	λ	i	λ	i	
3639.72	1	39.71	6			Pb
83.60	1	83.60	6			Pb
3706.17	1			06.16	2.3	Mn
19.05	1			19.04	2.3	Mn
3933.77	3	33.83	6	33.77	—	Ca
44.13	6	44.16	6			Al
61.65	5	61.68	6			Al
68.59	1	68.63	6	68.62	—	Ca
4033.19	3			33.18	10	Mn
34.62	2			34.60	10	Mn
57.96	1	57.97	6			Pb
77.85	1	77.88	6			Sr
4226.87	4	26.91	6			Ca
4318.80	2	18.80	5			Ca
4425.59	1	25.61	6			Ca
54.94	1	54.97	6			Ca
56.07	1	56.08	5			Ca
4501.41	1			01.42	3	Ti
4715.49	1			15.46	2	Ti
5424.28	2	24.27	6			Fe
5535.66	1	35.69	6			Ba

We find here the principal lines of calcium and of lead, as well as of aluminium, and among those lines not occurring with me, probably a few other lines of the platinum metals.

I have already remarked that vanadium is only slightly represented in the general solar spectrum, faint absorption lines corresponding only to the strongest lines of the metal. In order to give a survey of this I have collected in the following table all the stronger vanadium lines (i = 3 or above) along with the estimated intensities of the coinciding solar lines.

Vanadium		⊙	Vanadium		⊙	Vanadium		⊙
λ	i		λ	i		λ	i	
3533.85	3	1	3818.37	3	2	3998.87	3	—
3553.43	3	—	28.66	3.4	2	4057.21	3	—
3673.55	3	1	40.88	3	1	90.70	3	1.2
76.86	3	—	55.50	3	2	92.83	3	2
80.26	3	?	56.00	4	2	95.64	3	1
83.27	3	—	65.02	3.4	2	4099.93	3.4	2
92.37	3	1	75.22	3	2	4102.32	3	1
3696.00	3	1	90.34	3	1.2	05.32	3	1.2
3703.71	3.4	2	3893.03	3	—	09.94	3.4	2.3
04.85	3	1.2	3902.40	3.4	2	11.93	4	2.3
3795.12	3.4	?	10.01	3	2.3	15.32	3.4	2
3813.63	3	?	34.16	3.4	—	16.65	3	1.2

Vanadium λ	i	⊙	Vanadium λ	i	⊙	Vanadium λ	i	⊙
4123.65	3	2	4416.63	3	1+	4578.92	3	?
28.25	3.4	2.3	21.73	3	1	80.57	4	2
32.14	3.4	2.3	26.17	3	1	86.54	4.5	1
4134.60	3.4	2.3	28.67	3	1	4594.27	4.5	2
4232.62	3	1	29.95	3	—	4770 66	4	—
33.09	3	1.2	36.31	3.4	?	76.54	3	1.2
34.12	3	—	38.02	3.4	1	4786.70	3	3
68.78	3	1	41.88	3.4	1.2	4807.70	3.4	1
71.70	3	—	44.40	3.4	1	27.62	3.4	1
77.13	3	1	52.19	4	1	31.80	3 4	1+
84.19	3	—	59.65	3.4	3	32 59	3	1
4291.96	3	—	59.93	4	2	52.65	4	1.2
4309.95	3	?	60.47	4.5	2.3	64 93	4	1.2
30.18	3	1	62.56	3.4	—	75.66	4	1+
32.98	3	1	69.88	3.4	—	4881.75	4	1.2
41.15	3	1	74.88	3.4	—	4904.59	3	2
53.02	3.4	1	89.06	3.4	2	5592.67	3	—
79.38	4.5	2.3	4496.26	3	—	5627.86	3.4	1
84.87	4.5	2.3	4502.12	3	—	71.10	3.4	?
90.13	4.4	2	24.42	3	—	5698.74	4	?
4395.40	4.5	2	45.60	3.4	?	5703.83	3.4	1
4400.74	4	1.2	49.82	3	2.3	07.26	3.4	1.2
06 80	4.5	2	60.90	3	?	27.25	4	1.2
07.85	4.5	3	71.96	3	—	31.48	3.4	—
08.35	4	2	77.36	4	1	5737.28	3	—
08.67	4.5	3						

It cannot be doubted from this table that vanadium enters into the general composition of the solar atmosphere, but apparently in only slight quantities. In this respect the conditions are quite different from those prevailing in Sun-spots, in the spectra of which, according to a communication by Young, many lines of vanadium invisible in the general solar spectrum attain a considerable intensity and breadth. The occurrence of vanadium in the Sun is not at all surprising in view of its very wide dissemination among the terrestrial minerals, and this adds interest to the fact that the metal similarly enters into the composition of a number of meteorites. I have frequently had opportunity to observe this in investigations of the arc spectra of these bodies on which I am at present at work. I shall soon report more fully upon this question.

ON THE DEVIATIONS FROM THE LAW OF RECIPROCITY FOR BROMIDE OF SILVER GELATINE.[1]

By K. Schwarzschild.

The so-called law of reciprocity states that sources of light of different intensity I produce an equal degree of blackening in their photographic images under different exposures t if the product $I \times t$ has the same value in the different cases. Laboratory experiments by Abney, Miethe, and Michalke have demonstrated that there are deviations from the law of reciprocity. These, however, reveal themselves most clearly in astrophotographic work. Scheiner, in 1891, proved that the increase in the number of fainter stars on prolonging the exposure fell far below what would be expected according to the law of reciprocity. In determinations of stellar brightness by the photographic method I have recently been able to confirm once more the existence of such deviations, and to follow them up in a quantitive way, and to express them in the following rule, which should replace the law of reciprocity: Sources of light of different intensity I cause the same degree of blackening under different exposures t if the products $I \times t^{0.86}$ are equal. In these experiments Schleussner's gelatine emulsion plates were employed. The exposures ranged from 3 to 5000 seconds, the intensity from one to a thousandfold, the blackening from the slightest degree up to almost complete opacity.

The results of a research by A. Schellen[2] are contradictory to the above result, as he found the law of reciprocity exactly confirmed for these same Schleussner plates, although this was after he had brought them to the maximum of a preliminary exposure. The contradiction cannot be explained, however, by the absence of a preliminary exposure in my experiments, as I

[1] *Photographische Correspondenz*, 1899.

[2] "Ueber die Giltigkeit des Bunsen-Roscoe'schen Gesetzes für Bromsilbergelatine." Münster, 1898.

obtained the same deviations for slight as for heavy degrees of blackening, and an appreciable effect of preliminary exposure on intense blackening is not to be thought of.

It was therefore desirable to repeat the laboratory experiments. This was made possible by the courtesy of M. Eder, who placed at my disposal the necessary appliances in the *k. k. graphische Lehr und Versuchsanstalt*, and assisted me with many suggestions. The procedure consisted simply in making plates with different (continuous) exposures at different distances from the normal benzine lamp which is used with the Scheiner sensitometer. All of the plates to be compared were cut from the same plate and were developed in the same bath at the same time.

It will suffice here to give a specimen of the results.[1] Equal degrees of blackening were produced in one series of experiments by the following combinations of intensity I and the exposure t:

1		2	
I	$= 81$	I	$= 1$
t	$= 4.8$ secs.	t	$= 785$ secs.
$I \times t$	$= 389$	$I \times t$	$= 785$
$I \times t^{0.86}$	$= 312$	$I \times t^{0.86}$	$= 309$

We see how striking the deviation is from the law of reciprocity. On diminishing the intensity to $\frac{1}{81}$, twice the amount of light is necessary to produce the same blackening. The products $I \times t^{0.86}$, on the contrary, come out equal within the limits of the errors of observation. For the intensity 1 the first trace of blackening appeared with an exposure of 10 seconds. A preliminary exposure of this amount did not alter the results of the experiment, as was expected, and the most dissimilar developers gave the same deviation.

Thus the laboratory experiments confirm the formula found from the star plates. The bromide of silver gelatine investigated had also the property of employing so much the less of the incident energy for the photographic work, the slower the influx of energy. The diminution of the action of the light

[1] The detailed account of my experiments will soon appear in the publication of the von Kuffner Observatory.

under intermittent exposure, which has been noticed by Abney and more recently by R. Englisch, and which I also found for these highly sensitive plates, is connected with this. In a simple experiment with the aid of Scheiner's sensitometer[1] equal degrees of blackening were produced by
continuous exposures of

96, 72, 48, 24, 12 secs.:

and intermittent exposures of

99, 80, 54, 30, 16.5 secs.

Here the single durations of exposure were to the period of inter ruption, as

1 : 2.5; 1 : 3.1; 1 : 4.6; 1 : 8.2; 1 : 15.0.

The following illustration, which may be useful as a "mechanical analogy" for the further discovery and formulation of the laws applying here, is intended to exhibit the peculiar behavior of the plate: Suppose that a kite, instead of being held by the cord, carries a rather heavy rope, the end of which trails over the ground. If the wind is strong (corresponding to a strong intensity of light) the kite will rise high, only a short portion of the rope will remain on the ground, and the kite will fly rapidly before the wind without much friction for a long distance (corresponding to a strong degree of blackening). If, on the contrary, the wind is light, the kite will remain low and the slight energy of the wind will be for the most part lost in the friction of the long extent of the rope lying on the ground. If the wind is intermittent the kite has so much more time to settle down after every rise, and the motion will be accompanied on the average by the greater friction the longer the intervals between gusts of wind (corresponding to the behavior of the plate under intermittent illumination.)

Von Kuffner'schen Sternwarte,
Wien-Ottakring.

[1] This consists of a rotating disk, from which portions have been cut out so as to allow the light to fall on the plate only during a definite fraction of a revolution.

ON THE EFFECT OF INTERMITTENT EXPOSURE ON BROMIDE OF SILVER GELATINE.

By K. Schwarzschild.[1]

An intermittent exposure can be varied in the four following ways, with the limitation that the source of light is constant during the experiment and the interruptions of the exposure are of uniform interval:

1. By the change of the ratio of the pause to the single exposure q.

2 By the change of the single exposure and the following pause taken together t.

3. By the change of the effective intensity of the light J.

4. By the change of the total duration from the first to the last exposure T.

These four quantities, which were thus selected for the convenience of explanation, yield all the others which might subsequently be of interest, as for instance:

The duration of the single exposure, $t_2 = \frac{t}{1 + q}$;

The duration of the pause, $t_2 = \frac{tq}{1 + q}$;

The sum of the exposures, $T_1 = \frac{T}{1 + q}$;

The number of interruptions, $n = \frac{T}{t}$.

The effect of intermittent exposure has been investigated for the bromide of silver gelatine emulsions of C. Schleussner in these fourfold relations.

The series of experiments A was carried out with the aid of the Scheiner sensitometer of the *k.k. graphische Lehr- und Versuchsanstalt* which was most kindly placed at my disposal for this purpose by M. Eder. In this instrument, as is well known, a disk rotates between the source of light and the plate with sections

[1] *Photographische Correspondenz*, 1899.

cut out of different breadth which are numbered and so calculated that the ratio q of the covered portions of the circuit to the free part, and hence the ratio of the pause to the single exposure, is determined by the formula

$$q = 1.28^{N+5} - 1.$$

The normal benzine lamp belonging to the sensitometer was always placed at a distance of 1 meter (intensity of light $J = 1$), and the period of revolution of the disk, which was the same as the duration t of the pause plus the single exposure, always amounted to 0.08 secs. In the first series of experiments J and t were therefore constant. But q on the contrary varied for the different sections cut out of the disk, and the total duration of exposure T was varied from 1 minute to 12 minutes, so that from 720 to 8400 interruptions took place. One experiment ran as follows: A strip of the plate received a series of continuous exposures. Another strip of the same plate was illuminated for a certain time T through the rotating sensitometer disk, and then had a numerical scale of degrees of blackening. It could be easily calculated from the above formula to which sensitometer number each continuous exposure would have to correspond, if it depended only on the sum total of the time of exposure, and the interruption made no difference. In fact equality was observed to exist between the continuous exposure and a sensitometer number different from the calculated one; in case of only approximate equality the deviation being estimated in tenths of the scale interval. For example, in one experiment I gave the first plate the following continuous exposures T_1 and illuminated the second plate for 4 minutes in the sensitometer. The calculated sensitometer number is given under N, the observed under N'.

T_1	N	N'	$N-N'$
28.8 s.	3.9	3.6	0.3
14.4	6.8	5.8	1.0
7.2	9.7	8.2	1.5
4.8	11.3	9.5	1.8
2.4	14.2	12.4	1.8

The result of the entire series of such experiments is collected in the following table. The one argument is q, the ratio of the pause to the exposure, the other is the total exposure T in the sensitometer. The difference $N - N'$ between the calculated and the observed sensitometer number is given. A positive number indicates a stronger action of the continuous exposure than the intermittent exposure. A negative number indicates a weaker action. It should be remarked, further, that a falling off of 1 sensitometer number corresponds to a loss of something like 25 per cent. of the luminous energy.

q $T=$	1 m	2 m	4 m	6 m	12 m	Mean
1.5	—0.3	—0.1		—0.1		—0.2
4	0.1	—0.1		—0.2		—0.1
6.5	0.5	0.1	0.3	0.1	—0.4	+0.1
14	0.9	0.6	1.0	0.5	0.6	0.7
24	0.9	1.0	1.6	1.2	1.3	1.2
60		1.2	1.8	1.7	2.0	1.7
130				1.5	2.5	2.0
260					2.3	2.3

Result: (1) The effect of the intermittent action depends for a given q not at all or very little on the duration T of the whole exposure in the sensitometer. Since the blackening increases with the duration of exposure for each separate q, for each sensitometer number, we see that the degree of the final blackening is of slight importance in the phenomenon. We are always concerned here, moreover, with the region of medium degrees of blackening only, since the very strong degrees of blackening are too uncertain for comparison, and the very weakest are too much affected by accidental preliminary exposure of the plate. We may, therefore, take the mean of the values of each horizontal line and find: (2) the effect increases with q. The intermittent exposure falls further behind the corresponding continuous exposure, the longer the pause is in proportion to the single exposure. The small negative values for a small q, which would indicate a stronger action of the intermittent than a continuous exposure, are to be assigned to errors in the experiment, and only show that if the pause is not much longer than the

duration of the single exposure as many as 8400 interruptions (for $T = 12$ min.) exert no demonstrable effect.

Strictly this same series of experiments should now be performed for all other possible systems of values of the intensity J and durations t of single exposure plus pause. Two simplifications, however, may be permitted. We may generalize *a priori* that it would not depend on the final degree of blackening for the other values of J and t, and can therefore limit ourselves to the test of a single blackening, which of course should be chosen of medium degree. Secondly we need not test all the ratios q of pause to exposure since the general increase of effect with rise in q is recognized, but we may limit ourselves to two extreme cases, for which I have selected $q = 1$ (pause equals duration of single exposure) and $q = 23$.

Series B. $q = 1$. With the assistance of gears different velocities of rotation could be communicated by clockwork (a chronograph of the von Kuffner Observatory) to a disk from which twelve sectors of 15° aperture at equal intervals were cut out. The lamp was set up at a definite distance, and acted on the plate for a certain period with the rotating disk intervening and then for half the time without the disk, so that the sum of the exposure times was equal in the two cases. The degrees of blackening obtained were both estimated on a scale obtained with the Scheiner sensitometer and the difference of the estimates was taken. The following table with double argument gives the differences which were obtained in a series of experiments with different intensity J (distance of lamp 40, 100, 200, 450 cm) and with different velocities of rotation of the disk. Instead of the time of revolution its twelfth part is directly given under t, which expresses the sum of the single exposure and pause. Positive numbers indicate as above a stronger degree of blackening by continuous exposure.

t $\quad J=$	4	1	$\frac{1}{4}$	$\frac{1}{20}$
0.02^s	0.2	−0.1	0.5	
0.4	0.0			0.3
1.8	0.3	0.1	0.4	
3.2			0.3	0.0
6.4				0.1

Result. For very different intensities and durations of a single exposure ($t_2 = \frac{t}{2}$) only a very slight effect of the intermittence is indicated, on the average a diminution of 0.2 sensitometer numbers. Therefore if the pause is of the same length as the single exposure no appreciable effect is lost under the conditions here obtained. It is to be noted that the last three experiments ($t = 3.2$ *s* and 6.4; therefore the single exposures of 1.6 and 3.2 secs.) were performed without the rotating disk by illumination with the hand according to a clock which beat fifths of a second. These prove therefore only that exposures from one to two seconds can be exactly made by hand.

Series C. $q = 23$. This series ran like the others except that a disk was used which had only one sector cut out of 15° and the continuous exposure therefore had to be $\frac{1}{4}$ of the exposure with the rotating disk to produce an equal total duration of exposure. The following table gives the result of the experiments in this same way as before. The argument t, the sum of the pause and the exposure, correspond here directly to the time of rotation of the disk. The last experiments for $t = 38$ secs. were carried out by illumination with the hand, but may be regarded from what has been found before as sufficiently certain.

Result. With a pause twenty-three times longer than the single exposure a heavy loss of luminous energy up to 40 per cent. (corresponding to two numbers of the sensitometer) occurs. The loss is the greater, the quicker the velocity of rotation, the shorter the single exposure, and an equal loss occurs with a slower rotation and longer single exposure if the intensity falls. Following up the behavior of this last more closely, we find at intensity 4 a loss of 1 sensitometer number for a rotation in $t = 0.8$ secs., corresponding to a single exposure of 0.033 secs. With intensity of $\frac{1}{20}$ we find the same loss for a rotation of about 50 secs. corresponding to a single exposure of 2.1 secs. The quantity of luminous energy introduced during the single exposure is measured in the first case by $4 \times 0.033 = 0.13$, and in the second case by the closely similar value $\frac{2.1}{20} = 0.10$.

Therefore with varying intensity the same loss will occur when the amount of light introduced during the single exposure is the same. If this quantity reaches the so-called limiting value, the value which of itself produces a barely appreciable blackening, there will remain only a slight weakening of about 0.5 sensitometer units (12 per cent.) in consequence of the intermittence. For the 4 intensities this limiting value is respectively 0.1, 0.5, 2.5, 15 secs. Multiplying by 24 we obtain 2.5, 12, 60, and 360 secs., and if we enter the table with these values of *t* for the 4 intensities, we shall find on the average the above loss of 0.5 sensitometer numbers. We further see that the effect of the intermittence almost wholly disappears if the single exposure exceeds the limiting value by a multiple, and that it does not attain large values until the single exposure falls far below the limiting value.

With this all the variations of the intermittence have been experimented upon, and we reach the following conclusion. The effect of the intermittence depends upon two instead of four quantities; first, upon the ratio of the pause to the duration of the single exposure — the longer the proportion of pause the greater the weakening; and secondly, on the quantity of light which the single exposure sends to the plate. A definite ratio of the pause to the single exposure produces more weakening the further the quantity of light lies below the limiting value. On the other hand no appreciable influence is due to the degree of the blackening and the absolute magnitude of the exposure time, or the luminous intensity taken by itself.

The derivation of a quantitive formula from the above figures cannot be ventured, since the determination of the blackening by estimation on a scale must be considered a rather rough procedure in comparison with the delicacy of these effects. The qualitive character of the results, however, seems to be substantial, and is further confirmed by the experiments made several years ago by Abney.[1] He found that the retardation of the blackening under intermittent exposure was the more

[1] *Journal of the Phot. Society*, 1893-4, p. 63.

pronounced the longer the pause in comparison to the single exposure, the quicker the rotation of the disks employed, and the less the light intensity. Therefore if the intensity becomes stronger and hence the deviation less, we can evidently increase the deviation up to its original amount by raising the velocity of rotation, so that from Abney's experiments also the result, aside from the ratio of the pause to the duration of exposure, must chiefly depend upon the quantity of light admitted in the single exposure. Abney's experiments differ from those given above in that slow plates and shorter single exposures were employed. The more recent experiments of R. Englisch[1] would seem to indicate an effect of the same character.

The following experiment, which is included in the last table, is the most remarkable, and at the same time easiest to repeat. Let a fairly constant light be moved so far from the plate that in 15 secs. the first trace of a fog is produced (in the above case 450 cm) and with the eye on the watch expose with the hand and interrupt alternately 3 secs. and repeat this twenty times. At another place illuminate twenty times for 3 secs. with pauses of 1 minute. The second exposure will develop a markedly less degree of blackening than the first (in the above case 1 unit of the sensitometer) in spite of the equality of the total time of exposure. From this we may conclude that the process of weakening develops in a peculiarly slow manner in that it must complete its action wholly, or for the greater part, in the period from 3 secs. to 1 minute after the cessation of the exposure. It is particularly in view of this fact that it appears a quite difficult problem to explain the action of the intermittent illumination by other than an artificial hypothesis.

Remarks on the Scheiner sensitometer. It has been shown above that the action of a rotating disk itself changes with the velocity of rotation. In Scheiner's sensitometer the rotation is produced by hand, and is therefore not entirely uniform. Nevertheless the variation of the action with the velocity of

[1] Review in *Archiv fur wissenschaftliche Photographie*, 1899, Heft 1.

rotation is so slow that the alterations which could be obtained by hand rotation anyhow must remain inappreciable, according to the above figures, and according to experiments of my own this is the case.

The difference in the action of continuous and intermittent exposure has, however, the further consequence that the original intention of this sensitometer, of constructing an accurate scale of exposure times at constant light intensity, cannot be obtained with the rotating disk. The higher numbers are too much weakened by relatively longer pauses. If, however, we should construct a scale of intensities by weakening the light in definite ratio, as, for instance, in the Warnerke sensitometer, instead of a scale of exposure time, this would deviate in an equal sense from the precise scale of the times of exposure, as the scale of Scheiner's sensitometer ; for, in consequence of the deviation of the reciprocity law above proven, a diminution of the intensity will produce a less blackening in the same proportion as a shortening of the exposure, and the higher numbers of the scale of intensity will again come out less than those of the exact time scale.

The most suitable thing in practice is properly something intermediate between a scale of time and a scale of intensity, because we have to deal neither with constant intensities nor constant exposures, as, for instance, highly sensitive plates are used now for instantaneous pictures of bright objects, and again for long exposures of faint objects. Scheiner's sensitometer therefore furnishes in general suitable data as to sensitiveness in practice just on account of its tendency from the time scale to the intensity scale which the intermittent exposure introduces. If the theoretical meaning of these data does remain perhaps indefinite and certainly complicated, it should be remembered that an exact determination of sensitiveness cannot be expressed by a single number in consequence of the deviation from the law of reciprocity and the different grades of plates.

Minor Contributions and Notes

CHANGE IN TIME OF PUBLICATION.

The attention of subscribers to the Astrophysical Journal is called to the fact that hereafter the February and August numbers of each year will be omitted, instead of the July and September numbers, as formerly. Thus the first volume of each year will consist of the January, March, April, May, and June numbers, while the July, September, October, November, and December numbers will constitute the second volume.

THE GREAT REFRACTOR OF THE POTSDAM ASTROPHYSICAL OBSERVATORY.[1]

The recent dedication of the great refractor of the Potsdam Observatory was an event of the first importance in the progress of astrophysics. The principal address on this occasion was that of Director Vogel, who reviewed the advances which have been made in the determination of stellar motions in the line of sight, and referred to the important contributions to this work which we owe to the Potsdam Observatory. After Professor Vogel's address, the motions of the telescope and dome were explained and demonstrated by Professor Scheiner. The telescope has two objectives, one of 80 cm aperture and 12 m focal length, and another of 50 cm aperture and 12½ m focal length. Both objectives, for which the glass was furnished by Schott & Co., of Jena, were made by C. A. Steinheil Sons, of Munich; the larger of the two is corrected for the actinic rays, the smaller for the visual rays. The mounting by A. Repsold & Sons, of Hamburg, is of the so-called German form as modified by Repsold; the motions of the telescope in both coördinates are easily effected from the floor by means of two hand wheels supported on the column of the instrument. The weight of the moving parts is about 7000 kg.

[1] See frontispiece.

The dome is 22 m in diameter and 18 m high. The iron construction of the hemispherical movable part is by Bretschneider and Krügner, of Pankow; the inner lining of wood was put in place by Joester, of Potsdam. It rests on a system of twenty trucks, each containing three wheels, of which the middle one bears the dome, while the outer ones run on a track fastened to the masonry. The rotation of the dome, in which a weight of 200,000 kg is set in motion, can be effected by hand, without great labor, although very slowly; by the aid of electricity a complete revolution can be accomplished in five minutes. The driving mechanism was made by the firm of C. Hoppe, which also furnished the very ingeniously constructed movable platform for the observer. This movable platform, which was first suggested by Dr. J. Repsold, is suspended from the dome, with which it moves, directly opposite the observing slit. It can also be moved independently through a limited distance to the right and left. The stage on which the observer stands moves up or down on an inclined plane. This motion can be effected by hand, or with great ease from the platform itself by means of electric motors. The opening in the dome has a width of 3½ m and extends 1½ m beyond the zenith. The shutter can be operated by hand from a gallery on the inner wall of the tower or electrically from the observing platform. The lower part of the opening can be closed by means of two screens 5 m high, which are moved outward from the middle of the slit.

We are informed that the preliminary tests of the two spectrographs constructed for the new telescope by Toepfer have been in every respect satisfactory. It may be expected that the great work of determining the motions in the line of sight of some five hundred stars, for which the telescope is specially designed, will soon be in progress.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XI MARCH, 1900 NUMBER 2

AN ATTEMPT TO TEST THE NEBULAR HYPOTHESIS BY AN APPEAL TO THE LAWS OF DYNAMICS.

By F. R. Moulton.

1. *Introduction.* — One of the boldest and most attractive speculations ever offered in any science is beyond a doubt the hypothesis of the development of the solar system from a vastly extended nebula, which was formulated by Laplace a little more than a century ago. His unparalleled researches on the mechanical causes of the forms and intricate movements of the planets and satellites, and his unprecedented success in employing the law of gravitation as an instrument of investigation, gave him a deep faith in the sufficiency of Newton's law for the explanation of all the facts relating to the present constitution and past development of the solar system. He was struck by the really remarkable circumstance that all of the bodies of the system, both planets and satellites, so far as had then been determined, moved in nearly the same plane, and revolved and rotated in the same direction. He came forward with an array of figures derived from the Theory of Probability showing that such a

number of practical coincidences in so few chances could happen, at hazard, but once in countless millions of times. He concluded that the present arrangement is not fortuitous, but that it has had its cause in the original dynamical relations of the system and the laws that have been operating upon it. It must be remarked, however, that he propounded his hypothesis with "that distrust which everything ought to inspire that is not the result of observation or calculation."

Perhaps no other theory in the natural sciences, which has not been rigorously demonstrated to be true, has found so speedy and unanimous acceptance, or has stood so long without radical modifications. This is all the more noteworthy since it was the first explicit formulation of the theory of evolution, a theory which has been so hotly contested in other fields of thought. It has exercised a marked influence on most of the speculations in geology and astronomy until the present day. It has been accepted with a few slight modifications almost without question by the highest authorities, as Helmholtz, Kelvin, Newcomb, and Darwin, and it has been made the basis for the most sweeping conclusions. It has held such a dominant sway over the thoughts of investigators in the field of the grosser modes of inorganic evolution that doubtless in many instances facts have been warped and perverted, and questionable methods of reasoning employed, in order to bring experience into harmony with it. But this is an age characterized by a critical attitude toward fundamental assumptions in all fields.

In this paper it is proposed to question the exactness of the Laplacian hypothesis, not only in its original form, but also as modified by the discovery of new members of the solar system, and by the establishment of the law of the conservation of energy. In particular, the inquiry will be raised whether it is compatible with the fundamental laws of dynamics.

The methods of testing the theory will be divided into three categories: (I) comparison of observed phenomena with those which result from the expressed or implied conditions maintained by the hypothesis; (II) answers to the question whether the

supposed initial conditions could have developed into the existing system; (III) comparison of those properties of the supposed initial system with the one now existing, which are invariant under all changes resulting from the action of internal forces.

The first category will contain the objections which have been usually cited as contradicting the theory, and some new ones of the same type. The second category will deal with problems which are of the very greatest difficulty when we attempt to restrict ourselves to rigorous reasoning, and the results obtained are not all that could be desired; yet it is believed that they are sufficient to be of considerable interest. The troubles arise in attempting to prove that certain things cannot take place in any time, however long. Mathematically stated, it is difficult to find solutions of the differential equations involved which are uniformly convergent for all values of the time. The third category will make a comparison of those properties which must remain constant. They are defined by *integrals* instead of by solutions, as in the former case, and have the important property of being independent of the time. They are such as the mass of the system, the invariability of the plane of maximum areas, and the constancy of the moment of momentum.

In carrying out the numerical work involved some approximations have been made for the sake of simplicity, but it will be noticed that in every instance they have been chosen in such a manner that they favor the validity of the nebular theory. It follows, therefore, that the adverse conclusions arrived at are just as certain, so far as their bearing on the theory is concerned, as though numerical exactness had been insisted upon throughout.

The results contained in this paper have been arrived at jointly by Professor T. C. Chamberlin and myself as the result of a more or less continuous study of the subject during the last three years. In this time we have had many conversations in regard to this field of science where geology and astronomy meet. Our exchanges of ideas have been so frequent and so complete that it is not possible to divide the responsibility for

the various methods of attack, and the manner of carrying them out. Yet, as a general rule, it may be said that Professor Chamberlin has employed his keen perception of physical relations, and his exceptional powers of invention, in the formulation of problems relating to the general theory, while it has been my office to investigate their pertinency and bearing by appeals to the mathematical principles of dynamics. This order of procedure has not been universally followed; indeed, in some instances the results have been reached by successive steps made alternately by one and the other of us. In a paper in the *Journal of Geology* published nearly concurrently with this, by mutual understanding, Professor Chamberlin outlines some of the studies which have led up to the present inquiry and indicates their geological relations.

2. *Definition of the nebular hypothesis as treated.*—In order to remove all uncertainty in regard to the precise hypothesis which is under discussion, it will now be briefly outlined. This is the more needful because it will be considered in its broad outlines so as to avoid urging objections of a special nature founded on particular assumptions regarding temperatures and physical conditions of matter.

The nebular theory examined in this paper supposes that at some remote epoch the materials of which the solar system is composed were in a gaseous or meteoroidal state, filling a space approximately spheroidal in form, and extending at least to *Neptune's* orbit. If the gaseous condition is assumed, it does not insist upon any particular temperature, but merely supposes that the whole mass was in hydrodynamical equilibrium, rotating practically as a solid with an angular velocity equal to that of the outermost planet. If the meteoroidal condition is assumed, it supposes that the properties of gases were nearly fulfilled in accordance with the conclusions reached by Darwin in his memoir, "On the Mechanical Conditions of a Swarm of Meteorites and on Theories of Cosmogony," which he presented to the Philosophical Society November 15, 1888. Therefore the system may henceforth be spoken of as being a nebula. The

theory supposes that the nebula has been subject to no appreciable external forces, and that it has contracted under its own gravitation, either (*a*) leaving off rings at certain intervals, or (*b*) dividing by some fission process, and that these separated portions have contracted, forming the planets and the satellite systems. The comprehensive theory under discussion is thus seen to include a considerably wider range of possible initial conditions and modes of development than that postulated by Laplace. There is no limiting assumption here regarding the temperature or the original physical condition of the matter or the manner of separation. A few of the tests brought forward will bear especially upon particular assumptions, but for the most part they are general in their application.

I. METHODS OF THE FIRST CATEGORY.

3. *On the planes of the planetary and satellite orbits.*—There are certain objections to the nebular theory which at once appeal to the most superficial and careless critic. The most conspicuous and best known of these is that the revolutions of the satellites of *Uranus* and *Neptune* are in a retrograde direction. The retrograde revolutions in themselves would not be so significant (since the precise process of contraction into the planets is not described) if it were not for the additional fact that the planes of the orbits of the four satellites of *Uranus* are almost perpendicular to the plane of the planet's orbit around the Sun, while the orbit of the satellite of *Neptune* has an inclination of nearly 35°. One of the requirements of the theory seems to be that the planes of the planets' equators and of their satellite systems shall not differ much from the planes of their respective orbits. It was shown by Darwin in his memoir, "On the Secular Changes in the Elements of the Orbits of a Satellite Revolving about a Distorted Planet," which he presented to the Royal Society in 1880, that tidal friction would under certain circumstances increase the mutual inclinations of the planes of the orbits of a satellite and its planet. There is no suggestion, however, that it could change the inclinations more than a few degrees, and

Darwin says in the same memoir, on page 886: "The retrograde motion and high inclinations of the satellites of *Uranus* are, if thoroughly established, very remarkable." These facts point to a condition of heterogeneity quite incompatible with the distribution of density in a system in hydrodynamical equilibrium.

Another thing of the same nature which could be reasonably expected, under the hypothesis, is that the planes of the planets' orbits would become more nearly coincident as the Sun is approached; for the nebula would tend to rotate more nearly as a solid as the time increased and the secular results of friction accumulated. The observed facts are quite the opposite of this, the plane of *Mercury's* orbit presenting a greater deviation from the average of the whole system than that of any other planet.

Likewise it could be reasonably expected that the orbits of the planets near the Sun would be more nearly circular than the orbits of remote planets, but quite the reverse is true.

4. *On the distribution of mass in the solar system.*—The masses of the various planets are not what one would expect if the ring theory is true. If the masses of the several planets are divided by the radii of their respective orbits, numbers are obtained which are proportional to the products of the densities and the cross sections of the respective rings. Choosing the units so that the result shall equal unity for the Earth's orbit, we have:

Planet	Product of cross section and density	Planet	Product of cross section and density
Mercury	0.123	*Jupiter*	61.064
Venus	0.592	*Saturn*	9.938
Earth	1.000	*Uranus*	0 761
Mars	0.070	*Neptune*	0.566

If it be supposed that the ring from which any planet has been formed extended half way to the two adjacent orbits, then the numbers above, divided by the squares of those representing these respective distances, will give the numbers proportional to the mean densities of the various rings. Choosing the units so that the result shall equal unity for the Earth's orbit, we have:

Planet	Density of ring	Planet	Density of ring
Mercury	0.150	*Jupiter*	0.609
Venus	1.010	*Saturn*	0.028
Earth	1.000	*Uranus*	0.0012
Mars	0.003	*Neptune*	0.0008

The irregularities of these numbers are such that it seems probable that the nebula was very heterogeneous.

5. *On the inner satellite of Mars and the rings of Saturn.*—Since the discovery of the satellites of *Mars*, by Hall, in 1877, the rapid revolution of *Phobos* has been urged as an objection to the nebular hypothesis; but the difficulty has been explained by Darwin in a memoir presented to the Royal Society in 1881, as being the necessary result of tidal evolution. He says in this memoir, "On the Evolution of the Solar System," p. 534, "That the inner satellite of *Mars* revolves with a period less than a third of its planet's rotation is perhaps the most remarkable fact in the solar system. The theory of tidal evolution explains this perfectly, and we find that this will be the ultimate fate of all the satellites, because the solar tidal friction retards the planetary rotation without directly affecting the satellite's orbital motion."

The reasoning employed by Darwin is certainly correct qualitatively, and the quantitative conclusions may perhaps be accepted as sound also. They will at least be adopted here, and on the basis of this it will be shown that a more serious difficulty arises in considering the revolution of *Saturn's* inner ring, and that it cannot be explained away in the same manner.

The tides raised by the satellites of *Mars* have been so small that tidal reactions cannot have modified their orbits appreciably; and conversely, they cannot have been effective in reducing the planet's rotation. If it be assumed that *Mars* actually extended to the orbit of *Phobos*, and had a rotation equal to that of the revolution of the satellite, and that all the reduction in the moment of momentum implied by the shrinkage from this dimension has been brought about by the action of the Sun's tides, then we have a standard of comparison. In considering the magnitude of the effect of the Sun's tides a sufficiently

approximate result will be obtained by assuming that *Mars* has always been a homogeneous sphere. Carrying out the computation, it is found that the rotational moment of momentum of *Mars* was, when it extended to the orbit of *Phobos*, 24.61 times as great as at present.

It is clearly demonstrated by theory and spectroscopic observations that the rings of *Saturn* are made up of a great number of discrete masses, each pursuing practically an independent orbit. Because of the very small mass of the entire ring system compared to that of the planet, each body makes its own revolution in almost exactly the same period that it would if it were undisturbed by the others. Then by means of Kepler's harmonic law, and from the accepted dimensions of the ring, and the elements of *Titan's* orbit, we find that *the bodies constituting the inner edge of the inner ring perform their revolutions in 5.66 hours, while the period of the planet's rotation is known to be 10.23 hours.*

The dimensions of the ring cannot have been sensibly changed by tidal action, so it is fair to assume, in testing the nebular theory, that *Saturn* at one time extended to the inner ring and rotated in its period of revolution. Carrying out the computation under the supposition that *Saturn* has always been homogeneous, it is found that its angular moment of momentum has been reduced from 2.8 to 1. If the Laplacian law of interior density is assumed, it is found that the reduction has been from 1.8 to 1. This is much less than that which had to be accounted for in the case of *Mars;* but it must be remembered that *Saturn* is more than six times as far from the Sun as *Mars*, and that the retarding force of the tides varies nearly as the inverse seventh power of the distance from the disturbing body. The tidal retardation also depends upon the size and viscosity of the tidally distorted planet. In the memoir last quoted, in Table III, p. 526, Darwin gives in the column headed, "Numbers to which retardation is proportional," "*Mars*, 0.89" and "*Saturn*, 0.000020 to 0.000066." The difference is so great that the moment of momentum of *Saturn* could not have been reduced by the solar

tides unless they have been operating upon this planet between 5063 and 1534 times as long as they have been upon *Mars*.

It may be urged that *Saturn* has satellites which may have been the cause of the great reduction in the moment of momentum, but the appropriate computation shows that the total moment of momentum of *Titan*, which is the largest of all the satellites, is but 0.0061 of that of the planet's rotation. The masses of the satellites are so uncertain that the computation has not been carried further; but in Darwin's paper just quoted, on page 523, he says that the total moment of momentum of the whole satellite system is "$\frac{1}{30}$ or more" of that of the planet's rotation. From these figures it is apparent that the satellites cannot have retarded the planet greatly, even if the extreme supposition be made that they have acquired all of their momenta at the expense of the planet's rotation. Therefore the rapid revolution of the inner ring of *Saturn* must be considered as being more remarkable than "the most remarkable fact in the solar system."

II. METHODS OF THE SECOND CATEGORY.

6. *On the escape of the lighter elements.*—The first objection under this category applies particularly to the hypothesis of a gaseous nebula extending to *Neptune's* orbit. According to the laws of diffusion and convection of gases the molecules of all of the elements contained in the system would be scattered among the molecules of all the other elements, with a tendency on the part of those with small molecular weights and of great abundance, as hydrogen, to preponderate at the surface. The individual molecules would be moving on the average with high velocities, depending upon their temperature and their weights. Near the surface (which in the ordinary nebular theory is supposed to be somewhat distinctly defined) the velocities would in some cases, especially for the lighter elements, be so great and in such directions that the molecules would escape the system never to return, if present determinations may be trusted.[1] The

[1] Compare "On the Cause of the Absence of Hydrogen from the Earth's Atmosphere and of Air and Water from the Moon," by Dr. Johnstone Stoney, *Trans. Royal*

velocity requisite for permanent escape at the distance of *Neptune's* orbit is about 4.8 miles per second.

The difficult part of the question arises in attempting to determine the efficiency of this sort of waste in depleting the system of its lighter elements. For hydrogen and the lighter substances, the mean square velocities of the molecules would at least nearly equal the 4.8 miles per second for very moderate temperatures. It must be supposed then that they would escape at a very rapid rate, and that hydrogen, helium, and such light elements should be entirely absent, or at least very rarely found, instead of occurring in such great abundance.

While there may be ground for doubt as to the quantitative value of the rate of escape when applied to the whole system, the case becomes much clearer when the test is applied to individual planets, and the postulated rings from which they are supposed to have been formed under the Laplacian hypothesis. Every planet defines a velocity of permanent escape for its surface. According to the nebular hypothesis it must be assumed that the planets originally contained practically all of the elements and in approximately the same proportions in which they are now found in the Sun, the residual mass. Their atmospheres should, therefore, be alike and about equally extensive relatively. The facts are these: The velocity of escape in the case of the Moon is 1.48 miles per second; it has no atmosphere, and probably none of the satellites has an atmosphere. The velocity of the escape in the case of *Mercury* is 2.45 miles per second; it has no atmosphere, or at the most a very rare one. The velocity of escape in the case of *Mars* is 3.12 miles per second; it has a very limited atmosphere. The other planets, with much higher velocities of escape, have extensive gaseous envelopes probably composed of nitrogen, oxygen, and carbon dioxide, and perhaps lighter gases in the cases of the great planets. These facts

Dublin Society, 1892. Also "Of Atmospheres upon Planets and Satellites," by the same author, *Trans. Royal Dublin Society*, Vol. VI, part 13, October 25, 1897; also "A Group of Hypotheses Bearing on Climatic Changes," by T. C. Chamberlin, *Journal of Geology*, Vol. V, No. 7, October–November 1897.

correspond with the theory and support the doctrine of effective escape.

It is a noteworthy fact that none of the very light elements, as hydrogen, is found free in the atmosphere of the Earth. It may be objected that hydrogen is very active chemically, and that it would unite with something else to form compounds. This is undoubtedly true, but helium is not found in the atmosphere appreciably, as it should be if it did not continually escape, for it is very inactive and is constantly given forth from the interior of the Earth. Now, if such concentrated bodies as the Moon and *Mercury* are unable to hold atmospheres at present temperatures, is there any ground for supposing that the Earth-Moon ring, or the ring of *Venus*, or of *Mars*, could have held any of the atmospheric gases, or water vapor, at the temperatures necessarily assigned them by the Laplacian theory? Their power of control at any given point in the ring must have been very much less than that of the Moon. Computations given later in this paper will show that the self-gravitation of such rings would be less than the differential pull of the gravity of the Sun's mass, and hence they would offer essentially no opposition to molecular dispersion.

7. *On the rarity of the solar nebula and on the formation of rings.*—The fact that the solar nebula must have been exceedingly rare when it extended out to planetary distances seems to have been insufficiently considered by those who have speculated on the subject of the formation of the planets. Owing to its rotation, the equatorial diameter must have been somewhat greater than the polar, so the volume obtained by considering the nebula to have been a sphere extending to the outermost planet will be too great, and the average density determined by dividing the mass by this volume somewhat too small. On the other hand, the nebula must have been much denser at the center than toward the outer limits. We may, therefore, divide the mass by the volume of a sphere with radius equal to *Neptune's* distance from the center of the system and obtain a fair idea of what the density must have been in the outer regions. Owing

to the central condensation the density obtained in this way will undoubtedly be too great.

The density of a sphere of constant mass varies inversely as the cube of its radius. When the solar nebula extended to the orbit of *Neptune* it had a radius 6444 times that of the present Sun and a volume more than 267,000,000,000 times as great. The average density of the Sun, on the water standard, is about 1.4. Its average density when occupying the larger volume which we are considering would have been, therefore, $\frac{1}{191,000,000,000}$ of that of water. This would be increased by about $\frac{1}{700}$ of its value by the addition of the material which went into planets and satellites.[1]

By hypothesis, the volume was maintained partly by the molecules or meteorites impinging on each other and rebounding, and partly by the general rotatory motion of the whole mass. Suppose the mass to have contracted with the consequent acceleration of rotation until the centrifugal and centripetal forces nearly balanced. The question arises what sort of a rupture would have taken place when the rotation increased; or, in other words, in what part of the mass the centrifugal force would have first balanced the centripetal. By hypothesis the nebula rotated nearly as a solid. Even if it is supposed that it was a meteoric swarm it probably would have done so. Darwin states on page 65 of his memoir "On the Mechanical Conditions of a Swarm of Meteorites, and on the Theories of Cosmogony," which he read before the Royal Society November 15, 1888, "The investigation of section 11 also gives the coefficient of viscosity of the quasi-gas, and shows that it is so great that the meteor-swarm must, if rotating, revolve nearly without relative motion of its parts, other than the motion of agitation."

When a mass rotates as a solid the centrifugal force varies directly as the distance from the center. If the mass were spherical in form and homogeneous the centripetal force, or acceleration toward the center, would also vary directly as the distance from

[1] Estimates by Kelvin made the density somewhat less than that found here. See *Popular Lectures*, Vol. I, p. 419.

the center. In this case the equality of the two forces would occur throughout the whole equatorial plane simultaneously and there would be no rings formed with subsequent contraction of the interior parts. If the mass were flattened at the poles, and especially if the density increased towards the center, the centripetal force would not increase so rapidly as *directly as the distance from the center*, and the centrifugal force and the acceleration toward the center would, therefore, become equal first for the extreme periphery. Since both of these conditions must be supposed to have been fulfilled under the hypothesis, and since there cannot have been any appreciable cohesion, the conclusion is that *the matter would have been left behind continually after the process was started, and that there could have been no separate rings formed.* This conclusion is equally certain whether we suppose the nebula to have been gaseous or meteoroidal.

There does not seem to be a remote chance for the separation of a large mass by fission, and it may be in order to remark that none of the conclusions arrived at by the discussion of the figures of equilibrium of rotating *homogeneous fluids* can lay any claim whatever to exactness when *heterogeneous gases* are under consideration.

8. *On the contraction of a ring into a planet.*—It will now be assumed, without regard to fact, that a Laplacian ring has been left off and that the material of which it was composed has nearly all been gathered into a planet. It will be assumed, further, that the planet revolves in a *circle* around the Sun, and that the portion of the ring which remains is composed of infinitesimal masses neither disturbing each other nor affecting the motion of the planet. Neglecting the perturbations arising from the attractions of the other planets it is required to determine, so far as is possible, whether the infinitesimal remnant of the ring will be precipitated upon its planet or not. This problem is chosen because of its relative simplicity, and because it seems to be the case most favorable to aggregation.

Take the center of gravity of the planet and Sun as the origin, and refer the motion of the infinitesimal bodies to

rectangular axes rotating with the angular velocity with which the planet moves. Choose the origin of time so that the positive end of the x-axis shall pass through the planet. Let M represent the mass of the Sun, and m that of the planet, and choose the unit of mass so that $M + m = 1$. Choose the unit of distance so that the radius of the planet's orbit shall equal one, and the unit of time so that the Gaussian constant shall equal one. Then it follows that the angular velocity of revolution of the planet is also equal to unity. The coördinates of M and m are respectively the constants x_1, 0, and x_2, 0. Consider one of the infinitesimal bodies and let its coördinates be x, y, and its distances from M and m, R and r respectively. Then the differential equations defining the motion of the infinitesimal body with respect to the rotating axes are

$$\left.\begin{aligned} \frac{d^2x}{dt^2} - 2\frac{dy}{dt} &= x - \frac{M(x - x_1)}{R^3} - \frac{m(x - x_2)}{r^3}, \\ \frac{d^2y}{dt^2} + 2\frac{dx}{dt} &= y - \frac{My}{R^3} - \frac{my}{r^3}. \end{aligned}\right\} \tag{1}$$

From these equations the integral

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = x^2 + y^2 + \frac{2M}{R} + \frac{2m}{r} - C \tag{2}$$

is found, in which C is the constant of integration. This equation is the relation between the coördinates and velocities which will always be fulfilled, because it has been assumed that there are no perturbations. (It is probable that in the long run the perturbations of the other planets already formed would neither assist nor hinder the process of agglomeration.) In particular, it defines the curves at which the velocity of the infinitesimal body will always be zero and which it can never cross.[1] The equation of the curve is obtained from (2) by putting the right side equal to zero, and is

$$x^2 + y^2 + \frac{2M}{R} + \frac{2m}{r} = C. \tag{3}$$

[1] See Hill's memoir on The Lunar Theory, in the *American Journal of Mathematics*, Vol. I, and Darwin's memoir on Periodic Orbits, in the *Acta Mathematica*, tome 21.

It is necessary to assign particular values to M and m in order to obtain numerical results. For the sake of rendering the computation easy it will be supposed that $M = \frac{4}{5}$ and $m = \frac{1}{5}$. Then let particles be taken in different parts of the Laplacian ring, as at A_1, A_2, A_3, A_4, and A_5, and suppose they move in the same direction and with the same velocity as the planet. They define in succession the constant C, and the corresponding curves of relative velocity are given in the figure and denoted by C_1, C_2, C_3, C_4, and C_5, respectively.

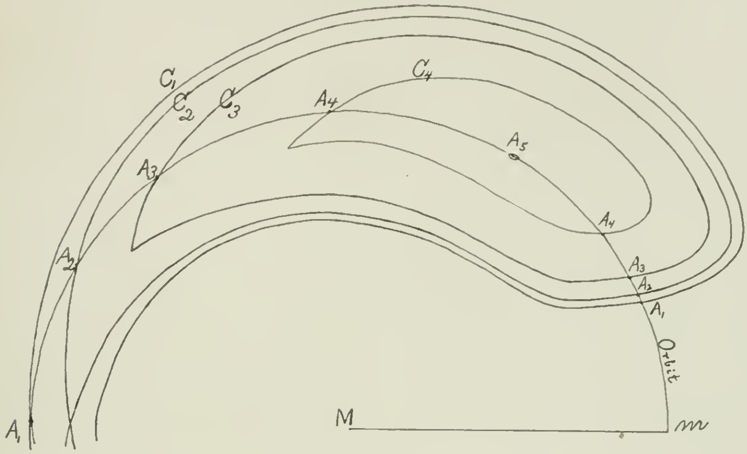


FIG. 1.

It must be borne in mind that the system is referred to rotating axes so that the Sun and planet have constant coördinates. Only the upper half of the figure is given, as the lower part is similar and oppositely placed.

The particle starting at A_1, on the left cannot get to the planet without passing *entirely around* one side or the other of the horseshoe, which is the region of imaginary velocities. This investigation does not show whether it will actually recede to a sufficiently great distance from the circle to pass around this forbidden region, but it seems very improbable that it will do so. On the other hand, there is no apparent reason why the particle at A_1 on the right cannot move along the circle to the planet.

For the initial conditions A_2 the horseshoe is on the point of dividing at the toe, and the limitations upon the motion of the infinitesimal body are similar to those in the case of A_1. There is a difference in the case of A_3 on the left, for the infinitesimal body may get to the planet by passing around the horseshoe, or by passing through the opening at its toe. There is no hindrance to A_3 on the right. Precisely similar remarks apply to the particles at A_4, the differences being evident from the figure

The horseshoe vanishes at the point A_5, and the particle may move in any direction so far as the curves of zero relative velocity are concerned. But this is the point which forms an equilateral triangle with the Sun and planet, and is one of the Lagrangian solutions of the Problem of Three Bodies. The particle will, therefore, always remain at that relative point unless disturbed by forces other than those considered.

From the considerations advanced here it seems probable that the planet could gather up material lying on the circle in which it moves, and extending in each direction to the points of the equilateral triangular Lagrangian solutions; beyond these points the conditions seem to be adverse to the aggregation of the particles of the ring with the planet. Thus, if it is supposed that the evolution has already advanced until only an infinitesimal ring remains, *it seems extremely improbable that it could complete itself except for an arc of the circle extending 60° on each side of the planet.* These results are not quite so conclusive as might be desired; but the excuse for their appearance is that they are the first arrived at in a subject fraught with great difficulties, and that they show in a decisive manner that the development of a planet out of a Laplacian ring is not so simple a matter as has been generally supposed. If the mass of the planet were taken very much smaller than the Sun, as is the actual case in the solar system, its power of gathering up the ring would be less; but on the other hand, the region of imaginary velocities would be much thinner.

9. *On the starting of a planet from a ring.*—A question of as great importance as the one which has just been considered, is

whether any condensation could begin in a ring of tolerable homogeneity. The ring as a whole would have as a resultant attraction upon any particle, a force acting in a line passing through the center of the Sun. Moreover, the intensity would be small and the same for all particles at equal distances from the center. In short, the conditions for equilibrium which have been found by Mme. Kowalewsky and M. Poincaré in the discussion of *Saturn's* rings, would be approximately verified, and there would be no sudden accumulation of material at any one point. It seems that the condensations would have their immediate causes in local irregularities in the distribution of the matter.

The first question is naturally whether bodies of considerable size would not rush together as a result of their mutual attractions. If the velocities were such as they would be when the nebula is in convective equilibrium Darwin found in his memoir, "On the Mechanical Conditions of a Swarm of Meteorites and on Theories of Cosmogony," in 1888, that two bodies with a mean density of 10 would need to have masses one twentieth of that of the Earth in order that their attractions might deflect the directions of their motions by 10° when they passed near enough to each other just to graze.

It may be supposed that the particles are moving in parallel directions with velocities depending upon their distances from the center of the Sun, as in undisturbed motion. In order to get an idea of what may happen, it will be well to investigate the amount of acceleration that small masses exert upon each other. For this purpose the following problem will be solved: Having given two spheres with diameters of 10 feet each and specific gravity 10; it is required to find how long it will take them to come together, when they are started from rest 100 feet apart, and are subject to no forces except their mutual attractions. Each one would weigh 164 tons at the surface of the Earth, and each has to move only 45 feet, yet the appropriate computation shows that it would take them 11.66 hours to come together. This illustrates the extreme feebleness of the force of gravitation between small masses, and shows that

two bodies of the maximum size that could be supposed to exist in the nebula could not unite in consequence of their own gravitation unless they were moving in orbits at very nearly the same distance from the Sun. As bodies of any considerable size would occur at very rare intervals under the Laplacian hypothesis, and as very particular initial conditions would be necessary in order that collision might ensue, it is seen that the union of two bodies by mutual attraction would be a comparatively rare event, and that the relative velocity at impact would be very small. If for other reasons they collided with high velocities there would be danger of fracture ; and if with low, there would be at least a slight rebound. It cannot be supposed that there would be much cohesion or adhesion between two bodies just having come into contact.

Moreover, the disturbing action of the Sun would have been very marked in the tenuous masses which would have existed after separation, and it is conceivable that these forces would have been sufficient to have disrupted a globular body of the density which must have prevailed if it had been brought to within planetary distances from the Sun. This possibility emphasizes the desirability of making an investigation, more refined than a mere guess, in regard to whether a fluid of this low density could have maintained itself by the gravitation of its parts in a continuous figure of equilibrium. This question will be considered in the next section.

10. *On the application of Roche's limit and a new criterion of a somewhat similar character.*— It is doubtless true that a fluid mass subject to disturbances will tend to assume the figure of most stable equilibrium. It is implied in the nebular theory that this figure is the one which is most nearly spheroidal in form ; for, otherwise, the rings would have remained as they were or developed into something different from the planets. It will be shown in this section, by the application of Roche's limit and a certain new criterion, that these nearly spherical forms of equilibrium would not be possible at the respective distances of the planets for fluids so rare as the rings must certainly have been.

It will be inferred then that the rings could not have contracted into these forms without first having become very much condensed.

The rings, when first separated, must have been of the same density as the solar nebula near its limits; for at the instant before separation they constituted a part of it, although exerting no pressure on the surface where division was about to take place. Considering the extreme tenuity of the material at the planetary distances, it cannot be supposed that there were any violent changes in form or density at the time of separation, as there could have been no appreciable cohesion. The objections about to be advanced will apply with equal force if the genesis of the planets is supposed to have been by the fission process.

Roche's limit.—It has not been possible to obtain a complete exposition of Roche's theory, so there is some uncertainty in regard to the precise conclusions reached by him. From the more or less popular accounts of his work, and from that given in Tisserand, tome II, chap. 8, it is inferred that Roche proved that a certain ellipsoid with unequal axes is a figure of equilibrium for an incompressible fluid, revolving in a circle around a spherical mass; that the figure is stable if the fluid body is at a sufficiently great distance from the other; that if the densities of the two bodies are equal, this limit is 2.44 times the radius of the central body, and that it varies inversely as the cube root of the density of the revolving fluid.

If the solar nebula were homogeneous, its density would vary inversely as the cube of its radius. If it is supposed that there was a pronounced condensation toward the center, as must certainly have been true, its density in the outer regions would be less than that found by computing it on the hypothesis of homogeneity. On the other hand, as has just been seen above, the inferior limit of density for the existence of Roche's figure varies inversely as 2.44 times the cube of the distance. Therefore the density of the rings must have been less than $\frac{1}{2.44}$ of that necessary for the equilibrium of the ellipsoidal figure. Neither the whole mass nor any portion of it could have assumed

the ellipsoidal figure unless there had been a heterogeneity giving local regions of very much greater density than the average. This, however, is contrary to the nebular theory as we have defined it.

A new criterion.—It is extremely difficult to express physical problems with perfect rigor in mathematical terms. It is almost invariably true that certain approximate conditions have to be replaced by others which are purely ideal. It does not matter then with what rigor the deductions are carried out, the conclusions can have no greater approximation to the truth than the idealized conditions do to the actual.

In the derivation of Roche's limit the assumption was made that the satellite was a perfectly homogeneous incompressible fluid, and that its rotation and revolution were performed in the same period. It is certain that the first assumption is not valid in dealing with gaseous bodies, and it can scarcely be considered as being approximately true. The problem of determining the figures of equilibrium of compressible gases in motion lies in an untouched field, and is evidently one of very great difficulty. Even in the case where the fluid has been assumed to be homogeneous, the results obtained have been of the nature of shrewd guesses which have been verified by showing that they fulfill the sufficient conditions. The difficulties in solving the equations for equilibrium arise from the fact that the relations which must be fulfilled involve the form of the body implicitly, since they depend upon the attraction of the body for all points upon the surface.

The method employed here will be to shift the point of weakness. Roche made the violent assumption of homogeneity of the fluid, and then verified, by the help of Lagrange's investigations of the attractions of ellipsoids, that a certain ellipsoid would fulfill the conditions for equilibrium; in this work the law of density is not specified, but the law of attraction for points throughout the fluid mass is assumed. The forms of equilibrium are then determined, and the limiting cases in which the bodies cease to have closed surfaces.

It is certain that the bodies will be densest in their central portions, and that the surfaces of constant density will become more nearly spherical as the center is approached, and it seems probable that the bodies as a whole will be more nearly spherical than they would be if they were homogeneous.

For the sake of simplicity in the discussion, it will be assumed that the attractions depend only upon the distances from the center of gravity of the mass. This is equivalent to assuming, in the computation of the forces arising from the mutual gravitation of the parts, that the masses are made up of homogeneous concentric layers. Since these forces are feeble compared to those arising from the revolution of the fluid and from the attraction of the central mass, the errors introduced will be relatively unimportant. The law of attraction upon the different parts will depend upon the distribution of the mass, and will vary between that given for a homogeneous body and that in which the mass of the outer regions is infinitesimal compared to that of a central core. If the body were homogeneous the attraction would vary directly as the distance from the center, which is one extreme, and is nearly that considered by Roche. If the finite part of the mass were all condensed into a central core the attraction would vary inversely as the square of the distance from the center. This is the other extreme, and the limits will be developed under this assumption.

Let M represent the mass of the central body and m that of the fluid satellite. Choose the unit of mass so that $M + m = 1$. By hypothesis they describe circles around their common center of gravity. Take the origin of the coördinates at the center of gravity of the fluid satellite, and let the negative end of the x-axis pass through the center of gravity of the large body. Choose the unit of length so that the distance of the two bodies apart shall equal unity, and the unit of time so that the Gaussian constant shall equal one; then it follows from the theory of circular motion that the angular velocity of revolution is also unity. Let X, Y, and Z be the sums of the resolved components of the accelerations along the x, y, and z-axes respectively.

Then if the body is in equilibrium the fundamental hydrodynamical equation

$$Xdx + Ydy + Zdz = 0 \tag{4}$$

must be fulfilled for every point of the surface. Let the distances from any point to M and m be denoted by R and r respectively. Then with the units chosen it is easy to verify that the following are the expressions for X, Y, and Z.

$$\left.\begin{aligned} X &= M + x - \frac{mx}{r^3} - \frac{M(1+x)}{R^3}, \\ Y &= \qquad y - \frac{my}{r^3} - \frac{My}{R^3}, \\ Z &= \qquad\quad - \frac{mz}{r^3} - \frac{Mz}{R^3}. \end{aligned}\right\} \tag{5}$$

Substituting in (4) and integrating we find

$$x^2 + y^2 + 2Mx + \frac{2m}{r} + \frac{2M}{R} = C. \tag{6}$$

This equation defines the *surfaces de niveau*, or level surfaces. The fluid body will be in equilibrium as long as the small fold around its center of gravity is a closed surface; but when this fold unites with others which are exterior, the body will begin to disintegrate. Therefore, the question of the equilibrium of a fluid mass which is very dense at its center is settled by the form of the surface defined by equation (6).

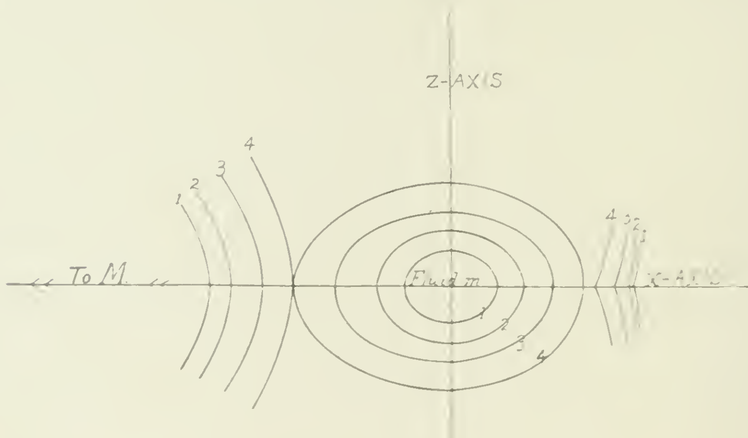


Fig. 2

For brevity, the detailed discussion of the surfaces will be omitted. The conclusion is that for sufficiently large values of C they consist of a closed fold somewhat ellipsoidal in form around each of the bodies M and m, and of an approximate cylinder whose axis is parallel to the z-axis and passes through the center of gravity of the whole system. The following sections of the xz and xy-planes with the surfaces give a clear idea of their form.

It is seen that the oval around the center of gravity of m will have its first contact with exterior folds in the xy-plane, and even on the x-axis. At the point where instability sets in the pressure at the surface becomes zero. If we express (6) in terms of r and the polar angles, and write it $F(r, \phi, \theta) = C$, then the condition for zero pressure is $\frac{\delta F}{\delta r} = 0$. This point will occur first on the x-axis between m and M; therefore y and z are zero, and the point is found by taking the derivative of (6) with respect to r, and setting the result equal to zero. Performing these operations and clearing of fractions we obtain

$$r^5 - (2 + M)r^4 + (1 + 2M)r^3 - mr^2 + 2mr - m = 0. \tag{7}$$

In all the applications which will be made m will be very small compared to M, and the value of r satisfying (7) will consequently also be small. Then a sufficiently approximate solution of (7) is

$$r^3 = \frac{m}{(1 + 2M)}. \tag{8}$$

Let σ represent the mean density of the mass m; then the condition that its radius shall just equal r, as defined by (8) is

$$r^3 = \frac{3m}{4\pi\sigma} = \frac{m}{(1 + 2M)}, \tag{9}$$

or solving

$$\sigma = \frac{3(1 + 2M)}{4\pi}. \tag{10}$$

Since m is very small, M is very nearly equal to unity, and $\sigma = \frac{9}{4\pi}$, approximately.

Let σ_0 represent the mean density of the solar nebula. Then in the units chosen

$$\sigma_0 = \frac{3}{4\pi}.$$

Therefore, $\sigma_0 = \frac{\sigma}{3}$ and the mean density of the fluid mass must have been three times as great as that of the solar nebula when they separated in order that this figure of equilibrium might

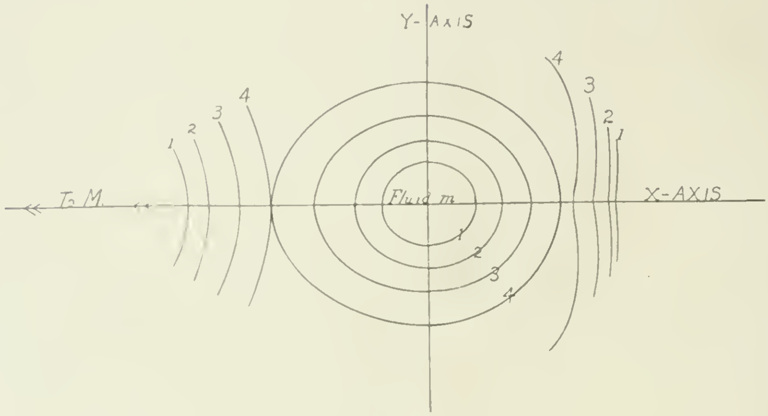


FIG. 3.

have been stable. A simple computation shows that if m and M have the same density, then m must be distant at least 1.35 times the radius of M from its center of gravity in order that it may be in stable equilibrium. The discussion of these limits shows that a Laplacian ring could not have contracted into a planet, and that the condition of the solar nebula must have been one of great heterogeneity instead of homogeneity in concentric layers.

III. A METHOD OF THE THIRD CATEGORY.

11. *On the moment of momentum of the solar system.*—It is known from the elementary principles of dynamics that the moment of momentum of a system which is subject to no external forces is constant. We shall have occasion in this section to compute the moments of momenta of the solar system at

different points of its supposed development. If a sphere and an oblate spheroid having equal masses and equal equatorial radii are composed of the same material and rotate in the same period, then the moment of momentum of the oblate spheroid will be the greater; for the greatest difference between it and the sphere is in the polar regions where the effect upon the moment of momentum of the displacement of matter is small, while it must be denser in the equatorial regions to preserve the equality of masses, since it does not seem in the least probable that a rotating body would be denser at the center than the same body would if it were stationary. Therefore, if we compute the moment of momentum which the solar nebula had when it extended to *Neptune's* orbit under the supposition that it was spherical in form we shall get a result which is too small. The moment of momentum of the present system can be determined with almost perfect accuracy, because the planets present no difficulties while the Sun is nearly an exact sphere.

Let M represent the moment of momentum of a sphere of the radius R, rotating with the angular velocity ω. Let σ represent the density; then we have

$$M = \omega \int_0^{\pi} \int_0^{2\pi} \int_0^{R} \sigma\, r^4 \sin^3\theta\, d\theta\, d\phi\, dr\,. \tag{11}$$

Suppose the density depends upon the distance from the center; then $\sigma = f(r)$. Substituting in (11), and integrating, we have

$$M = \frac{8}{3}\pi\omega \int_0^{R} f(r)\, r^4\, dr\,. \tag{12}$$

The law of density of a gaseous sphere has been determined by Ritter in a paper in Wiedemann's *Annalen* (New Series) Vol. XVI, 1882, p. 166, entitled "Untersuchungen über die Höhe der Atmosphäre und die Constitution gasförmiges Weltkörper." The problem has also been treated by Dr. G. W. Hill in a paper in the *Annals of Mathematics*, Vol. IV, 1888. Professor Darwin has given a complete discussion of the same problem in a memoir read before the Philosophical Society November 15, 1888, "On the Mechanical Condition of a Swarm of Meteorites, and on the

Theories of Cosmogony." The law of density used in the following computations is that of Darwin's "Isothermal-Adiabatic" sphere given in *loc. cit.* p. 25. The density is given there for different values of the radius and agrees substantially with the results found by Ritter and Hill. We have computed the integral of (12) by a somewhat rough quadrature. The results are the following:

When the nebula extended to *Neptune's* orbit,	$M =$	32.176
When the nebula extended to *Jupiter's* orbit,	$M =$	13.250
When the nebula extended to the Earth's orbit,	$M =$	5.690
When the nebula extended to *Mercury's* orbit,	$M =$	3.400
In the system at present, - - - - -	$M =$	0.151

Instead of being a constant, the moment of momentum is found to vary in a remarkable manner. On account of the approximations made the first number is somewhat too small while the last is too large, as the Sun was assumed to be homogeneous in computing the moment of momentum which enters into it. Notwithstanding these errors in opposite directions, the moment of momentum in the first case is 213 times that in the last. It follows from these figures that if the mass of the solar system filled a spheroid extending to *Neptune's* orbit, and rotated with a velocity sufficient to make its moment of momentum equal to that of the present system, and if it then contracted with the law of density always that adopted above, the centrifugal force would not equal the centripetal until it had shrunk far within *Mercury's* orbit. Such an enormous difference cannot be ascribed to uncertainties in the law of density or to the approximations in the mechanical quadratures; but it points to a mode of development quite different from, and much more complicated than, that postulated in the nebular theory under discussion.

12. *Conclusions.*—It has been the purpose of this paper to gather up and to present in a connected form the greater part of the dynamical considerations bearing upon the nebular hypothesis which we have thus far been able to discover. In attempting to outline the whole problem in the limits of one paper the discussion of many points has been left in an incomplete state,

yet it is believed that the method of attack has been sufficiently clearly indicated, and that the conclusions are plain. It is believed that the nebular hypothesis treated is substantially the one held by astronomers to be true, and the one which has served as a basis for all the computations regarding the heat that the Sun has radiated and the age of the solar system.

Under the methods of the first category certain phenomena are enumerated which contradict the hypothesis so flatly that candid minds must admit that its validity in the form considered is open to serious question. In less exact sciences such objections would overthrow a theory or lead to its reconstruction. The objections are that the planes of the planets' orbits present considerable deviations, while four satellites revolve in planes making practically right angles with the average of the system; that the distribution of mass in the planets is unaccountably and suspiciously irregular; and that there is an unexplainable anomaly in the motion of the inner ring of *Saturn*.

Under the methods of the second category, it is shown that the development of a system of planets and satellites from an extended nebula is by no means a simple matter, and that in the system under consideration the conclusions which it was possible to make were invariably adverse to the theory. In subjects where perfectly rigorous mathematical processes cannot be employed, such a uniform agreement of conclusions, when so various methods of attack are employed, is sufficient to establish a proposition. The objections are that the lighter elements would have escaped; that matter would have been left off continually instead of in rings at rare intervals; that if a ring were all contracted into a planet except an infinitesimal remainder distributed in its path, the process of aggregation could not complete itself; that the gravitation between the masses occurring in the rare media would be so feeble that they would seldom come in contact, and that Roche's limit and a similar new criterion show that fluid masses of the density which must have existed would be disintegrated by the disturbing action of the Sun.

The one objection which is advanced in the methods of the third category is of great simplicity and leads to certain conclusions. It is of such a character, and the numerical discrepancies are so great, that it seems to render the nebular hypothesis in the simple form in which it has usually been accepted absolutely untenable unless some fundamental postulates now generally accepted are radically erroneous. It seems a necessary inference from the results of the discussion that the solar nebula was heterogeneous to a degree not heretofore considered as being probable, and that it may have been in a state more like that exhibited in the remarkable photographs of spiral nebulæ recently made by Professor Keeler.

If the above conclusions are well founded, it follows that the age of the Earth computed from the theory of the Sun's contraction as a gaseous sphere arranged in homogeneous concentric layers can be accepted only with great reservation, and that it is folly to attempt to state what was the temperature of the Sun's surface when it extended to the orbits of the various planets. If it is established that the Laplacian hypothesis is only partially true, and that we do not yet know the precise mode of development of the solar system, geologists will feel freer to interpret geological phenomena in accordance with geological principles, and astronomers will have one of the most attractive fields of investigation still demanding their efforts for its complete exploration.

University of Chicago,
February 1900.

THE ORBIT OF THE SPECTROSCOPIC BINARY, χ *DRACONIS.*

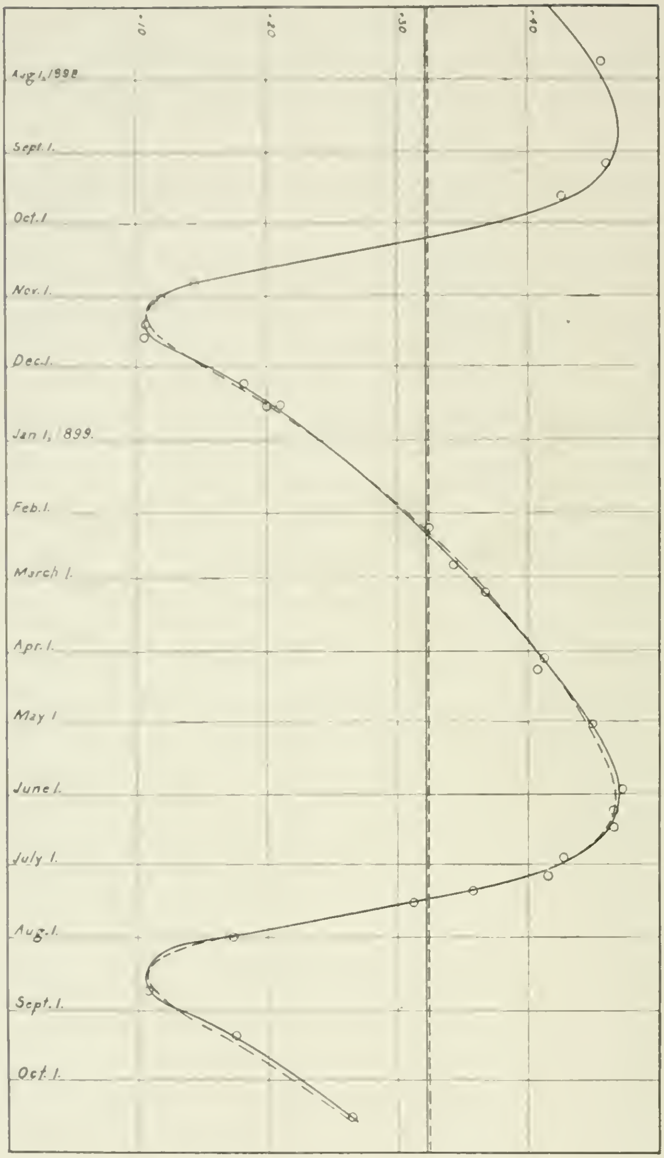
By W. H. Wright.

The variable velocity in the line of sight of χ *Draconis* was announced by Professor Campbell in this Journal (6, 291). Since July 1898 twenty-eight spectrograms have been secured with the Mills spectrograph, the variations in velocity meanwhile running through somewhat over a period. Data are therefore at hand for a fairly accurate determination of the star's orbit, and the computation has been assigned to the writer.

In the accompanying table the first column contains the numbers of the observations, the second the numbers of the plates, the third the dates of observation, the fourth the assigned weights, and the fifth the measured velocities.

	Plate No.	Date	Wt.	Vel. (observed)	Vel. (comp.)	O–C I	Vel. (comp.) II	O–C II
1	839 C	1898, July 25.8	1	+ 45.6	+44.0	+ 1.6	+44.7	+ 0.9
2	24 A	Sept. 5.7	1	46.0	46.8	— 0.8	46.1	— 0.1
3	51 B [1]	19.7	⅔	42.6	44.7	— 2.1	43.2	— 0.6
4	1038 A [1]	Oct. 26.6	1	14.5	14.3	+ 0.2	14.3	+ 0.2
5	52 A	Nov. 1.6	⅔	11 9	11.5	+ 0.4	11.6	+ 0.3
6	61 A	5.6	⅔	11.3	10.7	+ 0.6	10.9	+ 0.4
7	83 A	12.6	⅔	10.7	11.0	— 0.3	11.0	— 0.3
8	99 A	18.6	⅔	10.6	12.2	— 1.6	12.1	— 1.5
9	1122 A	Dec. 7.6	⅔	18.3	17.4	+ 0.9	17.4	+ 0.9
10	23 A	17.1	⅔	21.0	20.1	+ 0.9	20.2	+ 0.8
11	24 A	17.6	⅔	20.0	20.2	— 0.2	20.3	— 0.3
12	61 B	1899, Feb. 7.1	1	32.4	31.5	+ 0.9	32.2	+ 0.2
13	93 B	23.0	1	34.3	34.3	0.0	35.0	— 0.7
14	1203 A	Mar. 7.0	1	36.7	36.2	+ 0.5	37.0	— 0.3
15	09 D	Apr. 4.0	1	41.2	40.4	+ 0.8	41.1	+ 0.1
16	16 B	9.0	1	40.7	41.1	— 0.4	41.8	— 1.1
17	38 C	May 1.9	1	44.9	44.0	+ 0.9	44.5	+ 0.4
18	66 A	29.9	⅔	47.2	46.7	+ 0.5	46.6	+ 0.6
19	72 A	June 8.0	1	46.5	47.0	— 0.5	46.6	— 0.1
20	88 C	15.0	1	46.5	46.7	— 0.2	46.1	+ 0.4
21	1310 D	27.9	⅔	42.7	44.6	— 1.9	43.4	— 0.7
22	19 A	July 5.7	1	41.5	41.4	+ 0.1	40.0	+ 0.5
23	26 D	11.9	1	35.8	37.1	— 1.3	35.8	0.0
24	37 C	16.9	1	31.2	32.5	— 1.3	31.5	— 0.3
25	60 C	31.8	1	17.3	16.4	+ 0.9	16.9	+ 0.4
26	1417 B	Aug. 23.8	1	10.8	11.5	— 0.7	11.4	— 0.6
27	58 B	Sept. 11.7	⅔	17.5	16.6	+ 0 9	16.4	+ 1.1
28	1510 B	Oct. 16.7	⅔	26.4	25.6	+ 0.8	25.8	+ 0 6

[1] Measured and reduced by Professor Campbell.


·10
·20
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Aug. 1, 1898
Sept. 1.
Oct. 1
Nov. 1.
Dec. 1.
Jan 1, 1899.
Feb. 1.
March 1.
Apr. 1.
May 1
June 1.
July 1.
Aug. 1.
Sept. 1.
Oct. 1.

FIG. 1.

The observations of November and December 1898 were made necessarily directly after dark, a time very unfavorable for photographing spectra, on account of the rapid changes of temperature then occurring. They were accordingly given smaller weight than those made under normal conditions. In some other cases the observing conditions were very poor, wind, clouds, and bad seeing interfering with the work. In these also the weight was reduced.

The observations were platted in the usual manner, as shown in the accompanying figure, and through the points thus obtained the full-line curve was sketched as smoothly as possible. From this curve the following provisional elements were computed:

ELEMENTS. I.

$V = +32.2$ km, velocity of system in line of sight.
$e = 0.45$, eccentricity of orbit.
$w = 114^\circ.99$, position of periastron.
$U = 281^d$, period.
$T =$ 1899, July 27.0, time of periastron passage.
$K = 18.15 = \frac{f}{1 \quad p} \sin i.$

Using these elements as a basis, the velocities corresponding to the times of observation were computed, and are given in column 6 of the table. A least-squares adjustment of the elements was then undertaken. On account of their juxtaposition, the following observations were combined: (5–6), (7–8), (12–13–14), (15–16), (21–22); these, with the other observations used singly, gave twenty-one equations of condition. The form of the equations was that given by Lehmann-Filhés, except that the correction to the velocity of the system was introduced as an unknown. The following are the resulting elements:

ELEMENTS. II.

$V = +32.38$ km $\pm$ 0.09 km.
$a \sin i = 62{,}020{,}000$ km $\pm$ 410,000 km.
$e = 0.423 \pm 0.006$.
$w = 119^\circ.0 \pm 1^\circ.1$.

$U = 281^d.8 \pm 0^d.7.$

$T = 1899$, July $28^d.3 \pm 0.5$.

$r_0 = \pm 0.4$ km per sec. probable error of a single observation of weight unity.

$[pvv]_{\text{elements I}} = 16.4.$

$[pvv]_{\text{elements II}} = 7.1.$

The dotted curve has been sketched in to represent these elements.

The spectrum of χ *Draconis* is very suitable for accurate measurement. It closely resembles that of *Procyon*. The $H\gamma$ line is well defined, and the metallic lines are exceedingly sharp and well separated.

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,
December 1899.

A PRISM OF UNIFORM DISPERSION.[1]

By Charles G. Abbot and Frederick E. Fowle, Jr.

In many bolometric researches it would be extremely advantageous if there were some method of producing a spectrum on the normal or wave-length scale, and without attendant large losses or inequitable distribution of the energy of the source. It is well known that neither the grating nor the usual prism combines these three desiderata. Indeed, although the grating perfectly fulfills the first condition, in that it produces a normal spectrum, yet its fortuitously irregular distribution and great waste of radiations have generally prevented its use for bolometric purposes. Prisms have therefore been employed very largely in connection with the bolometer, and indeed also for most stellar spectroscopic researches, on account of the comparatively small losses of energy attending their use, although their dispersion varies enormously between the violet and red or infra-red ends of the spectrum.

It occurred to us, while comparing the dispersion curves of rock-salt and flint glass, that there might be a possibility of combining prisms of different dispersions and different angles in such a way as to give a spectrum in which equal increments of deviation should correspond approximately to equal increments of wave-length. Preliminary computations with a parallel sided combination of three prisms, of which the central one was rock-salt and the two outer ones glass, satisfied us that a nearly direct vision prism of these materials would be a very great improvement in uniformity of dispersion over either prism taken separately.

Recently ten small sample prisms of glass have been obtained from Mr. Brashear, and their dispersion curves have been roughly determined at this Observatory between the wave-lengths

[1] Published by permission of the Secretary of the Smithsonian Institution.

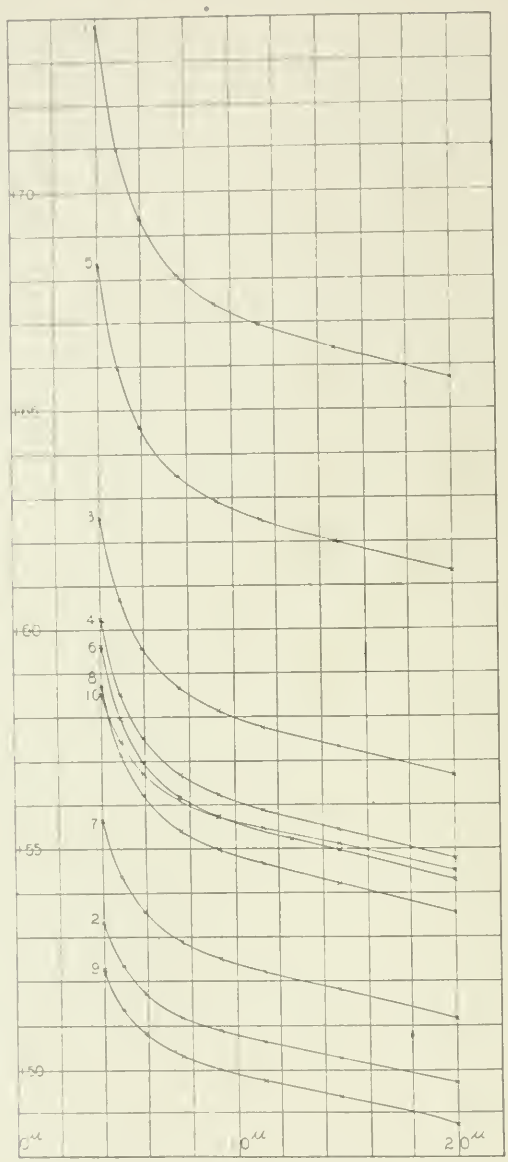


Fig. 1.

n and λ Curves of Glass Prisms.

0.4μ and 2.0μ. The n and λ curves are given in the accompanying illustration (Fig. 1).[1]

It will be seen that there is a gradual modification in form from first to last among the ten curves. In general the first point of inflection is in the red, and a second point of inflection at about 1.8μ has begun to appear in the later members of the series. The earlier curves show, both actually and relatively to the later curves, far greater dispersion in the visible than in the infra-red spectrum. Comparing the first prism with the last, we see that both have excessive dispersion in the visible spectrum, but the first has a far more excessive dispersion here than the last. Hence a small angle prism of the first kind opposed to a large angle prism of the last ought to correct this inequality approximately.

Computations were made for several such combinations, and it was found that very good results could be secured. Fig. 2 illustrates three arrangements of two prisms of $5°\ 10'$ and $20°$ angle respectively, called prisms 1 and 9 in what follows. In the first case the beam *AAA* is assumed to enter the thin prism at zero incidence, thus emerging from the thick prism at an angle of emergence of about 22°. Next the beam *BBB* is supposed to enter the thick prism at about 22° incidence, emerging nearly normal to the face of the thin prism. Third the beam *CCC* enters the thick prism as in the second case, but is reflected at the last face of the thin prism (supposed silvered) and emerges nearly as it entered.

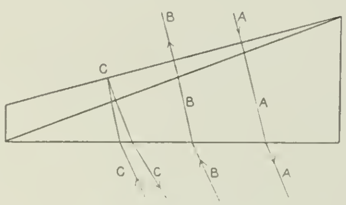


FIG. 2.

The results of these three arrangements, *A*, *B*, and *C*, are shown in curves *A*, *B*, and *C* of Fig. 3, and for the sake of comparison, curves *D* and *E* are added to show how much less uniform is the dispersion of either of the separate prisms. A

[1] The order of numbering conforms to a series of arbitrary numbers given to distinguish the prisms prior to investigating their dispersion.

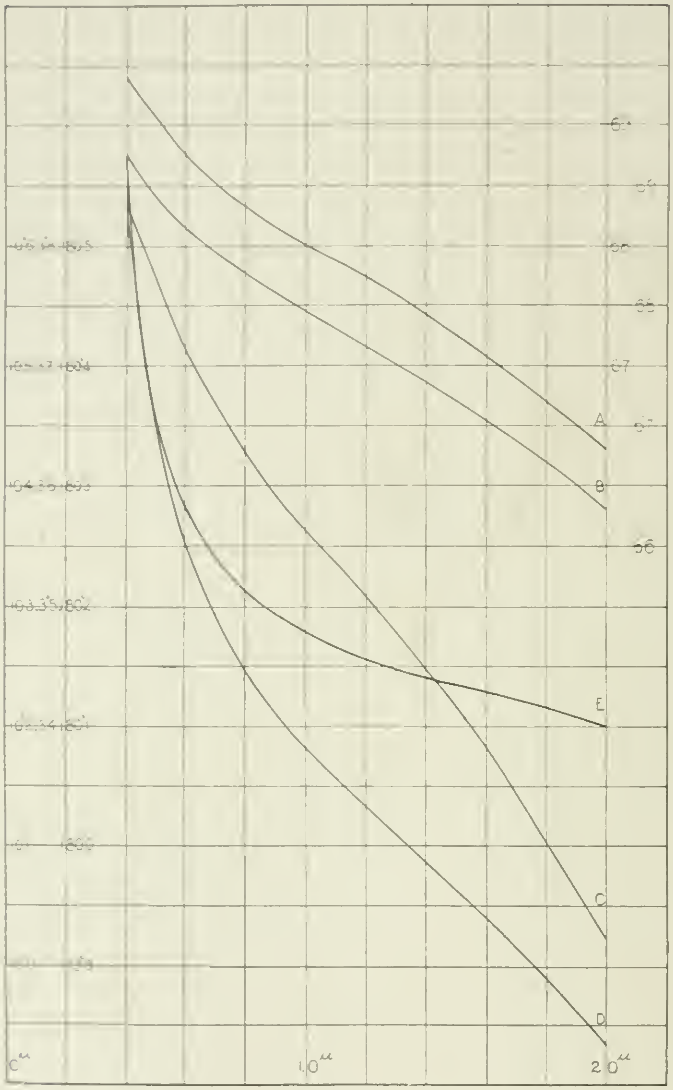


Fig. 3.

Deviation and Wave-Length Curves of Glass Prisms.

summary of the optical properties of these prisms and their combinations is given in the following table:

COMPARISON OF PRISMS.

Arrangement	Refracting angle	Angle of incidence	Mean deviation, θ	Total dispersion, $\Delta\theta$, from 0.4μ to 2.0μ	Limiting values of the dispersion $\frac{d\theta}{d\lambda}$	
					Maximum	Minimum
Prism 1	5° 10′	For minimum deviation	3° 37′	25′ 14″	180.0	3.3
Prism 9	20° 0′	" " "	10° 17′	43′ 30″	225.0	14.2
Combination *A*		00° 00′	6° 47′	18′ 30″	19.5	9.0
Combination *B*		21° 30′	6° 47′	17′ 34″	18.9	9.6
Combination *C*		21° 30′	180° 14′	36′ 50″	37.5	15.9

From an inspection of the table and the curves it appears that the combination *C* gives twenty-four times as close an approximation to uniformity of dispersion as prism 1 and seven times as close an approximation as prism 9. Long stretches of curve *C* might be chosen where the dispersion is practically uniform, and even throughout its whole extent from 0.4μ to 2.0μ the greatest dispersion is no more than twice the smallest. The convenience of the arrangement *C* for many researches leaves little to be desired, for the collimating mirror or lens could also be used to bring the spectrum to a focus, since the beam is practically returned upon itself.

In conclusion it may be said that while no general mathematical treatment of the problem of a uniform dispersion prism has been attempted by us, yet we have obtained what will be for many purposes, we hope, a sufficiently close approximation. It seems quite possible that objective prisms for photographic stellar work may be made in a similar way, so closely normal in dispersion through the limited range of wave-lengths employed as to require no correction whatever to compare the photographs with grating spectra.

ASTROPHYSICAL OBSERVATORY,
SMITHSONIAN INSTITUTION, January 1900.

THE VARIABLE VELOCITY OF β *HERCULIS* IN THE LINE OF SIGHT.

By W. W. Campbell.

The third plate of this star was measured in July 1899, and since the result differed 9 km from those given by the two previous observations, its velocity was assumed to be variable. Additional plates amply confirm the variation.

1897 June	1,	— 32.	km. ±, underexposed
1898 June	13,	— 31.9,	measures by Wright
1899 February	26,	— 23.3,	" " Campbell
July	31,	— 30.9,	" " Wright
August	21,	— 30.8,	" " Campbell
1900 January	10,	— 21.0,	" " Campbell

The period is undetermined, but it appears to be long.

Note.—I am indebted to Miss Clerke's interesting article in the *Observatory*, November 1899, p. 389, for a reference to the Harvard College Observatory's list of stars whose spectra are apparently composite. This portion of Miss Maury's work on the classification of star spectra was unfamiliar to me; otherwise, in my papers announcing the discovery of the variable velocities of o *Leonis*, β *Capricorni* and υ *Sagittarii*, I should have called attention to her recognition of the composite character of their spectra. Our collection of spectrograms includes two or more plates each of the majority of the stars on Miss Maury's list of composites; but no evidence of variable velocity has been detected except for the three stars mentioned above.

Lick Observatory,
January 11, 1900.

THE DETERMINATION OF THE MOON'S THEORETICAL SPECTROGRAPHIC VELOCITY.

By W. W. CAMPBELL.

INASMUCH as *Venus* and *Mars* are often inaccessible, it is sometimes desirable to obtain spectrograms of the Moon, in order to test the accuracy of radial velocity determinations. The Moon's apparent velocity in the observer's line of sight may on occasion amount to nearly two kilometers per second,[1] and is made up of several components, as follows:

1. The radial velocity V_1 of the Earth's center with reference to the Sun.

2. The radial velocity V_2 of the Moon with reference to the Earth's center.

3. The component V_3 of the velocity V_2 in the line joining the Sun and Moon.

4. The component V_4 of the Moon's orbital velocity (referred to the Earth's center), in the great circle passing through the Sun and Moon.

5. The component $-v_d$ in the line of sight of the observer's velocity due to the Earth's diurnal rotation. (The component of the Moon's rotational velocity with reference to the Sun will not exceed the half of one hundredth of a kilometer per second, and may be neglected.)

The values of these components may be computed quickly by the following methods, which have been developed with reference to utilizing data tabulated in the Nautical Almanac.

1. Let log D_1 be the logarithm of the radius vector of the Earth, and let a be the hourly change in log D_1. It can be shown by elementary methods[2] that the radial velocity of the Earth's center in kilometers per second is given by

$$V_1 = \frac{149{,}500{,}000}{\omega m} D_1 a = [4.9805]\, D_1 a. \qquad (1)$$

[1] Among the first to call attention to this subject were Ranyard (in the *Observatory*, January 1892, pp. 39–40), and Deslandres (in *Comptes Rendus*, 120, 419, 1895).

[2] See paper on "The Mills Spectrograph," this JOURNAL, 8, 151–154.

The quantities D_1 and a are tabulated in the Nautical Almanac.

2. Let

R = the equatorial radius of the Earth in kilometers,
D_2 = the Moon's distance from the Earth's center,
p = the Moon's horizontal parallax, and
δp = the change in the parallax in 12 hours;

then

$$D_2 = \frac{R}{\sin p} = \frac{[3.8047]}{\sin p}, \tag{2}$$

and

$$V_2 = \frac{\delta D_2}{\delta t} = -\frac{R \sin 1''}{3600} \cdot \frac{\cos p}{\sin^2 p} \cdot \delta p = -[4.934] \operatorname{cosec}^2 p \,.\, \delta p \,. \tag{3}$$

The quantities p and δp are tabulated in the Nautical Almanac.

3. Let E be the angular distance between the Sun and Moon. Then

$$V_3 = -V_2 \cos E \,. \tag{4}$$

The "lunar distance" E is tabulated in the Nautical Almanac.

4. Let δE be the variation in E in 1 second,—being positive or negative according as the distance is increasing or decreasing. Then

$$V_4 = \sin 1'' \, D_2 \sin E \,.\, \delta E = [4.6856] \, D_2 \sin E \,.\, \delta E \,. \tag{5}$$

The value of D_2 is obtained from equation (2). The lunar distance E and the arithmetical complement of log (δE) are tabulated in the Nautical Almanac.

5. The value of $-v_d$ is given by[1]

$$-v_d = [9.672] \sin t \cos \delta \cos \phi \,. \tag{6}$$

The apparent velocity of the Moon in the line of sight is given by

$$V = V_1 + V_2 + V_3 + V_4 - v_d \,. \tag{7}$$

Example.—The Moon was observed with the Mills spectrograph at

Mt. Hamilton sidereal time, 1900, January 9^d 4^h 30^m;
Greenwich mean time, " " 9 17 20.

[1] See *Astronomy and Astro-Physics*, 11, 319.

Determine the Moon's theoretical spectrographic velocity at this time.

(1)

$$\begin{aligned} \text{Constant} &= 4.980 \\ \log D_1 &= 9.993 \\ \log a &= 3.820 \\ \hline \log V_1 &= 8.793 \\ V_1 &= +0.06 \text{ km} \end{aligned}$$

(2)

$$\begin{aligned} \text{Constant} &= 3.805 \\ \sin(57' \, 27'') &= 8.223 \\ \hline \log D_2 &= 5.582 \end{aligned}$$

(3)

$$\begin{aligned} \text{Constant} &= 4.934 \\ \operatorname{cosec}^2(57' \, 27'') &= 3.554 \\ \log(-1.45) &= 0.161_n \\ \hline \log V_2 &= 8.649 \\ V_2 &= +0.04 \text{ km} \end{aligned}$$

(4)

$$\begin{aligned} \log V_2 &= 8.649 \\ \cos 114.7 &= 9.621_n \\ \hline \log V_3 &= 8.270 \\ V_3 &= +0.02 \text{ km} \end{aligned}$$

(5)

$$\begin{aligned} \text{Constant} &= 4.686 \\ \log D_2 &= 5.582 \\ \sin 114^\circ.7 &= 9.958 \\ \log(\delta E) &= 9.709 \\ \hline \log V_4 &= 9.935 \\ V_4 &= +0.86 \text{ km} \end{aligned}$$

(6)

$$\begin{aligned} \text{Constant} &= 9.672 \\ \sin 26^\circ.5 &= 9.650 \\ \cos 19.1 &= 9.975 \\ \cos 37.3 &= 9.901 \\ \hline \log(-v_d) &= 9.198 \\ -v_d &= +0.16 \text{ km} \end{aligned}$$

(7)

$$\begin{aligned} V_1 &= +0.06 \text{ km} \\ V_2 &= +0.04 \\ V_3 &= +0.02 \\ V_4 &= +0.86 \\ -v_d &= +0.16 \\ \hline V &= +1.14 \text{ km} \end{aligned}$$

Measures of this spectrogram by two observers gave, as the mean observed velocity, + 1.46 km.

LICK OBSERVATORY,
January 18, 1900.

A PROPOSAL FOR THE PHOTOMETRIC OBSERVATION OF THE PLANET *MERCURY* DURING THE TOTAL SOLAR ECLIPSE ON MAY 28, 1900.

By G. MÜLLER.

DURING the years 1878–1888 I observed the planet *Mercury* photometrically as often as its visibility in the morning and evening twilight made this at all possible, and I deduced from these observations with sufficient accuracy the connection between its brightness and the magnitude of the phase for phase-angles from 50° to 120°. The discussion of these observations showed that for the range of phase in question the light-curve of *Mercury* is almost absolutely identical with the light-curve found by other observers for the Moon. It further appeared that, if we may conclude as to the brightness of the planet at smaller phase-angles from the portion of the curve investigated, the behavior of *Mercury* is very similar to that of a number of the minor planets, and that therefore approximate values of the diameters of these small bodies could be deduced on the assumption of an equal albedo.

These conclusions, which are not without interest in themselves, could be subjected to a considerably sharper test if it should be possible to determine directly the brightness of *Mercury* at phase-angles between 0° and 50° by photometric measurements. At other stations more favorably situated than the Potsdam Observatory in respect to the purity of the air, as, for instance, at mountain stations, and perhaps also in more southern latitudes, it would certainly be possible to follow *Mercury* closer to the conjunctions than I was able to do; but under any circumstances it could only be a question of the gain of very few degrees of the phase-angle, and it would always be necessary to take into the bargain the disadvantage that the observation would have to be made at very low altitudes.

But as I have already indicated in my memoir on the Brightness of the Planets (*Potsdamer Publicationen*, Band VIII, p. 311), total solar eclipses under some circumstances offer an excellent opportunity to fill out the gap in the light-curve of *Mercury*, and to observe the planet photometrically during the daytime, and even when it is high above the horizon. In this respect the approaching total solar eclipse of May 28 will be especially favorable, and I therefore desire to respectfully recommend that this opportunity should not be allowed to pass unutilized.

The eclipse will be visible in the southern part of North America, in Portugal, Spain, and North Africa. According to the Nautical Almanac, *Mercury* reaches superior conjunction with the Sun on May 29, at 19^h G. M. T. The following values are computed for the phase-angle:

May 28, 0^h G. M. T. $7^\circ.3$
May 29, 0^h G. M. T. $3^\circ.6$

Totality will begin at $1^h\ 32^m$ G. M. T. for an observing station on the eastern coast of America, and at $4^h\ 2^m$ G. M. T. for a place on the coast of Portugal. The phase-angle of *Mercury* therefore amounts to 7° at the eclipse, and its angular distance from the Sun to something over 2°.

As to the brightness of the planet, this (expressed in stellar magnitudes) may be computed, according to my investigations, between the range of phase from 50° to 120° for any moment, according to one of the two following formulae:

$$\text{(I)}\quad h = -1.041 + 5(\log r + \log \Delta - \log r_0) + 0.03679(\alpha - 50)$$

$$\text{(II)}\quad h = -0.901 + 5(\log r + \log \Delta - \log r_0) + 0.02838(\alpha - 50) + 0.0001023(\alpha - 50)^2,$$

where α is the phase-angle, r and Δ the appropriate distances of the planet from the Sun and Earth, r_0 is the mean distance of *Mercury* from the Sun. If the formulae are applicable for smaller phase-angles than 50°, we obtain for the brightness of *Mercury* at the time of the total eclipse the value -2.5 according to formula (I), and -1.8 magnitudes according to formula (II).

The brightness would therefore be just about the same as that of the planet *Jupiter* at its mean distance.

Venus is the most suitable object for comparison, being about 40° east of the Sun at the time of the eclipse. Her angle of phase is 113°, and she happens to be exactly at her greatest brilliance (brightness = — 4.2 magnitudes), and she can be perceived with the naked eye in full sunlight.

The question might be raised whether the short duration of totality, being only $1^m\ 45^s$ on the eastern coast of America, and only $1^m\ 33^s$ in Portugal, is sufficient for a precise determination of brightness. To this may be replied that inasmuch as *Venus* can be observed both before and after totality in consequence of her favorable position, the whole duration of totality can be employed for measurements on *Mercury* alone. A practiced observer, however, is easily able to make from four to six settings of the photometer in one and a half minutes, which are quite sufficient for furnishing an accurate measurement of the brightness. During the day previous to the eclipse the most favorable aperture for the observation of *Venus*, as well as the necessary shade glasses, may be tested, and since the difference of brightness to be measured between *Venus* and *Mercury* amounts to about two magnitudes, the conditions may be realized in advance under which the measurements must be carried out during the eclipse. An accurate value of the brightness of *Venus* can be made, if necessary, on the days before and after the eclipse by comparison with bright stars. It would be advisable to employ small objectives of very short focus, as I have always done in my observations of planets, in order to obtain images of the planets in the form of points; it is also desirable to employ only those photometers with which, as in the case of Zöllner's, the effect of the different brightness of the background of the sky is eliminated.

Unless entirely unexpected difficulties should arise, I feel convinced that it should be possible to obtain a thoroughly reliable determination of the brightness of *Mercury* during the eclipse. I hope myself to be in a position to make an attempt of this sort at a station in Portugal; but as this experiment might

possibly fail on account of unfavorable weather, or for other reasons, I beg to direct the attention of those astronomers who intend to observe the approaching eclipse—and especially those who are experienced in photometric measurements—to the problem here proposed, and to request their kind coöperation in order that so favorable an opportunity for extending our knowledge of the conditions of illumination of the planets may not be unutilized.

POTSDAM, ASTROPHYSIKALISCHES OBSERVATORIUM,
February 8, 1900.

ON THE LAW OF DIURNAL ROTATION OF THE OPTICAL FIELD OF THE SIDEROSTAT AND HELIOSTAT.

By M. A. Cornu.

The well-known instruments known as the *Heliostat* and the *Siderostat* permit a beam of light coming from a heavenly body sharing the diurnal motion to be sent in a constant direction by means of a movable mirror. The geometrical theory of these instruments is very simple; the heavenly body is considered as a luminous point and the incident beam is regarded as a straight line which in twenty-four hours describes a cone of revolution around the polar axis of the instrument, which is itself parallel to the Earth's axis. In order to secure the fixity of the reflected beam it is necessary and sufficient that the normal to the mirror shall remain parallel to the bisector of the angle between the ray coming from the heavenly body and a given fixed direction. This is the condition which is more or less perfectly realized in the instruments invented by S'gravesande, Gambey, Silbermann, Foucault, and others.

If the beam reflected by the mirror thus guided is received by a telescope along its principal axis, the focal image of the heavenly body will remain fixed in the center of the field of view in spite of the angular displacement of the celestial sphere. But this condition of fixity, geometrically realized for the object under examination, is no longer fulfilled for the neighboring regions: it is easily shown that the field of view turns about its center in such a way as to effect a complete revolution n twenty-four hours. The velocity of rotation is not uniform, and thus the angular displacement of the field varies with the time in accordance with a law which it is important to determine.

Let us represent the celestial sphere by a sphere of unit radius, and each line of sight on the sky by the projection on

this sphere of a straight line passing through the center parallel to this direction.

Let $NESW$ be the circle of the true or apparent horizon (Fig. 1); P the celestial pole; Z the zenith; PZS the meridian of the place; PD the hour circle of the star D, and D' the point of the true or apparent horizon toward which the reflected beam is constantly directed.

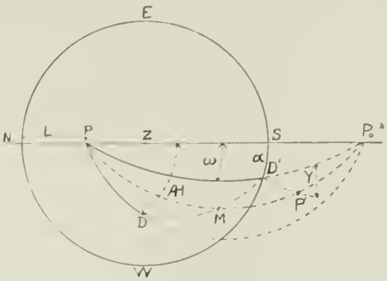


Fig. 1.

The position of the star D is defined at any instant by its polar distance $\delta = PD$, and its hour angle $AH = SPD$ counted positively in the direction of the diurnal motion, from the east E toward the west W. Similarly the point D' is determined by its polar distance $\rho = PD'$ and by the angle $\omega = SPD'$, which the plane PD' makes with the meridian. We will call this plane SPD, which is the hour circle passing through the point D' extended, the *reference plane*.[1]

If in place of ρ and ω there were given the azimuth $\alpha = SD'$ and the arc PS, the supplement of the latitude L, ρ and ω would be calculated by the aid of the two following expressions furnished by the right triangle PSD':

$$\cos \rho = \cos \alpha \cos L, \quad \tan \omega = \frac{\tan \alpha}{\sin L}.$$

In order that the beam coming from the star D shall be constantly reflected to D', it is necessary and sufficient, according to the laws of reflection, that the projection M of the normal to the mirror shall be maintained by the mechanism at the middle of the arc of the great circle DD'. Knowing at any instant the

[1] The use of stereographic projection on the circle of the horizon permits all of these arcs of circles to be rigorously traced out: it is well to adopt it in order to verify graphically the size and direction of the calculated angles. But this manner of projection has the inconvenience of so greatly distorting the sides of the spherical triangles which rise from the circle of the horizon that the use of these projections is more troublesome than useful for the clearness of the demonstrations. For this reason schematic figures are employed here instead of any regular system of projection.

projection M of this normal, we can draw the projection of the direction in which a ray emitted from any point whatever of the celestial sphere is reflected by the mirror: it is only necessary to join this point to the point M by an arc of a great circle and to prolong this arc through an equal distance. Thus the image P' of the pole P is on the arc PM prolonged to the point P', so that $MP' = MP$. The reflected spherical image of the various points of the celestial sphere is thus at any instant symmetrical to their direct position with respect to the point M.

It follows from this that the orientation of the field of view is wholly determined by the knowledge of the reflected image of any given point except that occupied by the star at the center. The pole P, because of its fixed position on the celestial sphere, is particularly adapted for this purpose, and its image P' constitutes a most simple and convenient standard of comparison.

We will now calculate for each instrument the distance and the orientation of the image P' of the pole, *i. e.*, the length of the arc $D'P'$ and the angle V made by this arc with the great circle $PD'P_0$, the projection of the *reference plane*.

Siderostat.—This name is used to designate the apparatus specially constructed for the purpose of sending a reflected beam toward the southern horizon.

The advantage of this arrangement, which was devised by Léon Foucault, is to reduce as far as possible the angle of incidence $DM = D'M$ of the rays coming from stars which at their upper transit culminate near the zenith or the equator: the aberrations of the reflected image caused by imperfections of the mirror are thus materially reduced. Fig. 1 represents the course of the beam coming from the star D and sent by the siderostat in a horizontal direction; it makes with the southern meridian an angle α, which is counted positively toward the west; α is generally a small fraction of a right angle.

1. *Distance $D'P'$ from the image P' of the pole to D', the center of the field.* The arc $D'P'$ is the side of the triangle $MD'P'$ symmetrical with the triangle MDP, since $MD' = MD$

and $MP' = MP$. These two triangles are equal, as they have an equal angle at M included between two equal sides. The two sides $D'P'$ and DP opposite the equal angle are thus equal: $D'P' = DP = \delta$. Thus *the distance $D'P'$ from the image of the pole to the image of the star (center of the field) is equal to the polar distance of the star observed.* From this it follows that *the image of the pole describes about the center of the field a circle with a radius equal to the polar distance of the star observed.*

2. *Orientation of the arc $D'P'$.*—Let Y be the angle which the arc $D'P'$ makes with $D'P_0$, the prolongation of the projection of the reference plane DP. $Y = P_0D'P' = \pi - PD'P' = \pi - (PD'D + DD'P') = \pi - (PD'D + PDD')$, for $DD'P' = PDD'$ on account of the equality of the triangles MDP and $MD'P'$. The desired angle Y is thus the supplement of the angles at the base of the triangle PDD', the apex of which is at P. From Neper's formula

$$\tan \frac{1}{2}(B + C) = \frac{\cos \frac{1}{2}(b - c)}{\cos \frac{1}{2}(b + c)} \cot \frac{A}{2},$$

we have, by substituting $A = DPD' = AH - \omega$, $b = \rho$, $c = \delta$,

$$\tan \frac{1}{2} Y = \frac{\cos \frac{1}{2}(\rho - \delta)}{\cos \frac{1}{2}(\rho + \delta)} \tan \frac{1}{2}(AH - \omega),$$

an expression which gives the orientation of the arc $D'P'$ and hence the law of rotation of the field of view, as AH varies proportionally to the time.

If we take as the origin of time the moment when the observed star is in the reference plane, $t = 0$ for $AH - \omega = 0$, and for the unit of time the sidereal or solar day (depending upon the object observed), we have $AH\omega = 2\pi t$ and the expression for Y takes the symmetrical form

$$\tan \frac{1}{2} Y = K \tan \frac{1}{2} 2\pi t,$$

where

$$K = \frac{\cos \frac{1}{2}(\rho - \delta)}{\cos \frac{1}{2}(\rho + \delta)} \quad \text{and} \quad AH - \omega = 2\pi t.$$

It immediately follows :

a. That the rotation of the field has the same period as the diurnal motion.

b. It is continuous and always in the same direction, direct or inverse, according to the sign of K.

c. The reference plane is a plane of symmetry, for the angle Y takes equal values with contrary signs for equidistant epochs on opposite sides of the origin of time.

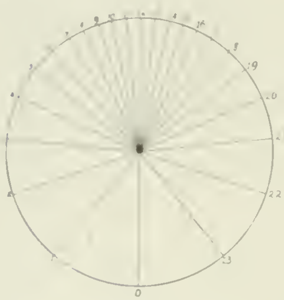


Fig. 2.

This law of rotation might be represented geometrically by a curve plotted with the time as abscissa and the angle Y as ordinate. But a more direct configuration of the rotation of the field may be obtained by considering the arc $D'P'$ as the moving radius vector of the circle described by P', the image of the pole, and by tracing the successive divisions of this radius vector at equidistant epochs, aliquot parts of a day. Fig. 2 is a representation of this character on the plane tangent to the sphere at D': the twenty-four successive positions of $D'P'$ are projected as straight lines ; they correspond to a subdivision of the day into twenty-four hours. The origin of time $t = 0$ corresponds to $D'P_0$, the projection of the reference plane and axis of symmetry.

3. *Expression for the angular velocity.*—The angular velocity of rotation at the epoch t is obtained by taking the derivative of the expression for Y with respect to t; after making the necessary reductions we obtain the formula

$$\frac{dY}{dt} = 2\pi \frac{K}{\cos^2 \pi t + K^2 \sin^2 \pi t}.$$

The denominator is necessarily positive, the velocity always

having the sign of K; it varies periodically between the value $2\pi K$, corresponding to the epochs $t = 0, 1, 2, \ldots$ and the value $\frac{2\pi}{K}$, corresponding to the intermediate epochs $t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$ passing through the value 2π, the angular velocity of the diurnal motion, at the epochs given by the condition

$$\cos^2 \pi t + K^2 \sin^2 \pi t = K$$

or

$$\tan \pi t = \frac{\pm 1}{\sqrt{1 + K}}.$$

As the positions of the star which are most favorable for observation (upper transit) are near the reference plane $t = 0$, the velocity of rotation may be considered as constant and equal to $2\pi K$, for this velocity varies but little in the neigborhood of $t = 0$, since it corresponds to a maximum or a minimum. The velocity $\frac{2\pi}{K}$ is never realized with the siderostat, with which lower transits cannot be observed.

The unit of angular velocity is evidently 2π or one circumference per day; if another unit is preferred, for example, to express the velocity in minutes of arc per minute of time or more generally in n^{ths} of a circumference per m^{ths} of a day, it is only necessary to substitute $\frac{n}{m}$ for 2π. This change of units amounts to the same thing as placing

$$\frac{Y}{2\pi} = \frac{y'}{n}, \qquad \frac{t'}{1} = \frac{\pi}{m},$$

whence

$$\frac{dy'}{d\pi} = \frac{1}{2\pi} \frac{n}{m} \frac{dY}{dt}.$$

As there are $n = 360 \times 60$ minutes of arc in the circumference and $m = 24 \times 60$ minutes of time, the velocity $2\pi K$, or $\frac{n}{m} K$, is here equal to $15 K$; we thus find $15'$ of angle per minute of time for the angular velocity of the diurnal motion $K = 1$.

4. *Direction of rotation of the field of view.*— Let us suppose that the observer is receiving the luminous beam; he thus looks

toward the center of the sphere along the radius which terminates at D', whence it results that the direction of the motion of rotation will be that which an observer placed in the direction D' outside of the sphere will attribute to the motion of the arc $D'P''$. From the expression for Y it is evident that Y and $RA - \omega$ will be of the same sign if the coefficient K is positive. The direction of the diurnal motion, *i. e.*, the direction of the positive variation of RA, is known; it is seen in the figure that when the right ascension of the star D increases the arc PD, seen from outside the sphere, turns in the direction of the hands of a watch; thus for a positive value of K, Y varies in the same manner. The condition under which K is positive is evidently

$$\cos\frac{1}{2}(\rho + \delta) > 0, \quad \frac{1}{2}(\rho + \delta) < \frac{\pi}{2}, \quad \delta < \pi - \rho.$$

Whence we conclude

When the polar distance of the observed star is less than the supplement of the polar distance of the reflected direction, the apparent direction of rotation of the field of view of the siderostat is that of the hands of a watch.

It is in the contrary direction if the polar distance of the star is less than this supplement. Observation with an astronomical telescope does not change the direction of rotation: the *reversal* of the image is confined to turning through 180° the direction of the origin $D'P_0$.

5. *Critical polar distance: fixed field of view.*—The point of transition between these two cases corresponds to the condition $K = 0$, that is $\cos\frac{1}{2}(\rho + \delta) = 0$; the value of Y remains constantly zero, whatever be the right ascension of the star. Hence

The field of view of the siderostat remains rigorously fixed when the polar distance of the observed star is equal to the supplement of the polar distance of the direction of reflection.

This case of absolute immobility of the field has a corresponding geometrical peculiarity which renders the result evident. It is in fact easily shown that if $\rho + \delta = \pi$, the arc $PM = \frac{\pi}{2}$; the normal to the mirror becomes normal to the line joining the

poles, and thus the mirror is parallel to the Earth's axis. Furthermore the arc PM bisects the angle DPD', and consequently the mirror turns through an angle equal to half the change in the hour angle. These are the two characteristic conditions of the *Coelostat* of M. Lippmann, a very simple instrument which gives an absolutely fixed image of the sky. It consists of a mirror turning about an axis parallel to its own plane and to the Earth's axis with an angular velocity equal to half that of the diurnal motion and in the same direction.

The siderostat may thus replace the coelostat for a region of the sky surrounding a star of polar distance δ. It is only necessary to send the reflected beam in a direction such as to satisfy the condition $\rho + \delta = \pi$, *i. e.*, along one of the generators of the cone of revolution which makes with the Earth's axis the angle $\pi - \delta$, the supplement of the polar distance. It is well to be acquainted with this property of the siderostat, for in certain cases it can be used without serious inconvenience.

6. *Siderostat oriented in the meridian.*—This is the most common arrangement of the siderostat: the horizontally reflected beam is directed exactly toward the south. It follows from this that $\omega = 0$, $\rho = \pi - L$, L being the latitude. The angle Y is the angle which the arc $D'P'$ makes with the meridian, which now becomes the plane of reference and of symmetry. The expression for Y takes the form

$$\tan \frac{1}{2} Y = K \tan \frac{1}{2} AH,$$

where

$$K = \frac{\sin \frac{1}{2} (L - \delta)}{\sin \frac{1}{2} (L + \delta)}.$$

The above propositions then become very simple.

With the siderostat oriented in the meridian, the field of view is fixed when the polar distance of the observed star is equal to the latitude of the place of observation. The rotation of the field is in the direction of the hands of a watch if this polar distance is less than the latitude; in the inverse direction if it is greater.

The coefficient K, which defines the practically constant value of the velocity of rotation of the field as a function of the diurnal rotation, is always less than unity.

The following numerical values are for the latitude of Paris $L = 48°\ 50'$, δ increasing by successive steps of 10° (upper transit).

δ	K	δ	K
0 (pole)	1.000	70	-0.213
10	0.677	80	-0.298
20	0.440	90	-0.376
30	0.258	100	-0.448
40	0.105	110	-0.518
48 50 L	0.000	120	-0.585
50	-0.013	130	-0.651
60	-0.120		

Heliostat.—This instrument sends the reflected beam in the direction of the northern horizon, rarely beyond N. E. or N. W.

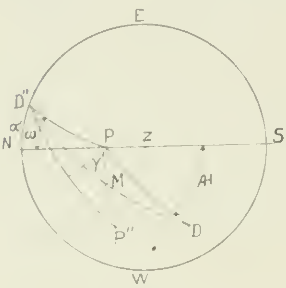


Fig. 3.

Fig. 3 represents the beam coming from the star D and sent in the horizontal direction D'', making with the north meridian an angle $NPD'' = a'$, counted positively toward the east. The pole is at P, SPD is the hour angle and δ the polar distance of the observed star D. We shall designate by $\omega' = NPD''$ and $\rho' = PD''$ the hour angle and the polar distance of the direction of reflection D'', which may be deduced as before from the azimuth a' and the latitude L by means of the right triangle NPD''. The normal M to the mirror is at the middle of the arc DD'', and the image P'' of the pole is on the arc PM produced so that $MP'' = MP$.

The angle of rotation of the field will be determined by the angle made by the arc $D''P''$ with the projection of the reference plane PD''.

Let $Y' = PD''P''$ be this angle; it might easily be deduced from the expression for Y for the siderostat, which is similarly defined. But it is simpler to calculate it directly.

The triangles PDM and $P''D''M$ are equal, as they have an equal angle at M included between two equal sides $DM = D''M$, $PM = P''M$. Thus $D''P'' = PD = \delta$.

Thus, as in the case of the siderostat, *the image of the pole reflected by the heliostat describes about the center of the field a circle with a radius equal to the polar distance of the observed star.*

Furthermore,

$$Y' = PD''P = PD''D + DD''P'' = PD''D + D''DP',$$

in consequence of the equality of the two triangles PMD and $P''MD''$. The angle Y' is thus the sum of the two angles at the base of the triangle PDD'', whose apex is at P. From Neper's formula referred to above we obtain, after substituting

$$b = \rho'', \ c = \delta, \text{ and } A = \pi - AH + \omega',$$

$$\tan \frac{1}{2} Y' = \frac{\cos \frac{1}{2} (\rho' - \delta)}{\cos \frac{1}{2} (\rho' + \delta')} \tan \frac{1}{2} (AH - \omega').$$

Y' is counted positively in the direction of the hands of a watch. This expression may also be put in the form

$$\tan \frac{1}{2} Y' = K' \tan \frac{1}{2} 2\pi t,$$

where

$$K' = \frac{\cos \frac{1}{2} (\rho' - \delta)}{\cos \frac{1}{2} (\rho' + \delta)} \quad \text{and} \quad AH - \omega' = 2\pi t.$$

Thus we again obtain the three conclusions (a), (b), (c), demonstrated above for the siderostat. It is unnecessary to repeat the discussion, which would be quite similar; but we wish to call special attention to the practical points of difference between the two instruments. With the heliostat the coefficient K' is always greater than unity and retains the positive sign under the conditions in which the heliostat is ordinarily employed, *i. e.*, in the observation of the upper transit of heavenly bodies near the zenith or the equator, reflected in a

direction which does not greatly differ from that of the northern horizon.

By developing the value of the cosine we may write K' in the form

$$K' = \frac{1 + \tan\frac{1}{2}\rho' \tan\frac{1}{2}\delta}{1 - \tan\frac{1}{2}\rho' \tan\frac{1}{2}\delta}.$$

The + sign of the coefficient K' here corresponds, as may be seen in the figure, to a variation in V' contrary in direction to that of the hour angle AH. Hence we conclude that

The field of view of the heliostat, under the actual conditions of observation, turns with an angular velocity which is always greater than that of the diurnal motion; the direction of rotation is that of the hands of a watch.

This conclusion puts in evidence a new cause for the inferiority of the heliostat as compared with the siderostat. To the inconvenience arising from the reflection at great angles of incidence from the mirror of the heliostat must be added that of a great velocity of rotation of the field of view. These two conditions are unfavorable for observations which require in the images both great perfection and complete stability. On account of this fact the siderostat is to be preferred for astronomy of precision.

But this rapidity of rotation of the field is not always an inconvenience; for certain astrophysical observations it is on the contrary advantageous, in that it dispenses with the necessity of employing complex and delicate optical arrangements; here is an example.

Suppose we project, with the aid of a suitable objective, the solar image reflected by a heliostat on to the slit of a spectroscope of high dispersion for the purpose of studying the displacement of lines due to the motions of the solar surface. The most favorable condition occurs when the solar equator is normal to the slit; if the image is oscillated in such a way as to make the opposite limbs of the disk successively tangential to the slit, one may obtain twice the maximum displacement due to

the difference of the radial velocities at the equator (method of oscillating lines).

Except under unusual circumstances the image of the solar disk will not be found in this favorable azimuth and will have little chance of attaining it if a siderostat is employed, since with this apparatus the velocity of rotation of the field of view is zero or very small.

In order to bring the equator to the required azimuth it is necessary to make use of an auxiliary apparatus consisting, for example, of an isosceles total reflection prism, movable about an axis parallel to its base; rotation of this prism changes the azimuth of the Sun's disk by twice the angle, which permits the equator to be placed perpendicular to the slit in the two successive positions 180° apart, which give the maximum double displacements in the inverse order. But the prism must be very perfect both as to material and flatness of the surfaces. Moreover, the rotating mounting which carries it is rather difficult to construct and adjust.

With the heliostat, the natural rotation of the field of view renders this auxiliary apparatus unnecessary; it is sufficient to await the effect of this rotation and to allow the solar equator to place itself perpendicular to the slit. At certain times of the year and for certain orientations of the slit and of the beam reflected by the heliostat, this condition of perpendicularity occurs twice in the same day within an interval of a few hours, the image of the solar equator turning through 180°.

This result, which I discovered experimentally and observed on several occasions, greatly surprised me at first; I had supposed that about twelve hours would be required for the reflected image of the solar disk to turn 180° about its center. The search for an explanation of this phenomenon led to the preparation of this paper. The complete discussion would require rather extended developments. I shall confine myself here to indicating the principle of the demonstrations.

The explanation is based upon the relative velocity of the field of the heliostat when the observed star is near the equator

($\delta = 90^\circ$). The coefficient K', which measures it as a function of the diurnal rotation, varies between 2 and 5 for positions of the Sun between the two solstices.

The accompanying table gives the values of K' in the usual case where the heliostat is oriented in the meridian, the reflected beam being directed horizontally toward the north; we substitute in the formula $\omega' = 0$, $\rho' = L = 48^\circ\ 50'$, δ increasing by steps of 10° (upper transits).

δ	K	δ	K
0 (pole) - -	1.000	70 - - - -	1.932
10 - - -	1.083	80 - - -	2.231
20 - -	1.174	90 (equinox) - -	2.663
30 - - -	1.277	100 - - -	3.358
40 - - - -	1.396	113 37 (winter solstice)	5.489
50 - - -	1.537	110 - - -	4.687
60 - - - - -	1.710	120 - - - -	8.359
66 38 (summer solstice)	1.849		

The value of K' approaches infinity, which it attains when the star is at the southern horizon; this is in fact a critical polar distance $\rho' + \delta = \pi$, which corresponds to grazing incidence at the mirror.

If we know the value of K' we can calculate the time which elapses between the epochs t_1 and t_2, between which the image of the field has turned through 180°. Let Y_1' be the value of the angle Y' at the epoch t_1, when the solar equator, for example, is normal to the slit of the spectroscope, and $Y_2' = Y_1' + \pi$, the value of Y_1' increased by 180° at the epoch t_2. We shall then have the two conditions

$$\tan \frac{1}{2} Y_1' = -K' \tan \pi t_1 \quad \cot \frac{1}{2} Y_2' = K' \tan \pi t_2.$$

Multiplying member by member we finally obtain

$$\tan \pi t_1 \tan \pi t_2 = -\frac{1}{K'^2}.$$

The minus sign shows that the two epochs t_1 and t_2 (supposed to be as near together as possible) are of contrary sign, which signifies that the two corresponding positions of the star are situated on opposite sides of the reference plane (in this

case the plane of the meridian); it is necessary to except the limiting cases for which $t=0$ and $t=\frac{1}{2}$. Let $\theta=t_2-t_1$ be the difference between the two epochs; if t_1 is given t_2 may be calculated. It is especially interesting to find the two epochs for which this difference is a minimum.

Let us therefore make $d\theta=0$, that is, $dt_2-dt_1=0$, and let us differentiate the expression which connects t_1 and t_2; we obtain, after completing the operation,

$$\sin \pi (t_2 + t_1) \cos \pi (t_2 - t_1) = 0 .$$

This is the solution $t_2 + t_1 = 0$ which gives the desired minimum; the other, $t_2 - t_1 = \frac{1}{2}$, gives the 12-hour maximum, which is of no interest.

The two desired epochs symmetrical with respect to $t=0$ are of equal length and contrary sign; substituting, in order to obtain their absolute values, we have

$$\tan \pi t = \frac{1}{K'} .$$

If we give K' increasing values starting from $K'=1$ (uniform rotation), which gives $t=\frac{1}{4}$ of a day, or 6 hours, and $t_2-t_1=$ 12 hours, we find that the interval $\theta=t_2-t_1$ grows smaller and smaller. Making the calculation to determine this difference θ for the three most interesting epochs relating to the Sun we obtain

	θ
Summer solstice - - - - - - -	$7^h\ 54^m$
Equinox - - - - - - - -	5 30
Winter solstice - - - - - - -	2 45

results which demonstrate the possibility of seeing the solar equator turn through 180° in much less than 12 hours.

It is, however, unnecessary that the rotation should be exactly 180° in order to show successively the two inverse effects of the oscillating lines, for the absolute velocity of the solar parallels diminishes only $\frac{1}{10}$ up to ± 25° heliocentric latitude, so that a displacement of 180° — 50° = 130° is sufficient to show the double phenomenon in the clearest manner.

It remains to determine the orientations of the reflected beam which are most favorable for observation; but this problem is rather complex and deserves to be treated separately.

What precedes is sufficient to show that even in those peculiarities of instruments which at first sight appear to be unfortunate imperfections, possibilities exist which may be turned to account in other classes of work. The complete study of the geometrical properties of instruments commonly reveals some peculiarity capable of rendering unexpected services.

PARIS,
January 1900.

Minor Contributions and Notes

ON THE PROBABLE ORIGIN OF SOME OF THE LINES OBSERVED IN THE SPECTRA OF STARS AND OF THE CHROMOSPHERE.

Sir J. N. Lockyer, in his "Photographic Spectra of Some of the Brighter Stars" (*Phil. Trans.*, **184**, 677, 1893), gives a line λ 4172 ,as one which occurs in γ *Cygni*, in *Bellatrix* γ *Orionis*, and also in α *Orionis*. In the spectrum of this latter star (*loc. cit.*, Plate 28), 4172 is represented as being an iron line observed in the flame spectrum of iron.

The oxhydrogen[1] flame spectrum of iron has been photographed as it is emitted from very pure metallic iron and by a large number of compounds, including such as yield metallic iron in the highest degree of purity, and with various degrees of dispersion, inclusive of that obtained with a Rowland grating of 21.5 feet radius, belonging to the Royal University of Ireland, but in no instance have photographs been obtained showing an iron line at or sufficiently near to λ 4172 to account for the existence of a line to which this wave-length could be attributed.

As, however, the flame spectra usually observed in the working of the basic Bessemer process, or obtained from meteorites, samples of commercial iron, about seventy iron ores from various localities and of different kinds, and from various rocks and minerals, have been found to show the *gallium line* with a wave-length according to recent measurements of 4172.214 (Hartley and Ramage), it appears most probable, considering the volatility of the metal, that the line observed was that of gallium, and that it is to this the wave-length 4172 has been assigned. The recognition of gallium in terrestrial matter and in the solar spectrum has been confirmed by Mr. Lewis E. Jewell (this Journal, **9**, 229, 1899) by an examination of Professor Rowland's photographs of aluminium, lead, silver, the solar spectrum, and the spectra

[1] Hartley, "Flame Spectra at High Temperatures," *Phil. Trans.*, **185**, 161, 1894; this Journal, **9**, 214 and 221, Hartley and Ramage.

of meteorites, and the wave-lengths of the gallium lines have been determined to be 4172.211 and 4033.224 *Fe-Mn* (Rowland). The ends of the gallium lines on the photographs of the solar spectrum, taken by Mr. Ramage and myself, were observed to be quite distinct from these, but in the center, where the lines are thicker and of greater intensity, this fact was not noticed, though a shorter exposure or a narrower slit would have made it evident. The relative intensities of the gallium lines were the same on all the photographs, whether taken from the oxhydrogen flame, the arc (bright and reversed), or spark spectra; and they are fairly represented by 1 and 00 on Rowland's scale. In the spectrum of α *Orionis* there is a group of four lines attributed to manganese, lying between 4030 and 4034. There is a manganese line given by Rowland as occurring in the solar spectrum at wave-length 4033.224, attributed both to iron and manganese; the second or weaker line of gallium lies at 4033.112. The latter is so frequently present in iron and iron ores, along with the stronger and less refrangible line, that unless its absence is proved its presence must be suspected. But the fact is to be particularly noted that in cases where the manganese was strong this gallium line was obscured by one of the broader lines of manganese, probably that indicated above. Finally, attention may be drawn to the fact that the line 4172 has been recorded as occurring in the following stars:

α *Cygni*	Intensity 4, where the maximum is 6.
β *Orionis*	Intensity 1, where the maximum is 6.
β *Tauri*	
67 *Ophiuchi*	
δ *Cygni*	
γ *Orionis*	
β *Canis majoris*	
γ *Cygni*	
α *Orionis*	

It appears therefore very probable that in α *Orionis* the line 4172 and one of those lying between 4030 and 4040 are gallium lines; and that the less refrangible of the gallium lines also occurs in the other eight stars mentioned above. The *Phil. Trans.*, 187, 606, 1896, contains an account of the comparison of the spectrum of the chromosphere with the Fraunhofer lines between wave-length 410 and 430, being a preliminary investigation of the results obtained by observations of the total eclipse of the Sun of April 16, 1893. Sir J. N.

Lockyer there gives the wave-lengths of the two lines in the chromosphere and prominences as 4172.2 and 4033.22 (see pp. 612 and 613), with their intensities at different heights; the maximum number representing the intensity is 10:

λ 4172.2						
Photographs	(7)	(8)	(21)	(22)		
Miles	1660	2000	1650	Base		
Intensity	1	3	1	5		
λ 4033.22						
Photographs	(7)	(8)	(9)	(10)	(21)	(22)
Miles	1660	2000	3000 to 3800	5480 to 9600	1650	Base
Intensity	1	1	1	1	3	3

Another line is given, λ 4171.9, taken from the African photograph No. 20. Its intensity in the chromosphere arc is 1, at the base of the chromsphere, 3. This is probably identical in its origin with λ 4172.2, and it appears in the highest degree probable that 4172.2 and 4033.22 are the lines of gallium corresponding to the solar lines 4172.211 and 4033.112 in Rowland's revised table of wave-lengths in this region (this JOURNAL, p. 229, 1899).

This metal being very easily volatilized, it is quite conceivable how its vapor may be separated from that of iron by a process resembling fractional distillation, the more volatile metal being carried to a greater elevation.

W. N. HARTLEY.

MME. CERASKI'S SECOND *ALGOL* VARIABLE.[1]

ANOTHER remarkable variable star of the *Algol* class has been discovered by Mme. Ceraski, and is announced in the *A. N.*, **151**, 223. The position for 1900 is R. A. $= 19^{h}\ 42^{m}.7$, Dec. $= +\ 32^{\circ}\ 28'$. From an examination of the Draper Memorial photographs of this star, it appears that while the star has its full brightness on 45 of them, on several of the early photographs it is so faint that they must have been taken when the star was near minimum. The Moscow photographs furnish the means of determining the period from an interval of four years, the Harvard photographs increase this interval to nine years.

[1] *Harvard College Observatory Circular* No. 47.

The following table gives, in the first seven lines, the results derived from the Harvard photographs; the next four, the results of the Moscow photographs; and the last line gives the estimate of M. Blajko. The times of minima as found by Professor Ceraski may be expressed by the formula *J. D.* 2,415,004.971 + $6^d.0065$ *E*. Measures of four Harvard photographs when the star had its full brightness gave the photographic magnitudes, 11.00, 10.80, 10.74, and 10.79; mean 10.83. The value of *E*, derived from the above formula; the year, month, and day; the Greenwich mean time of the middle of the exposure; the corresponding time expressed in Julian days and decimals; and the duration of the exposure in minutes are given in the first five columns of the table. The sixth column gives the photographic magnitude, and the seventh, the phase computed by means of the formula mentioned above. The error in the ephemeris is given in the eighth column, and is derived from an approximate light curve. It appears that the period is too long by about $0^m.6$, and if this correction is applied, the errors have the values given in the ninth column. The tenth column gives the mean photographic magnitude, during the entire time of exposure, derived from the corrected ephemeris and light curve.

E	Date	G. M. T.	J. D	Ex.	Mag.	Phase	O-C	O-C1	C. M.
	y m d	h m		m					
−580	1890 6 2	16 32	1521.689	27	11.81	+.488	+.25	+.01	11.7
−570	1890 8 1	14 20	1581.597	13	12.75	+.331	+.24	+.02	12.7
−570	1890 8 1	14 57	1581.623	20	12.22	+.357	+.16	−.06	12.5
−557	1890 10 8	11 53	1659.495	10	<12.4	+.145	.....	.....	12.8
−508	1891 8 8	16 07	1953.672	20	<11.7	+.003	.....	.....	12.1
−443	1892 9 2	14 44	2344.614	16	10.96	+.522	+.18	.00	11.0
−325	1894 8 11	15 07	3052.630	10	10.90	−.228	+.13	−.01	10.9
−257	1895 9 24	10 11	3461.424	102	<12	+.124	.....	.....	12.9
−254	1895 10 12	6 45	3479.281	300	<12	−.039	.....	.....	12.5
−97	1898 5 12	10 12	4422.425	105	<12	+.085	.....	.....	12.9
−84	1898 7 29	10 22	4500.432	125	<12	+.008	.....	.....	12.9
−0	1899 12 16	3 33	5005.106	125	Ft.	+.135	.....	.....	12.5

It appears from this table that while the formula of Professor Ceraski satisfies all the later observations, it is not confirmed by the early observations. For instance, according to this formula the star should have had nearly its full brightness on the first three photographs. On the other hand, all the observations are satisfied by the corrected formula, in which the period is $6^d\ 0^h\ 8^m.8$ As soon as we

obtain accurate observations of subsequent minima, these combined with the photographs taken in 1890, will give a much more precise formula. A comparison of the sixth and tenth columns shows that the observed and computed magnitudes differ in one case only by more than one tenth. A slight defect partially covers the image of the variable on the second plate taken August 1, 1890, and thus renders the measured value too bright. The period differs so little from exactly six days that for a long time the minima cannot be observed in certain longitudes. Accordingly, while valuable observations may be obtained next autumn in Europe, or better still in Asia, minima cannot be observed in America until the following year.

Five stars of the *Algol* class, S *Cancri*, U *Cephei*, W *Delphini*, $+45^\circ 3062$, and the star here discussed, are especially interesting, owing to the large variation in their light, which amounts to about two magnitudes in each case. It is remarkable that two of these were found by Mme. Ceraski, and one by her distinguished husband.

EDWARD C. PICKERING.

February 12, 1900.

ON THE PROBABLE ERROR OF A RADIAL VELOCITY-DETERMINATION.

THE practice of following the radial velocity of a star by what purports to be its probable error, computed from the measures of only one plate, seems to me to be incorrect; and, in the absence of special interpretation, misleading. Such a determination of probable error is based on only a small part of the observation. It assumes that the plate is perfect: containing no injurious effects of temperature variation, flexure, jarring, imperfect adjustments, etc. Save for peculiarities in the distribution of the silver grains, it is affected by little beyond the errors in the assumed intervals between the lines, and by accidental errors in the micrometer bisections of the lines.

A meridian circle observer, in determining the right ascension of a star, notes its times of transit over a number of lines in the reticle. From these and the assumed intervals between the lines, he can compute the probable error of the mean of the observed times of transit. If x is this probable error, it would be incorrect to say that the observed right ascension of the star is

$$11^h\ 12^m\ 13^s.14 \pm x,$$

since this would disregard many important factors in the complete observation. Yet such a practice would be fairly analogous to that discussed above for the spectrograph.

It seems to me that the probable error of a single velocity-determination should be derived from the results of several complete observations, unless it has been shown by long experience that the probable error yielded by the measures of one plate is in fair agreement with the value obtained from a comparison of the results from several plates. The substantial agreement of the values obtained by the two methods would indicate: first, that the plates were free from the accidental effects of flexure, temperature changes, etc.; and second, that the observer's habit of measurement remained constant. The first of these desiderata has, I believe, been practically attained, for the best stars, with the Mills spectrograph. The second is, from its nature, more difficult to control, and necessitates frequent checks.

W. W. CAMPBELL.

LICK OBSERVATORY,
January 15, 1900.

CATHERINE WOLFE BRUCE.

ASTRONOMERS the world over will learn with deep regret of the death of Miss Catherine Wolfe Bruce, news of which reaches us just as we go to press. For years confined to her room by an ever increasing illness, and personally known by but few of the many who have benefited by her bounty, Miss Bruce has nevertheless endeared herself to men of science at home and abroad, aiding as perhaps no other has done the progress of research. Recognizing no national boundaries, giving assistance where it was most needed, and seeking no fame for herself, Miss Bruce may well be regarded as one of the most sympathetic and generous patrons astronomy has ever known. Many a project, which without her assistance would have come to naught, has been successfully developed through her aid. Many an advance in our knowledge of the heavens is due directly to the help she gave. The appeals that came to her from far and wide were received with the kindliest consideration, both by herself and also by her sister, Miss M. W. Bruce, who often acted in her stead. Astronomers in almost every country of the civilized world know from their own experience how prompt was

the response, and how often it took the affirmative form. In common with many others who have received less direct advantage from her gifts to science, they will sincerely mourn her loss.

ERRATA.

Vol. VIII, p. 221, in the formula, *for* 404.251 *read* 464.251.

Vol. X, p. 189, in the formula, for $R_0 - R$ read $R - R_0$.

Vol. X, p. 277, *for* 0.0043053 *read* 0.0043083; *for* 4325.57 *read* 4235.27. In formula (1) *for* $[\lambda_3]$ *read* $[\lambda_3 - \lambda_0]$.

Vol. X, p. 278, *for* 4425.52 *read* 4442.52; *for* 4425.36 *read* 4442.36.

Vol. X, p. 350. The entire passage in Professor Hasselberg's article, beginning "It appears, first, that numerous exact coincidences" etc., on p. 350, and ending ". . . . this is also independent of the titanium line," on p. 351, should immediately follow the table ending at the bottom of p. 357.

Reviews

PROFESSOR J. J. THOMSON'S WORK ON THE STRUCTURE OF THE ATOM.

1. *The Discharge of Electricity Through Gases.* Princeton Lectures. J. J. Thomson. (Scribner's, 1898.)

2. *Cathode Rays.* J. J. Thomson. *Phil. Mag.*, 44, 293–316, 1897.

3. *On the Charge of Electricity carried by the Ions produced by Röntgen Rays.* J. J. Thomson. *Phil. Mag.*, 46, 528–545, 1898.

4. *On the Masses of the Ions in Gases at Low Pressures.* J. J. Thomson. *Phil. Mag.*, 48, 547–567, 1899.

Those who listened to the lectures of Professor J. J. Thomson at Princeton in the autumn of 1896 will recall the surprise — not to say astonishment — with which was received his clearly stated experimental evidence for thinking that the carriers of electrification, through gases at ordinary pressures, are aggregations of atoms, large compared with a molecule of the gas, so large, indeed, that they can be filtered out with glass wool.

In striking contrast with this stands the principal result of his investigations during the three years that have elapsed since the Princeton Sesqui-centennial, viz., that in gaseous discharges at low pressures the negative electrification is carried by particles of matter — "corpuscles" he calls them — whose mass is of the order of one thousandth of the mass of a hydrogen atom.

Not for many years — perhaps never — has anyone doubted the divisibility of the chemical atom; but the experimental evidence offered by Thomson is so convincing as compared with the arguments of Prout, or as compared with any *a priori* considerations, that his work becomes of the highest interest to all students of physical science, especially those of chemistry and spectroscopy.

The general method of attack, which may be roughly described as *a quantitative study of the charged ion*, is outlined in his paper on "Cathode Rays." The ratio $\frac{m}{e}$, where m is the mass and e the charge of an ion, is a well-established electrolytic constant, an invariant for

any one conducting solution. Indeed, the constancy of this ratio may be considered as a mathematical expression for one of Faraday's Laws of Electrolysis. For hydrogen this ratio has the value

$$\frac{m}{e} = 0.0001035 = 10^{-4} \text{ approximately}.$$

But within the last few years this ratio has been determined for conducting, or "ionized," gases. For this determination Professor Thomson has employed three independent methods, to which may be added a fourth, due to Lorentz. These are briefly and roughly as follows:

1. The deflection of cathode rays by means of a magnetic field.

Here $\frac{m}{e} = \frac{I^2 Q}{2W}$

where
- I is the intensity of a magnetic field multiplied by a radius of curvature,
- Q is a quantity of electrification measured by an electrometer, and
- W is a quantity of heat energy measured by aid of a thermopile, balances, etc.

2. The deflection of cathode rays by means of an electrostatic field.

Here $\frac{m}{e} = \frac{H^2 l}{F\theta}$

where
- H denotes the strength of a magnetic field,
- F, the intensity of the electrostatic field,
- l, a measurable distance, and
- θ, the deflection produced by the field F.

3. The effect of a magnetic field in diminishing the gaseous conduction between electrodes separated by more than a certain critical distance, an effect discovered by Elster and Geitel.

Here $\frac{m}{e} = \frac{H^2 d^2}{2V}$

where
- H is the intensity of a magnetic field,
- d, the critical distance, determined by experiment, and
- V, the potential difference between the electrodes.

4. H. A. Lorentz has shown (*Phil. Mag.*, 43, 226, 1897) that from the Zeeman effect we have

$$\frac{m}{e} = \frac{H(\lambda_1 - \lambda)}{4\pi V}$$

where $\left\{ \begin{array}{l} H \text{ is the intensity of the magnetic field producing the Zeeman effect,} \\ \lambda_1 - \lambda, \text{ the change in wave-length produced on any ray by the magnetic field, and} \\ V, \text{ the speed of light.} \end{array} \right.$

Very striking is the general agreement among the various values of $\frac{m}{e}$ obtained by these diverse methods. Whether employed by Lenard (*Wied. Ann.*, 64, 279, 1898), and Kaufmann (*Wied. Ann.*, 61, 544, 1897 ; 65, 432, 1898) on the continent, or used with many variations by Thomson at Cambridge, or made to depend upon the Zeeman effect, all these methods lead to a value approximately 1.0×10^{-7}. While the values of this ratio vary, roughly speaking, to the extent of 25 per cent. on either side the mean, and while these differences will doubtless lead to future investigation and to new and interesting results, we may, for the present, say that they are in practical agreement. And what is quite as remarkable is the fact that the value of $\frac{m}{e}$ appears to be entirely independent of the gas employed, a result in marked contrast to that obtained in the case of liquid electrolytes, where $\frac{m}{e}$ varies from one element to another directly as the atomic weight and inversely as the valency.

Observing that in the case of liquids, this ratio is nearly one thousand times larger than in the case of gases at low pressures, Thomson at once set himself the problem of determining whether this discrepancy arises from the fact that the *mass* of the liquid ion is *greater* than that of the gaseous ion, or from the fact that the *charge* of the liquid ion is *smaller* than that of the gaseous ion, or from both facts. As the sequel shows the first of these alternatives is confirmed by experiment. For Thomson succeeded in measuring in C. G. S. units the charge of electricity carried by a negative ion in gas ionized by Röntgen rays (*Phil. Mag.*, December 1898).

The highly original method and carefully executed experiments which led him to his final results, namely,

$$e = 7.3 \times 10^{-10} \text{ electrostatic units}$$

must be followed in the author's own paper.

For space will here permit us merely to say that, by using the negative ions as centers of condensation for water vapor, he determined the number of ions in a cubic centimeter of the gas, and then measuring what practically amounts to the total charge of a cubic centimeter of the gas, he at once obtains the value of e by division. But let no one imagine that these processes, so scantily described, are simple; on the contrary, they require manipulative skill of a high order, and depend upon important results previously obtained by Rutherford, Wilson, and others.

Knowing e and $\frac{m}{e}$ for a gas, we have at once the value of m for that gas. But we have, as yet, no means of getting m for a liquid electrolyte. The one step necessary was supplied by Townsend (*Proc. Roy. Soc.*, **65**, 192, 1899), who showed, by a study of the diffusion of ions into gases, that the charge on an ion produced by Röntgen rays is the same as that carried by the hydrogen atom in ordinary liquid electrolysis. Since the e's are the same in each case, it follows that the divergence in the values of $\frac{m}{e}$ between hydrogen gas and hydrogen in a liquid electrolyte, say H_2SO_4, is due to the fact that the mass of the gaseous ion is about one thousand times smaller than the mass of the hydrogen atom.

The demise of the atom has been long expected; only the hypothetical atom has been regarded as indivisible. But it is doubtful whether many outside the Cavendish laboratory expected the division to come in this way. The outlook along spectroscopic lines appeared more hopeful, especially in the direction of Michelson's powerful separation of so-called homogeneous radiations. The Echelon spectroscope and analysis make a powerful combination and one which probably has much to tell us concerning that complicated dynamical system which gives us the series of Kayser and Runge and which has so long been miscalled an "atom."

Since the mass of Thomson's "corpuscle" appears to be the same for all gases, it would seem that we are here confronted by a veritable *Urstoff*. But the most disappointing feature about it all is that these corpuscles cannot be collected in quantities large enough for one to study their chemical properties. Nevertheless, the work under review is full of suggestion for the chemist. For instance, the fact that practically all lines in the spectrum of an element exhibit the Zeeman effect leads to

the inference that in each atom there are *many* of these corpuscles. While, from the manner in which Lenard rays are absorbed, it is evident that the mean free path of these ions depends only upon the density of the medium, and not at all upon its chemical composition. In facts such as these Thomson finds his warrant for saying that "atoms of ordinary elements are made up of corpuscles and holes, the holes being predominant" (*Princeton Lectures*, p. 198).

Among all chemical problems, it would not be easy to find one which is more fundamental, more profoundly interesting, or more hopelessly difficult, than the dynamical explanation of the periodic law. Unusual interest, therefore, attaches to the slight hint which Thomson has given as to a possible manner in which atoms may be built up out of corpuscles so as to satisfy the demands of Mendelejeff's table. Briefly, the arrangement of the corpuscles in the atom is that of Mayer's groups of magnets floating in a magnetic field. If, in addition, we imagine that certain chemical properties are functions of the number of magnets (corpuscles) in the center of the group, we have a system for which these chemical properties, at least, are periodic functions of the atomic weight. Such an atom, it is pointed out, is not inconsistent with what we know of spectral series; it furnishes a possible mode of conceiving electric conduction in gases, it offers a most plausible explanation of the law of absorption for Lenard rays, and, finally, it brings comfort to those who still regret that Prout's hypothesis — in its original form — is untenable.

Your reviewer's conscience is not quite at rest until he has warned any chance reader not to infer from the above that the director of the Cavendish laboratory has, for a moment, forgotten to draw a sharp line between the results of experiment and the results of imagination — even of the "scientific imagination." Any such impression is the fault of the reviewer, and should be removed at once by a careful reading of the original. For, doubtless, no one has sought so ably as Professor Thomson, or would welcome more warmly than he, any explanation of these phenomena involving an assumption less violent than that of a corpuscle with a mass one thousand times smaller than that of the hydrogen atom.

Henry Crew.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XI APRIL, 1900 NUMBER 3

CALCULATION OF ELLIPTIC ELEMENTS OF THE SYSTEM OF *Y CYGNI*.

By N. C. Dunér.

In spite of the very bad weather that prevailed almost continuously in the autumn months, Dr. Bergstrand and I succeeded in observing the following minima of *Y Cygni* in the year 1898:

Epoch	G. M. T.	Observer
	d h m	
2774	1898 April 26 12 5.9	D.
2776	April 29 11 39.4	D.
2854	Aug. 24 8 31.6	D.
2861	Sept. 3 13 30.4	D.
2895	Oct. 24 12 23.6	B.
2902	Nov. 4 5 44.5	D.
2902	Nov. 4 6 5.7	B.
2906	Nov. 10 6 1.7	B.
2907	Nov. 11 11 56.7	B.

On comparing these observations with the ephemeris which I communicated in the *Vierteljahrschrift der Astronomischen Gesellschaft* for 1897, I found the following differences between observation and prediction:

EVEN EPOCHS

Epoch	O—C
	h m
2774	—0 53.1
2776	—1 14.3
2854	—0 54.8
2902	—1 21.2
2902	—1 42.3
2906	—1 15.7
Mean 2852	—1 13.6

ODD EPOCHS

Epoch	C—O
	h m
2861	+1 0.2
2895	+1 22.6
2907	+1 26.4
Mean 2887	+1 16.4

From these the following two normal minima were derived:

Epoch = 2852; minimum = $1886.0 + 4616^d.349$; $O—C = —0^d.052$

Epoch = 2887; minimum = $1886.0 + 4668^d.529$; $O—C = +0^d.053$

The considerable deviations of the earlier formulae, which are negative for the even normal minimum and positive for the odd minimum, and are numerically almost precisely equal, show that the observations can no longer be satisfied by formulae which include only the first power of the epoch number. I have therefore derived the following formulae by the method of least squares:

$$\left.\begin{aligned} \text{Even minima} &= 1886.0 + 343^d.4670 + 1^d.498276\,E \\ &\qquad - 0^d.000\,000\,0255\,E^2 \\ \text{Odd minima} &= 1886.0 + 343^d.4131 + 1^d.498076\,E \\ &\qquad + 0^d.000\,000\,0190\,E^2 \end{aligned}\right\} \quad (1)$$

The coefficients of E^2 in the two formulae have therefore almost the same numerical value but opposite signs, as was to be expected.

In my earlier essays on this star[1] I have expressed the opinion, now probably generally adopted, that the system of *Y Cygni* consists of two equally large and bright components whose mutual occultations produce the light variation of the stars. At the same time I have, however, also asserted that the line of apsides of the orbits of the two stars turns in the plane of the orbit in such a way that the angle between these two lines has constantly been increasing since the discovery of the variability of the star by Chandler in 1886, when the line of apsides

[1] For instance, *Sur les éléments de l'étoile variable Y Cygni*, Stockholm, 1892.

nearly coincided with the line of sight. The last terms in the equations (1) prove that this angle was in 1886 a straight line and now exceeds 90°, so that the line of apsides now approaches the line of sight. Since the last term should evidently have one and the same coefficient in the two formulae (1), I have taken the mean of them, adopting therefore for these terms the values $-0^d.000\,000\,022\,E^2$ and $+0^d.000\,000\,022\,E^2$. After introducing these values in the equations of condition I again solved and thus obtained:

$$\left.\begin{array}{l} \text{Even minima} = 1886.0 + 343^d.4686 + 1^d.498267E \\ \qquad\qquad\qquad\qquad\qquad\qquad - 0.000\,000\,022\,E^2 \\ \text{Odd minima} = 1886.0 + 343^d.4175 + 1^d.498068\,E \\ \qquad\qquad\qquad\qquad\qquad\qquad + 0^d.000\,000\,022\,E^2 \end{array}\right\} \quad (2)$$

Direct comparison with the normal minima results as follows:

EVEN MINIMA		ODD MINIMA	
Epoch	O—C	Epoch	O—C
12	$-0^d.008$	191	$-0^d.013$
418	+0 .014	705	+0 .014
1184	—0 .021	1187	+0 .007
1640	+0 .019	1311	+0 .007
2208	—0 .006	1913	—0 .010
2858	+0 .003	2371	—0 .012
		2887	+0 .006

The agreement between the formulae and the normal minima is therefore as good as could be asked for. Nevertheless it is clearly possible that this year's minima will no longer agree well, for formulae are not the natural expression of these phenomena and the more terms of higher order that we employ, so much the sooner will terms of a yet higher order be necessary, until finally the formulae are wholly insufficient for furnishing even a rough approximation.

I prefer, therefore, to adopt now the only rational method and to calculate the actual orbital elements. It is obvious at the beginning that a perturbation term, namely the revolution of the line of apsides in the plane of the orbit, must be taken into account in the computation. On the other hand it is equally

obvious that the longitude of the node and the inclination of the orbit plane to the line of sight disappear: the latter must be regarded as zero, and the former has no effect on the light variation of the star and can therefore never be determined.

In order to find a method of determining the orbit I shall show first how the epochs of minima can be computed when the orbital elements are known. It has already been stated that the line of apsides coincided with the line of sight a short time before the first observed minimum. Whether the even or the odd minima then occurred near the perihelion passage, it is quite impossible to decide, and for the present research is not of over much importance. We shall assume, however, that in 1886 the even minima occurred near perihelion, since according to Chandler's observations in 1886 and 1887 there is some probability that the occultations were of longer duration in 1887 than in 1886.

Let Σ be the sidereal period of the star and μ be its mean daily sidereal motion, while U denotes the anomalistic period, and n the mean daily anomalistic motion. Further, let A be the number of tropical years in which the line of apsides turns through 360°. Then $\frac{365.24222}{\Sigma} A$ and $\frac{365.24222}{U} A$ are respectively the number of sidereal and anomalistic revolutions which will occur in A years. The former number is evidently greater by one unit than the latter, and consequently

$$\frac{A}{\Sigma} - 1 = \frac{A}{U}, \quad \text{or} \quad \frac{A - \Sigma}{\Sigma} = \frac{A}{U},$$

whence

$$U = \frac{A}{A - \Sigma} \Sigma . \tag{3}$$

This equation gives us U when Σ and A are known. Further, let 2ω be the angle which the line of apsides turns in the same direction as the stars during one sidereal revolution, t_0 be the time when the line of apsides coincided with the line of sight, e the eccentricity of the orbit, ϕ the angle of eccentricity, and let M, E, v, r, and a have the ordinary significance, as in Gauss'

Theoria Motus, or in Oppolzer's *Lehrbuch zur Bahnbestimmung der Kometen und Planeten.*

At the time t_0, $v = 0$; but at a minimum occurring at the time t, the angle between the line of apsides and the line of sight was equal to $\frac{t - t_0}{\Sigma} \cdot 2\omega$, and the true anomaly at that time was equal to $-\frac{t - t_0}{\Sigma} \cdot 2\omega = -2ma$. But we have

$$dM = \frac{r^2}{a^2 \cos\phi} dv = \frac{a^2 \cos^4\phi}{a^2 \cos\phi (1 + e\cos v)^2} dv = \frac{\cos^3\phi}{(1 + e\cos v)^2} dv\,.$$

In order to determine M from this, the equation must be integrated between the limits 0 and $-2m\omega$, or, what is the same thing, between $2m\omega$ and 0. Consequently

$$M = \cos^3\phi \int_{2m\omega}^{0} \frac{dv}{(1 + e\cos v)^2}\,. \tag{4}$$

But we have

$$\int \frac{dx}{(a + b\cos x)^n} = \frac{A \sin x}{(a + b\cos x)^{n-1}} + \int \frac{B + C\cos x}{(a + b\cos x)^{n-1}} dx\,,$$

where

$$A = \frac{1}{n-1} \cdot \frac{b}{b^2 - a^2}\,, \quad B = \frac{a}{a^2 - b^2}\,, \quad C = \frac{n-2}{n-1} \cdot \frac{b}{b^2 - a^2}\,,$$

whence

$$\int \frac{dv}{(1 + e\cos v)^2} = \frac{e}{e^2 - 1} \cdot \frac{\sin v}{1 + e\cos v} + \frac{1}{1 - e^2} \int \frac{dv}{1 + e\cos v}\,. \tag{5}$$

We have, further, (for $b < a$),

$$\int \frac{dx}{a + b\cos x} = \frac{2}{a \sin\beta} \cdot \text{arc tan} \left[\tan\frac{1}{2}\beta \tan\frac{1}{2}x\right]\,,$$

where

$$b = a\cos\beta\,, \text{ or } \cos\beta = \frac{b}{a}\,;$$

therefore,

$$\int \frac{dv}{1 + e\cos v} = \frac{2}{\sqrt{1 - e^2}} \cdot \text{arc tan} \left[\sqrt{\frac{1 - e}{1 + e}} \tan\frac{1}{2}v\right]\,, \tag{6}$$

and by introducing the values from (5) and (6) in (4), there results

$$\cos^3\phi \int \frac{dv}{(1 + e\cos v)^2} = 2 \text{ arc tan} \left[\sqrt{\frac{1 - e}{1 + e}} \tan\frac{1}{2}v\right] - e\sqrt{1 - e^2} \cdot \frac{\sin v}{1 + e\cos v}\,.$$

On introducing the limits we finally obtain

$$M = -2 \text{ arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan m\omega\right]$$

$$+ \sin\phi\cos\phi\,\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,. \qquad (7)$$

It has been assumed in the above that $v = 0$ at the time t_0. This means that at that time a minimum should also occur. In view of the smallness of ω we may so assume t_0 that this time does coincide with a minimum. Since $\omega < 0.04$, no error in excess of $0^d.001$ can be thereby introduced in the calculated minima, as will be shown later. This accuracy is consequently more than sufficient for the first computation of the elements.

Now in equation (7) M by no means designates the actual total change of the mean anomaly occurring in the interval $t - t_0$, but on the contrary it is as much less than $\frac{t - t_0}{\Sigma}\,.\,360^\circ = m\,.\,360^\circ$ as the mean anomaly of the star has increased. If M_t is the actual total increase of the mean anomaly, then

$$M_t = m\,.\,360^\circ - 2\text{ arc tan}\left[\tan\left(45^\circ - \tfrac{1}{2}\phi\right)\tan m\omega\right]$$

$$+ \sin\phi\cos\phi\cdot\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,.$$

But

$$M_t = n\,(t - t_0)\,,$$

whence

$$n\,(t - t_0) = m\,.\,360^\circ - 2\text{ arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan m\omega\right]$$

$$+ \sin\phi\cos\phi\,\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,,$$

or,

$$t = t_0 + mU - \frac{2}{n}\text{ arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan m\omega\right]$$

$$+ \frac{\sin\phi\cos\phi}{n}\cdot\frac{\sin 2m\omega}{1 + e\cos 2m\omega}\,.$$

Here m is one half the number of the epoch reckoned from t_0. It will be more convenient to introduce in place of this the numbers of the epochs hitherto employed. Therefore let $-E_0$ be the number of the epoch corresponding to t_0, and E be the

epoch for which the computation is to be made. Then we have

$$t = t_0 + (E+E_0)\frac{U}{2} - \frac{2}{n} \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan (E+E_0)\frac{\omega}{2}\right.$$
$$\left. + \frac{\sin\phi\cos\phi}{n \sin 1''} \cdot \frac{\sin (E+E_0)\,\omega}{1+e\cos(E+E_0)\omega}\right. \tag{8}$$

This equation holds good for the even minima. We may proceed in a similar way for the odd minima. Let t_1 be the time at which the minimum occurs. Then at the time t_1 the angle between the line of apsides and the line of sight is $\frac{t_1 - t_0}{\Sigma} \cdot 2\omega$, and the true anomaly is

$$v = 180^\circ - \frac{t_1 - t_0}{\Sigma} 2\omega = \pi - 2m_1\omega \; .$$

In order to find M from this we have to integrate the following equation:

$$M = \cos^3\phi \int_0^{\pi - 2m\omega} \frac{dv}{(1+e\cos v)^2} \; . \tag{9}$$

Since, however, as before,

$$\cos^3\phi \int \frac{dv}{(1+e\cos v)^2} = 2 \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan\frac{1}{2}v\right]$$
$$- \sin\phi\cos\phi \frac{\sin v}{(1+e\cos v)^2} \, ,$$

we only have to introduce the new limits, and obtain

$$M = 2 \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \cot m_1\omega\right]$$
$$- \sin\phi\cos\phi \frac{\sin 2m_1\omega}{1 - e\cos 2m_1\omega} \; . \tag{10}$$

The M in this equation now indicates how much more than a certain multiple of 360° the mean anomaly has increased since the passage of perihelion which occurred at the time t_0. Since M is less than 360°, we therefore have to set

$$n(t_1 - t_0) = \frac{E_1 + E_0}{2} \cdot 360^\circ + 2 \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \cot m_1\omega\right]$$
$$- \sin\phi\cos\phi \frac{\sin 2m_1\omega}{1 - e\cos 2m_1\omega} \, ,$$

or, if we introduce the number of the epoch, here $\epsilon_1 = \epsilon + 1$, as before,

$$t_1 = t_0 + (E_1 + E_0 - 1)\frac{U}{2} + \frac{2}{n}\text{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\cot\left(E_1 + E_0\right)\frac{\omega}{2}\right]$$
$$- \frac{\sin\phi\cos\phi}{n\sin 1''} \cdot \frac{\sin(E_1 + E_0)\,\omega}{1 - e\cos(E_1 + E_0)\,\omega} \,. \qquad (11)$$

After the formulae for the prediction of the minima from the elements have been developed in this way, we must now show how the elements t_0, ω, n, and e may be computed. I start here from formula (2), which represents the observations very well indeed. We must note here that E cannot denote in both formulae one and the same number. In order that this may be the case for any selected epoch, we may transcribe the first formula (2) in the following way:

$$\text{Even minima} = 1886.0 + 343^d.4686 + 1^d.498267\,(E \mp 1)$$
$$- 0^d.000\,000\,022\,(E^2 \mp 2E + 1)\,,$$

$$\text{or even minima} = 1886.0 + 343^d.4686 \mp 1^d.4983 + [1.498267 \mp$$
$$0.000\,000\,044]\,E - 0.000\,000\,022\,E^2\,.$$

Consequently we have for the two even minima which immediately precede or follow the odd minimum whose epoch $= E$:

$$\text{Even minimum (Epoch} = E - 1) = 1886.0 + 341^d.9703$$
$$+ 1^d.498267\,E - 0^d.000\,000\,022\,E^2\,. \qquad (12)$$

$$\text{Even minimum (Epoch} = E + 1) = 1886.0 + 344^d.9669$$
$$+ 1^d.498267\,E - 0^d.000\,000\,022\,E^2\,. \qquad (13)$$

$$\text{Odd minimum (Epoch} = E) = 1886.0 + 343^d.4175$$
$$+ 1^d.498068\,E + 0^d.000\,000\,022\,E^2\,. \qquad (14)$$

Now let T_1 be the difference between the following even and the preceding odd minimum, or consequently the difference of equations (13) and (14), T_2 on the other hand be the difference between the following and the preceding even minimum, consequently (14) minus (12), and we obtain:

$$\left.\begin{aligned} T_1 &= 1^d.5494 + 0^d.000\,199\,E - 0^d.000\,000\,044\,E^2 \\ T_2 &= 1^d.4472 - 0^d.000\,199\,E + 0.000\,000\,044\,E^2 \end{aligned}\right\} . \qquad (15)$$

If we place

$$\theta = T_1 - T_2 \qquad (16)$$

we get

$$\theta = 0^d.1022 + 0^d.000\,398\,E - 0^d.000\,000\,088\,E^2\,. \qquad (17)$$

In order to find when the line of apsides was perpendicular to the line of sight, we must now determine the value of E for which θ is a maximum. Consequently, we differentiate equation (17)

$$\frac{d\theta}{dE} = 0^{d}.000\,398 - 0^{d}.000\,000\,176\,E\,.$$

If we now place the differential quotient equal to zero we get

$$E = 2261\,. \tag{18}$$

Introducing this in equation (17), we obtain the maximum value

$$\theta \max = 0^{d}.5522\,. \tag{19}$$

In order to determine the epoch when the line of apsides coincided with the line of sight, we must place θ equal to zero in equation (17). Thus we obtain

$$E_{0} = -244\,, \tag{20}$$

with a residual error of $-0^{d}.0001$. The line of apsides consequently turns through 90° during 2505 epochs, or, what is the same thing, through 360° during 5010 sidereal revolutions. In order to determine from this the length of the sidereal revolution we can employ formula (2), since in the neighborhood of the time when the line of apsides is perpendicular to the line of sight the intervals between two consecutive even or odd minima are equal among themselves, and equal to the sidereal period. The calculation gives us:

Even minima	Odd minima
$E = 2260$ Min. $= 3385^{d}.971053$	$E = 2261$ Min. $= 3387^{d}.244215$
$E = 2262$ Min. $= 3388^{d}.967388$	$E = 2263$ Min. $= 3390^{d}.240550$
$\Sigma = 2^{d}.996335$	$\Sigma = 2^{d}.996335$

In this computation terms not containing E in equations (2), and therefore having no influence on the result, were not included. We therefore have

$$\Sigma = 2^{d}.996335\,. \tag{21}$$

Hence we get

$$\left.\begin{aligned} A &= 15011^{d}.638 = 41.10 \text{ years} \\ U &= 2^{d}.996933 \\ 2\omega &= 0^{\circ}.071856 \end{aligned}\right\} \tag{22}$$

It now remains to determine the eccentricity of the orbit. If no motion of the line of apsides occurred in the interval from

$E = 2260$ to $E = 2262$, then $\frac{1}{2}(U - \theta)$ and $\frac{1}{2}(U + \theta)$ would represent the times during which the star moved respectively from $v = 270^\circ$ to $v = 90^\circ$, and from $v = 90^\circ$ to $v = 270^\circ$.

In fact the true anomalies change by $180^\circ - \frac{U - \theta}{2} \cdot \frac{2\omega}{U}$ and $180^\circ - \frac{U + \theta}{2} \cdot \frac{2\omega}{U}$ respectively from the epochs 2260 to 2261, and from 2261 to 2262.

But
$$dM = \frac{\cos^3 \phi}{(1 + e \cos v)^2} dv\,.$$

An approximate knowledge of the eccentricity is therefore necessary. I assume on the basis of preliminary calculations:

$$e = 0.1456\,; \quad \phi = 8^\circ.3518.$$

In view of the smallness of ω as well as of e we may in both cases place $v = 90^\circ$ or $v = 270^\circ$, and we get

$$dM = \cos^3 \phi\, dv\,,$$

while

$$\left.\begin{aligned} dv &= -\frac{U - \theta}{U}\omega \\ dv_1 &= -\frac{U + \theta}{U}\omega \end{aligned}\right\} \qquad (23)$$

The dM thus found must now be divided by the mean daily anomalistic motion, that is by $\frac{360^\circ}{U} = n$. I find for n the value

$$n = 120^\circ.1228. \qquad (24)$$

From this we obtain

$$\frac{dM}{n} = 0^d.0003 \text{ and } \frac{dM_1}{n} = 0^d.0002,$$

which are to be added to θ; consequently

$$\theta = 0^d.5527. \qquad (25)$$

From this there results

$$\frac{1}{2}(U - \theta) = 1^d.2221\,; \quad \frac{1}{2}(U + \theta) = 1^d.7748, \qquad (26)$$

which are the intervals during which the radius vector sweeps over the two parts of the orbital ellipse bounded by the parameter.

Now the areas are proportional to the times in which they are described. Therefore, if $2s_{\prime\prime}$ and $2s_{\prime}$ are the areas described in the above times,

$$2s_{\prime} : 2s_{\prime\prime} = \frac{1}{2}(U-\theta) : \frac{1}{2}(U+\theta) .$$

Further, if F be the area of the whole ellipse, we shall have the well known relations:

$$\left.\begin{aligned} 2s_{\prime} &= \frac{1}{2}\pi a^2 \cos\phi + \frac{1}{2}\pi a^2 \left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] \cos\phi , \\ 2s_{\prime\prime} &= \frac{1}{2}\pi a^2 \cos\phi - \frac{1}{2}\pi a^2 \left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] \cos\phi , \\ F &= \pi a^2 \cos\phi . \end{aligned}\right\}$$

Hence we have

$$\left.\begin{aligned} \frac{\frac{1}{2}(U+\theta)}{U} &= \frac{2s_{\prime}}{F} = \frac{1}{2} + \frac{1}{2}\left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] , \\ \frac{\frac{1}{2}(U-\theta)}{U} &= \frac{2s_{\prime\prime}}{F} = \frac{1}{2} - \frac{1}{2}\left[\frac{2\phi + \sin 2\phi}{180^\circ}\right] , \end{aligned}\right\}$$

or

$$2\phi + \sin 2\phi = \frac{\theta}{U} . 180^\circ . \tag{27}$$

From this is obtained

$$\begin{aligned} \phi &= 8^\circ.3577 \\ e &= 0.14535 \end{aligned} \tag{28}$$

It still remains to calculate the last element, $t_\circ$. As an even minimum occurred at this time, we employ formula (8):

$$t = t_\circ + (E + E_\circ)\frac{U}{2} - \frac{2}{n} \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan\,(E + E_\circ).\frac{\omega}{2}\right]$$
$$+ \frac{\sin\phi\cos\phi}{n \sin 1^\circ} \cdot \frac{\sin\,(E + E_\circ)\omega}{1 + e\cos\,(E + E_\circ)\omega} .$$

The zero epoch is used for E, whence by (2)

$$t = 1886.0 + 343^d.4686.$$

We thus obtain

$$t_\circ = 1886.0 + 343^d.4686 - 244\frac{U}{2} + \frac{2}{n} \text{ arc tan } \left[\tan\left(45^\circ - \frac{1}{2}\phi\right) \tan\,244\frac{\omega}{2}\right]$$
$$- \frac{\sin\phi\cos\phi}{n \sin 1^\circ} \cdot \frac{\sin\,(244\omega)}{1 + e\cos\,(244\omega)} ,$$

or

$$t_0 = 1886.0 + 343^d.4086 - 122U + \frac{2}{120.1228} \times$$
$$\text{arc tan}\left[\tan\left(45^\circ - \frac{1}{2}\phi\right)\tan(122\omega)\right]$$
$$\frac{\sin\phi\cos\phi}{120.1228\sin 1^\circ}\cdot\frac{\sin(244\omega)}{1+e\cos(244\omega)}. \qquad (29)$$

Therefore

$$t^0 = 1885.0 + 342^d.8968\,. \qquad (30)$$

I have compared all of the normal minima with the elements thus obtained, and have found the following differences between computation and observation:

Epoch	O—C	Epoch	O—C
12	$-0^d.007$	1640	$+0^d.019$
191	$-0\ .022$	1913	$-0\ .014$
418	$+0\ .022$	2208	$-0\ .012$
705	$+0\ .008$	2371	$-0\ .021$
1184	$-0\ .016$	2852	$-0\ .008$
1187	$+0\ .004$	2887	$-0\ .007$
1311	$+0\ .004$		

The deviations of the computations from the observations cannot be considered too large; but it cannot be denied that the negative values preponderate. I have therefore taken the mean of them all, and find for the correction to the element t_0

$$dt_0 = -0^d.0038.$$

The final elements therefore become:

ELEMENTS OF Y CYGNI.

Principal epoch, $t_0 = 1885.0 + 342^d.8930$
Motion of line of apsides, $\omega = 0^\circ.035928$
Eccentricity, $e = 0.14535$
Anomalistic period, $U = 2^d.996933$

On comparing these elements with the normal positions, I obtained the following result:

Epoch	Normal minimum	O—C	Epoch	Normal minimum	O—C
12	$361^d.440$	$-0^d.004$	1640	$2800^d.586$	$+0^d.023$
191	629 .536	$-0\ .018$	1913	3209 .292	$-0\ .010$
418	969 .756	$+0\ .025$	2208	3651 .529	$-0\ .008$
705	1399 .580	$+0\ .012$	2371	3895 .448	$-0\ .017$
1184	2177 .365	$-0\ .012$	2852	4616 .349	$-0\ .004$
1187	2181 .662	$+0\ .007$	2887	4668 .529	$-0\ .004$
1311	2307 .429	$+0\ .008$			

It is obvious that a still closer representation of the observation can be obtained by slight variations of the elements. Since, however, the residual errors hardly exceed the probable errors, and since we may expect to obtain observations of the even as well as of the odd minima perhaps as soon as in 1900, I prefer to await these observations before I proceed to improve the elements, which, of course, may be accomplished in a manner analogous to the case of the elements of a planetary orbit.

Of the elements, U and e are evidently already included within very narrow limits. The uncertainty in ω may be considered as somewhat greater in comparison to its less magnitude, and t_0 is somewhat uncertain. Moreover, as has been already remarked, it is not possible to decide whether an even or an odd minimum occurred at the perihelion passage at the time t_0. The former has been assumed above, but on rather uncertain premises.

Finally, a fifth element must be calculated in order to be able to predict the duration of occultation of the star during a given minimum. As such element we may select either the duration of occultation at that minimum when the distance of the stars was equal to the semi-major axis of the orbital ellipse; or the ratio between the diameters of the two stars and the semi-major axis; or, finally, the angle which one star subtends as seen from the other when at their mean distance. Since the knowledge of all three of these quantities is interesting, I shall now show how they may be determined. The observational data upon which this computation may be based are still extremely scant, and were only obtained on searching through my observations to see whether there were any which either began so early or ended so late that the half of the duration of the occultation could be inferred with some certainty. Omitting certain isolated and more unreliable data, I find the following nine observations, all of even minima and all in the year 1892, which appear fairly suitable:

Epoch	Half-duration	Epoch	Half-duration
	h m		h m
1152	2 21	1182	2 55
1162	2 31	1184	2 63
1168	2 33	1194	2 40
1168	2 27	1194	2 50
1180	2 54		

Mean $2^h\, 41^m$

The whole duration of the occultation was accordingly

$$T' = 0^d.225.$$

Now, since it is clear that occultation begins at the moment when the stars come into contact as seen from the Earth, and does not cease until the star disks are again tangential, and since the stars are of equal size, the one star must have described an arc of the orbit four times as large as its own radius seen from the other star, if we assume, as is universally done, that one star is stationary. Assuming that the radius is equal, at its mean distance, to π, and letting R be the radius vector at minimum, and taking the semi-major axis as unity, and calling B the arc described by the star, we shall get

$$B = \frac{4\pi}{R},$$

or

$$\pi = \frac{RB}{4}. \tag{31}$$

Since at the beginning of the occultation the moving star touches one of the tangents to the stationary star, parallel to the line of sight, the chord joining the points of the orbit where the center of the moving star is situated at the beginning and end of occultation is equal to the sum of the diameters of the two stars, or, which is the same thing, is equal to four times the radius of one star. If r is now the radius of the star we have

$$\frac{2r}{R} = \sin B$$

or

$$r = \frac{R}{2} \sin B. \tag{32}$$

For two different minima we accordingly have, writing dv in place of B,

$$Rdv = R_1 dv_1 .$$

According to Kepler's second law, however,

$$\frac{T}{T_1} = \frac{R^2 dv}{R_1^2 dv_1} = \frac{R}{R_1} .$$

If $R = a = 1$, and T is the duration of occultation,

$$T = \frac{T_1}{R_1} .$$

We may now introduce the factor $\frac{1}{1 + e \cos v}$ in place of R, and we get

$$T = T_1 (1 + e \cos v_1) . \tag{33}$$

We therefore obtain for *Y Cygni*

$$T = 0^d.245 . \tag{34}$$

In this case, however, $v = \omega (E + E_0) = 90° + \phi$. Hence, the corresponding mean anomaly is $M = 81°.67$. If M_1 and M_2 indicate respectively the mean anomalies at the beginning and end of the minimum for which $T = 0^d.245$, we have

$$M_2 - M_1 = \frac{0^d.245}{U} . 360° = 29°.44 .$$

Consequently

$$M_1 = 67°.95 \ ; \quad M_2 = 96°.39 .$$
$$v_1 = 84°.26 \ ; \quad v_2 = 112°.38 .$$

and

$$\left.\begin{aligned} \pi &= \frac{1}{4}(v_2 - v_1) = 7° \\ r &= \sin \pi = \frac{1}{8} \end{aligned}\right\} . \tag{35}$$

It is perfectly evident that this whole investigation of the elements T, π, and r is highly uncertain. For one thing, the quantities T', directly found from the observations, are not very sharply determined. But the principal uncertainty is whether a perihelion or an aphelion passage occurred at the chief epoch t_0; and this is of quite a different significance in the determination of the above elements than for those of others. But the values

found for T, π, and r are nevertheless to be regarded as approximations to the truth. The general result of the foregoing investigation can therefore be expressed as follows:

The variable star Y Cygni consists of two stars of equal size and equal brightness, which move about their common center of gravity in an elliptical orbit whose major axis is eight times the radius of the stars. The period of an anomalistic revolution is 2.996933 days, and the eccentricity is 0.145. A minimum occurred while the stars were at perihelion (?)[1] *at* $21^h\ 26^m$ *G. M. T., on December 8, 1885. The line of apsides of the orbit, which then coincided with the line of sight, completes one revolution in the plane of the orbit in 41.1 tropical years.*

These investigations may well claim some interest. The observations on which they are based were made with the very simplest apparatus, namely with ordinary telescopes, without photometers, but with the use of Argelander's famous method of estimation in grades. Nevertheless, the observations have proven the existence of a binary system hitherto unknown, whose period, shape and position of orbit, and position of line of apsides are now known with a degree of accuracy in part not inconsiderable. Moreover, we know approximately the ratio between the major axis of the orbit and the diameter of the stars, as well as the angular diameter of the one star as seen from the other. Finally, a force has been brought to light which causes the line of apsides to assume all possible positions in the orbital plane in a space of about forty years.

The line of apsides will again coincide with the line of sight in 1906, and then the even minima will occur at the time of aphelion passage, if we have been correct above. It will then be possible, and possibly even earlier, to decide with certainty from observations with photometers and by estimates whether an aphelion or a perihelion passage occurs at this time. As it is my intention to make observations of the even minima occurring at a very favorable season in this year, for the purpose of observing the duration of occultation of the star, I have predicted in the following ephemeris the times when the minima occur,

[1] The uncertainty is whether it was perihelion or aphelion.

but not the duration of the occultation, in order not to prejudice my observations thereby.

EPHEMERIDES FOR 1900.

Even minima		Odd minima	
Epochs	G. M. T.	Epochs	G. M. T.
	d h m		d h m
3186	Jan. 3 17 31	3185	Jan. 2 0 0
3206	Feb. 2 16 37	3205	Feb. 0 23 8
3226	Mar. 4 15 43	3225	Mar. 2 22 17
3246	Apr. 3 14 49	3245	Apr. 1 21 25
3266	May 3 13 55	3265	May 1 20 34
3286	June 2 13 1	3285	June 0 19 42
3306	July 2 12 7	3305	June 30 18 51
3326	Aug. 1 11 13	3325	July 30 18 0
3346	Sept. 0 10 18	3345	Aug. 29 17 9
3366	Oct. 0 9 24	3365	Sept. 28 16 18
3386	Oct. 30 8 29	3385	Oct. 28 15 27
3406	Nov. 29 7 35	3405	Nov. 27 14 36
3426	Dec. 29 6 40	3425	Dec. 27 13 45

It will be seen that the even minima may be observed in *Europe* under especially favorable circumstances from the beginning of July onward. The odd minima can be observed under equally favorable conditions at the same time in *America*.

UPSALA,
February 2, 1900.

ON THE MAGNITUDES OF 919 FIXED STARS DETERMINED FROM SEQUENCES OBSERVED BY SIR JOHN HERSCHEL DURING THE YEARS 1835 to 1838.

I.

By W. Doberck.

The value of astronomical observations increases with their age, especially in case of determinations of magnitudes of fixed stars. Old records would enable us to discover secular variations in those magnitudes.

Sir John Herschel's observations were made nearly seventy years ago with the naked eye, without any instrumental appliances, according to the method of sequences which, with the exception of the very brightest stars, is capable of furnishing results which are as accurate as the latest photometric determinations. Sir John Herschel not only made these observations, but he also partially reduced them and determined magnitudes of stars included in Table I, using as standards the magnitudes given in the catalogue published by the Astronomical Society in 1827, although he mentions that those latter magnitudes are not homogeneous and, at least in individual cases, frequently considerably in error. Gould mentions that the order of the magnitudes given by Herschel differs at present so widely from that recorded by him, that it is not always easy to decide what magnitude on our scale corresponds to his determination, and Sir John Herschel himself expressed the hope that his sequences might be definitively reduced at some future time, when accurate photometric determinations of the magnitudes of fixed stars were available for reference.

Sir John Herschel states that he chose perfectly clear nights (which for this purpose are quite indispensable), but it is found that the results on some nights, especially on December 29, 1837, are not so satisfactory as those obtained on other nights, which may possibly be due to the existence of faint patches of cirrus haze, which were not remarked at the time. Sir John

Herschel found it impracticable to limit himself to the comparison of stars of nearly the same altitude, nor could he always avoid working on moonlight nights. He states that he did not consider it worth while to bestow particular attention upon insignificant stars, not exceeding the sixth magnitude.

The order of the stars in Table I is as definitively arranged by Sir John Herschel in his corrected normal sequence. The same table shows the magnitudes according to the *Uranometria Argentina*,[1] and the equalized magnitudes, which are the means of the star's own magnitude as stated in the former column and those of the two stars immediately preceding and the two immediately following it in order. Taking now the numerical order of each star from the beginning of the table for an abscissa, and erecting, on paper divided into millimeter squares, an ordinate equal to its equalized magnitude, a series of points was obtained. A curve was now drawn as nearly as possible through (or rather among) those points. This curve was then read off, and the reading set down in the last column, the magnitudes shown in which have been used as standards in reducing the individual sequences.

Each of the forty-six sequences was then treated separately in a similar manner. The numerical order of each star from the beginning of the sequence was taken for an abscissa, and an ordinate equal to its provisional magnitude in Table I was dotted down in case of stars included in that table. No ordinate was entered in case of stars not included in the table, except in case of stars fainter than the stars in Table I. In case of the latter their magnitudes as given in the *Uranometria Argentina* were entered as the corresponding ordinates. Curves were then drawn, keeping as close as possible to all the dots as the condition of a continual increase of the ordinate would allow. In general this was very satisfactory as long as the provisional magnitudes were in question, but when it came to the magnitudes taken from the *Uranometria Argentina* the agreement was less satisfactory. The apparent magnitudes of the stars in every sequence were now

[1] When not found in the *U. A.* the magnitudes were taken from the *Uranometria Nova*, and are shown in parentheses.

read off on the curves, and the results are given in Table II. The stars are here distinguished by their numbers in Table III, which contains the final catalogue. The value of a step, or grade (the difference in magnitude between two adjacent stars), is very different in different sequences or in different parts of the same sequence. For stars of the first two magnitudes it amounted in general to about 0.20 of a magnitude, for stars of about the fourth magnitude it came to about 0.08 magnitude or less, while for stars of the sixth magnitude it varied between 0.02 and 0.40 of a magnitude. The ordinate rises quickly at the beginning of a sequence, a natural consequence of the fact that the brighter stars are scarce, then it gradually rises more slowly and either continues to approach parallelism with the axis of abscissae, or, what is more frequent, it again gradually rises steeply. The sequences 10, 14, 15, 17, 19, and 38 were represented by straight lines. The probable error of a magnitude shown in Table II is in case of the second magnitude ±0.075, of the third ±0.087, of the fourth ±0.121, and of the fifth ±0.204. Herschel observed stars of the first magnitude on an average 11 times, of the second 9 times, of the third 3 times, of the fourth twice, of the fifth once or twice, and of the sixth once only.

The identification of the stars has given great trouble, although I was ably assisted by Mr. J. I. Plummer and Mr. F. G. Figg, and I have not as yet been able to identify the following five stars: σ *Eridani* (1837.874, 3.87), ψ *Columbae* (1838.018, 6.08), *P. Carinae* (1836.179, 5.76), *Cent.* 154 *Bode* (1837.993, 6.32), and ι *Coronae Aust.* (1835.540, 5.91).

Herschel occasionally included the major planets in his sequences with the following results:

Mars, 1836.867: 1.12. *Saturn*, 1836.415: 0.95, 1837.188: 1.00, 1837.272: 1.28, 1838.149: 1.10. *Jupiter*, 1836.867: ÷0.62, 1837.990: ÷0.20, 1837.993: ÷0.33, 1838.149: ÷0.21, 1838.-283: ÷0.44.

Table III shows the mean magnitudes: column 1 indicates the number and column 2 the name of the star according to Gould's nomenclature. The third and fourth columns show the

coördinates of the stars for 1875. The fifth column shows the mean magnitude according to Sir John Herschel (h), and the sixth column shows how often he observed the star (n). The seventh column shows the magnitude which Behrmann (B) assigned to the star in autumn 1866, reduced to the *U. A.* scale by aid of the table in the *Uranometria Argentina* (page 117). The eighth column contains the *U. A.* magnitude. The ninth column shows the *S. M. P.* magnitude taken from *Annals of the Astronomical Observatory of Harvard College*, Volume XXXIV. The tenth column shows the magnitude observed by Mr. A. Stanley Williams in 1885-6 and published in his catalogue of the magnitudes of 1081 stars (London, 1898). The data in columns 9 and 10 have been reduced to *U. A.* scale by aid of the tables given by Mr. Williams on pages 8 and 10 of the introduction to his catalogue. In case the light of a star as seen by the naked eye is made up of several stars separately noted in the *U. A.* or *S. M. P.* their light has been added together and the resulting magnitude entered. Where this has not been done, the magnitude is that of the brightest star only and is placed in brackets [ ]. Where it was not possible to apply any reduction to *U. A.* the magnitude is placed in parentheses ( ). The eleventh column contains the mean of all the magnitudes, but this is given only in cases where it is most probable that the magnitude has not undergone any variation whatsoever during the past seventy years.

These mean magnitudes might serve as standards in future photometric work in the southern hemisphere. They are referred to the *U. A.* scale, and I am by no means satisfied that that is the best scale that could be selected, but then it is so well known that magnitudes observed on any other scale can be easily referred to it, or the magnitudes here given can conveniently be reduced to any other system. Stars possibly variable have been pointed out. They deserve attention. With reference to secular variation, more or less faintly indicated in case of some stars, it appears that nothing definite will be known till further observations are available.

My thanks are due to Mr. J. I. Plummer, and to Mr. F. G. Figg for the willingness with which they have assisted me in carrying out this investigation. The former gentleman especially has devoted much time to calculations in connection with this work.

TABLE I.

Provisional magnitudes based on Herschel's corrected sequences and the *Uranometria Argentina.*

Name	*U. A.*	Equalized Mag.	Provisional Mag.	Name	*U. A.*	Equalized Mag.	Provisional Mag.
α *Canis Majoris*	0.1	(0.1)	0.07	β *Ceti*	2.3	2.16	2.14
α *Argûs*	0.4	(0.4)	0.41	θ *Centauri*	2.2	2.16	2.15
α *Centauri*	0.7	0.64	0.64	β *Aurigae*	(2.0)	2.16	2.15
α *Bootis*	(1.0)	0.82	0.80	κ *Orionis*	2.3	2.10	2.16
α *Aurigae*	(1.0)	0.94	0.93	α *Ursae Minoris*	(2.0)	2.10	2.16
α *Lyrae*	(1.0)	1.04	1.00	α *Pegasi*	(2.0)	2.16	2.17
β *Orionis*	1.0	1.04	1.05	β *Canis Majoris*	2.2	2.10	2.18
α *Canis Minori*	1.2	1.08	1.07	δ *Orionis*	2.3	2.10	2.19
α *Eridani*	1.0	1.08	1.09	β *Leonis*	(2.0)	2.12	2.21
α *Orionis*	1.2	1.12	1.12	α *Coronae Bor.*	(2.0)	2.16	2.22
α *Tauri*	(1.0)	1.14	1.14	α *Ophiuchi*	2.1	2.20	2.24
β *Centauri*	1.2	1.22	1.18	γ *Centauri*	2.4	2.30	2.26
α *Crucis*	1.3	1.20	1.22	ι *Carinae*	2.5	2.36	2.28
α *Scorpii*	1.4	1.30	1.27	ζ *Puppis*	2.5	2.40	2.31
α *Aquilae*	1.1	1.34	1.34	β *Pegasi*	(2.3)	2.40	2.34
α *Virginis*	1.5	1.42	1.41	ε *Bootis*	(2.3)	2.36	2.36
α *Piscis Australis*	1.4	1.48	1.48	α *Phoenicis*	2.4	2.32	2.39
β *Geminorum*	(1.7)	1.52	1.55	ε *Scorpii*	2.3	2.38	2.42
β *Crucis*	1.7	1.60	1.61	γ *Ursae Majoris*	(2.3)	2.38	2.44
α *Leonis*	(1.3)	1.72	1.67	α *Lupi*	2.6	2.42	2.46
α *Gruis*	1.9	1.74	1.73	β *Ursae Majoris*	(2.3)	2.54	2.48
γ *Crucis*	2.0	1.80	1.77	ε *Centauri*	2.6	2.56	2.50
ε *Orionis*	1.8	1.94	1.83	η *Canis Majoris*	2.9	2.50	2.52
ε *Ursae Majoris*	(2.0)	1.92	1.87	δ *Scorpii*	2.4	2.58	2.53
λ *Scorpii*	2.0	1.92	1.91	δ *Leonis*	(2.3)	2.56	2.54
ε *Canis Majoris*	1.8	1.92	1.95	ζ *Centauri*	2.7	2.48	2.56
α *Ursae Majoris*	(2.0)	1.92	1.99	γ *Corvi*	2.5	2.48	2.57
ζ *Orionis*	1.8	1.92	2.02	η *Centauri*	2.5	2.54	2.59
β *Carinae*	2.0	1.90	2.05	α *Ceti*	2.4	2.48	2.60
η *Ursae Majoris*	(2.0)	2.10	2.07	β *Corvi*	2.6	2.52	2.61
γ *Orionis*	1.7	2.16	2.09	η *Ophiuchi*	2.4	2.64	2.62
γ *Velorum*	3.0	2.18	2.10	π *Puppis*	2.7	2.68	2.63
α *Pavonis*	2.1	2.18	2.11	γ *Virginis*	3.1	2.66	2.65
Carinae	2.1	2.28	2.12	ζ *Ophiuchi*	2.6	2.72	2.66
β *Tauri*	(2.0)	2.12	2.12	β *Scorpii*	2.5	2.74	2.68
α *Trianguli Aust.*	2.2	2.12	2.12	γ *Pegasi*	(2.7)	2.66	2.70
ε *Sagittarii*	2.2	2.12	2.12	δ *Centauri*	2.8	2.76	2.71
θ *Scorpii*	2.1	2.12	2.12	δ *Ophiuchi*	2.7	2.78	2.72
α *Hydrae*	2.1	2.10	2.13	ζ *Sagittarii*	3.1	2.78	2.73
β *Ursae Minoris*	(2.0)	2.10	2.13	κ *Scorpii*	2.6	2.68	2.74
δ *Canis Majoris*	2.1	2.16	2.14	κ *Velorum*	2.7	2.68	2.75
β *Gruis*	2.2	2.14	2.14	β *Herculis*	(2.3)	2.68	2.76
σ *Sagittarii*	2.4	2.14	2.14	α *Leporis*	2.7	2.76	2.76
α *Arietis*	(2.0)	2.16	2.14	β *Librae*	3.1	2.72	2.77
γ *Leonis*	(2.0)	2.18	2.14	η *Bootis*	(3.0)	2.84	2.78
δ *Velorum*	2.2	2.20	2.14	α *Columbae*	2.5	2.84	2.78
ε *Pegasi*	2.3	2.26	2.14	μ *Velorum*	2.9	2.84	2.79
λ *Velorum*	2.5	2.26	2.14	ε *Virginis*	(2.7)	2.84	2.79
γ *Geminorum*	(2.3)	2.28	2.14	α *Librae*	3.1	2.90	2.80
ζ *Ursae Majoris*	(2.0)	2.26	2.14	α *Canum Venat.*	(3.0)	2.84	2.80

TABLE I — *Continued.*

Name	*U. A.*	Equalized Mag.	Provisional Mag.	Name	*U. A.*	Equalized Mag.	Provisional Mag.
β *Eridani*	2.8	2.84	2.81	*q Carinae*	3.3	3.40	3.32
α *Serpentis*	2.6	2.76	2.82	o^2 *Canis Majoris*	3.4	3.40	3.34
λ *Sagittarii*	2.7	2.72	2.83	π *Hydrae*	3.6	3.46	3.36
β *Hydri*	2.7	2.76	2.84	ν *Puppis*	3.5	3.52	3.38
β *Lupi*	2.8	2.80	2.86	α *Pictoris*	3.5	3.54	3.40
ι *Centauri*	3.0	2.86	2.87	η *Scorpii*	3.6	3.52	3.42
δ *Sagittarii*	2.8	2.90	2.89	σ *Puppis*	3.5	3.54	3.43
δ *Corvi*	3.0	2.90	2.92	ε *Gruis*	3.5	3.46	3.45
θ *Carinae*	2.9	2.88	2.95	γ *Arae*	3.6	3.44	3.46
β *Arae*	2.8	2.98	2.98	α *Doradûs*	3.1	3.46	3.47
β *Leporis*	2.9	3.06	3.00	δ *Virginis*	3.5	3.38	3.48
ε *Corvi*	3.3	3.04	3.02	ω *Carinae*	3.6	3.38	3.49
π *Scorpii*	3.4	3.06	3.04	ε *Leporis*	3.1	3.52	3.49
α *Toucani*	2.8	3.12	3.05	*p Carinae*	3.6	3.54	3.49
α *Muscae*	2.9	3.10	3.07	δ *Crateris*	3.8	3.48	3.50
ρ *Puppis*	3.2	3.10	3.08	*c Puppis*	3.6	3.54	3.50
γ *Lupi*	3.2	3.12	3.09	α *Reticuli*	3.3	3.54	3.50
σ *Scorpii*	3.4	3.26	3.10	μ *Leporis*	3.4	3.46	3.50
α *Arae*	2.9	3.26	3.10	σ *Canis Majoris*	3.6	3.54	3.51
μ^1 *Scorpii*	3.6	3.28	3.11	γ *Phoenicis*	3.4	3.56	3.51
υ *Scorpii*	3.2	3.24	3.11	η *Virginis*	4.0	3.62	3.51
ι^1 *Scorpii*	3.3	3.28	3.12	μ *Centauri*	3.4	3.60	3.51
τ *Scorpii*	3.2	3.16	3.12	ν *Centauri*	3.7	3.60	3.51
π *Sagittarii*	3.1	3.16	3.12	φ *Eridani*	3.5	3.52	3.51
γ *Aquilae*	(3.0)	3.12	3.12	κ *Ophiuchi*	3.4	3.50	3.51
γ *Hydrae*	3.2	3.08	3.12	δ *Lupi*	3.6	3.46	3.52
β *Trianguli Aust.*	3.1	3.08	3.12	μ *Serpentis*	3.3	3.42	3.52
β *Arietis*	(3.0)	3.06	3.12	20 *Librae*	3.5	3.40	3.54
γ *Trianguli Aust.*	3.1	3.06	3.12	β *Serpentis*	(3.3)	3.42	3.55
α *Hydri*	2.9	3.06	3.12	ε *Ophiuchi*	3.3	3.50	3.56
τ *Puppis*	3.2	3.04	3.12	χ *Carinae*	3.7	3.60	3.58
ζ *Hydrae*	3.1	3.08	3.12	ε *Lupi*	3.7	3.68	3.60
ι *Orionis*	2.9	3.10	3.13	o *Velorum*	4.0	3.74	3.61
υ *Carinae*	3.3	3.14	3.14	β *Toucani*	3.7	3.80	3.63
ν *Hydrae*	3.0	3.12	3.15	ζ *Lupi*	3.6	3.80	3.65
δ *Crucis*	3.4	3.18	3.16	κ *Canis Majoris*	4.0	3.64	3.67
β *Canis Minoris*	3.0	3.14	3.16	η *Lupi*	3.7	3.66	3.68
N Velorum	3.2	3.22	3.17	γ *Ceti*	3.2	3.68	3.70
π^3 *Orionis*	3.1	3.14	3.18	α *Horologii*	3.8	3.66	3.71
λ *Centauri*	3.4	3.20	3.19	ξ *Hydrae*	3.7	3.68	3.72
γ *Gruis*	3.0	3.24	3.20	δ *Columbae*	3.9	3.82	3.72
κ *Centauri*	3.3	3.34	3.20	α *Carinae*	3.8	3.78	3.73
β *Muscae*	3.4	3.28	3.21	χ *Eridani*	3.9	3.78	3.74
ζ *Virginis*	3.6	3.32	3.22	ϕ^1 *Lupi*	3.6	3.76	3.76
α *Indi*	3.1	3.32	3.22	β *Virginis*	3.7	3.68	3.77
ζ *Canis Majoris*	3.2	3.22	3.22	ι *Lupi*	3.8	3.72	3.78
β *Phoenicis*	3.3	3.16	3.24	λ *Hydrae*	3.4	3.70	3.80
β *Columbae*	2.9	3.24	3.24	κ *Lupi*	4.1	3.72	3.82
ε *Hydrae*	3.3	3.12	3.26	β *Cancri*	3.5	3.76	3.84
α *Circini*	3.5	3.16	3.26	γ *Volantis*	3.8	3.96	3.86
θ *Eridani*	2.6	3.22	3.28	*k Puppis*	4.0	3.94	3.88
ξ *Puppis*	3.5	3.22	3.30	*l Carinae*	4.4	3.96	3.91
γ *Hydri*	3.2	3.20	3.31	α *Puppis*	4.0	4.00	3.93

TABLE I — *Continued.*

Name	U. A.	Equalized Mag.	Provisional Mag.	Name	U. A.	Equalized Mag.	Provisional Mag.
ζ^2 *Scorpii*	3·6	3·94	3·94	κ *Eridani*	4·2	4·32	4·21
γ *Toucani*	4·0	3·86	3·96	δ *Toucani*	4·8	4·26	4·22
ψ *Velorum*	3·7	3·84	3·98	δ *Ceti*	4·0	4·16	4·22
μ *Hydrae*	4·0	3·90	3·99	*P Velorum*	4·1	4·18	4·24
o¹ *Canis Majoris*	3·9	3·96	4·00	δ *Muscae*	3·7	4·02	4·24
ι *Hydrae*	3·9	4·04	4·01	ζ *Volantis*	4·3	4·04	4·26
π *Lupi*	4·3	4·00	4·02	γ *Muscae*	4·0	4·04	4·27
ϵ *Columbae*	4·1	4·06	4·02	μ *Crucis*	4·1	4·20	4·29
ϵ *Phoenicis*	3·8	4·10	4·03	ν^2 *Canis Majoris*	4·1	4·14	4·30
ζ *Phoenicis*	4·2	4·04	4·04	η *Phoenicis*	4·5	4·24	4·31
δ *Hydri*	4·1	4·18	4·04	*q Velorum*	4·0	4·26	4·32
η *Columbae*	4·0	4·20	4·05	ρ *Lupi*	4·5	4·40	4·34
μ *Lupi*	4·8	4·14	4·06	α *Chamaeleontis*	4·2	4·38	4·35
β *Pictoris*	3·9	4·16	4·06	ξ^2 *Centauri*	4·8	4·36	4·36
β *Doradûs*	3·9	4·14	4·06	β *Pyxidis*	4·4	4·52	4·37
ϵ *Hydri*	4·2	3·98	4·06	γ *Apodis*	3·9	4·54	4·38
ϕ *Velorum*	3·9	3·96	4·06	*f Centauri*	5·3	4·48	4·39
u Carinae	4·0	3·96	4·07	ϵ *Toucani*	4·3	4·40	4·40
α *Pyxidis*	3·8	3·96	4·08	ϵ *Volantis*	4·5	4·50	4·40
β *Volantis*	3·9	3·94	4·08	α *Apodis*	4·0	4·34	4·41
l Puppis	4·2	3·96	4·09	γ *Doradûs*	4·4	4·30	4·42
ν *Eridani*	3·8	4·10	4·10	δ *Chamaeleontis*	4·5	4·30	4·42
g Eridani	4·1	4·16	4·10	ζ *Toucani*	4·1	4·44	4·44
β *Hydrae*	4·5	4·12	4·11	δ *Doradûs*	4·5	4·48	4·48
υ^1 *Centauri*	4·2	4·14	4·12	θ *Chamaeleontis*	4·7	4·54	4·55
δ *Phoenicis*	4·0	4·24	4·13	β *Chamaeleontis*	4·6	4·64	4·61
β *Reticuli*	3·9	4·14	4·14	ζ *Doradûs*	4·8	4·66	4·66
ζ *Crucis*	4·6	4·08	4·14	λ^1 *Phoenicis*	4·6	4·74	4·73
ϵ *Crucis*	4·0	4·10	4·15	ϵ *Reticuli*	4·6	4·76	4·80
κ *Phoenicis*	3·9	4·26	4·16	ϵ *Doradûs*	5·1	4·86	4·86
ϕ *Centauri*	4·1	4·16	4·16	γ *Reticuli*	4·7	4·94	4·93
η *Crucis*	4·7	4·22	4·16	η *Toucani*	5·3	5·14	4·99
δ *Volantis*	4·1	4·28	4·17	κ *Reticuli*	5·0	5·12	5·04
σ *Centauri*	4·3	4·34	4·18	ζ^1 *and* ζ^2 *Reticula*	5·6	5·22	5·10
α *Volantis*	4·2	4·24	4·18	*s Eridani*	5·0	5·20	5·15
I Carinae	4·4	4·26	4·19	ζ *Hydrae*	5·2	(4·90)	5·20
ι *Eridani*	4·2	4·36	4·20	β *Horologii*	5·2	(5·20)	5·25

TABLE II.

Apparent magnitudes of stars observed by Herschel in sequences at Feldhausen from July 16, 1835, to February 25, 1838, and on board H. E. I. C. S. Windsor, at sea, from March 28 to April 15, 1838, inclusive, deduced by aid of the *Uranometria Argentina.*

No.	Ep.	Mag.
1	36.535	2.14
2	37.732	5.82
3	35.620	3.87
	36.905	3.82
	37.812	3.99
4	37.732	5.82
5	36.535	2.78
6	35.620	4.39
	36.905	4.15
	37.812	4.47
7	37.732	5.84
8	37.812	4.53
9	35.620	2.78
	36.179	2.76
	.535	2.86
	.541	2.84
	.905	2.77
	37.730	2.72
	.812	2.94
	.993	2.83
	38.015	3.17
	.018	3.23
	.034	2.96
10	35.620	3.97
	36.905	3.84
	37.812	4.36
11	35.620	2.37
	36.535	2.70
	.905	2.56
	37.730	2.61
	.812	2.46
12	37.732	5.57
13	35.620	4.86
	37.812	4.70
14	35.620	3.68
	36.541	3.67
	.905	3.64
	37.812	3.61
15	37.732	5.83
16	35.620	4.82
	37.812	5.07
17	37.812	5.33
18	36.535	1.99
	.905	2.42
	37.730	2.33
	.732	2.13
	.812	2.07
	.874	2.35
19	35.620	4.34
	37.812	4.34
20	37.732	4.88
21	37.732	5.74
22	37.732	5.70
23	37.732	5.78
24	37.732	5.90
25	37.732	5.66
26	37.812	5.19
27	37.732	5.88
28	37.732	5.88
29	37.732	4.02
30	37.732	5.79
31	37.732	5.88
32	35.620	3.39
32	36.905	3.43
	37.812	3.48
33	37.732	5.89
34	37.730	3.39
	.732	3.18
	.874	3.18
35	35.620	3.92
	37.812	3.95
36	37.730	5.15
37	37.812	5.11
38	38.283	2.14
	.286	2.22
39	37.730	3.64
	.732	3.55
40	37.730	5.05
	.732	5.64
41	35.620	3.47
	36.905	3.51
	37.812	3.56
42	37.732	5.22
43	37.732	5.87
44	37.732	5.62
45	35.620	3.81
	36.905	3.72
	37.812	4.31
46	37.732	5.91
47	37.732	5.45
48	35.620	1.16
	36.179	0.82
	.434	1.08
	.535	1.00

TABLE II.—*Continued.*

No.	Ep.	Mag.
48	36.601	1.06
	.867	1.19
	.905	1.24
	37.812	0.87
	.957	1.18
	.990	1.16
	.993	0.90
	38.004	1.05
	.015	1.10
	.018	1.04
	.034	1.13
49	37.732	5.92
50	37.730	3.58
	.732	3.31
51	37.732	5.41
52	37.730	4.95
	.732	4.95
	.874	4.98
53	37.730	4.45
54	37.732	5.80
55	37.812	3.07
56	37.812	4.90
57	38.018	5.07
58	35.620	4.73
59	37.730	5.25
	.732	5.37
60	35.620	3.74
	36.905	3.67
	37.812	3.78
61	35.620	5.16
	37.812	5.03
62	37.812	4.93
63	37.732	5.60
64	37.730	3.76
	.732	4.14

No.	Ep.	Mag.
65	35.620	2.95
	36.541	3.10
	.905	2.85
	37.730	3.17
	.812	3.01
	38.015	3.17
	.018	3.18
	.034	3.06
66	37.730	3.45
	.874	3.61
67	37.732	5.10
68	37.732	4.44
69	36.905	2.28
	37.730	2.21
	.812	1.87
	.874	1.88
70	37.732	5.03
71	37.732	5.48
72	35.620	3.55
	36.541	3.25
	.905	3.58
	37.812	3.66
	38.018	3.77
73	37.730	5.35
74	37.732	5.28
75	37.732	5.76
76	35.620	4.30
	36.905	4.12
	37.812	3.74
	38.018	3.99
77	37.730	4.61
78	37.874	4.12
79	35.620	4.02
	.905	3.77
	37.730	3.88
	.812	4.19
	38.018	4.45

No.	Ep.	Mag.
80	37.732	5.81
81	37.730	5.30
	.732	5.84
82	37.730	4.86
83	37.732	4.71
84	37.730	4.14
	.812	4.25
85	37.730	4.55
86	37.812	4.80
87	37.812	4.13
88	37.730	3.52
	.812	3.70
	.874	3.25
89	35.620	4.25
	36.905	4.05
	37.812	3.74
	38.018	4.04
90	37.874	4.44
91	37.730	4.09
	.874	4.02
92	37.730	4.51
	.874	4.38
93	38.018	5.72
94	35.620	4.64
	36.905	4.62
	37.812	4.83
	38.018	5.12
95	37.730	5.20
	.874	4.72
96	37.730	4.04
	.874	3.77
	.874	3.90
97	37.812	5.28
98	36.905	3.28

TABLE II—*Continued.*

No.	Ep.	Mag.
98	37.730	3.00
	.812	3.24
	38.018	3.34
99	37.732	5.90
100	37.730	2.92
	.732	2.58
	.812	2.55
	.874	2.61
101	36.905	5.17
	37.812	5.44
	38.018	5.72
102	37.732	5.77
103	37.730	4.24
	.874	4.07
104	37.812	5.39
105	37.732	5.68
106	37.730	3.93
107	37.730	4.91
	.874	4.78
108	37.730	4.81
109	37.732	5.72
110	37.730	3.99
	.874	3.38
111	38.018	4.56
112	36.905	5.08
	37.812	4.77
	38.018	5.05
113	37.874	3.56
114	37.874	3.66
115	37.874	4.91
116	37.732	5.82
117	37.730	3.82
	.732	3.80

No.	Ep.	Mag.
117	37.874	3.61
118	36.905	5.02
	37.812	4.67
	38.018	5.05
119	37.730	4.20
	.874	4.23
120	37.732	5.85
121	37.874	3.96
122	37.730	5.10
	.732	5.52
123	38.018	4.63
124	37.730	3.24
	.874	3.02
125	37.730	3.70
	.732	3.68
	.874	3.31
	.874	3.13
126	37.874	4.42
127	37.732	5.84
128	37.874	5.09
129	35.620	4.77
	37.730	4.76
	38.018	4.60
130	37.874	4.79
131	37.874	4.79
132	37.874	3.13
133	37.732	5.16
	.874	4.42
134	37.730	4.40
	.874	4.18
135	37.874	3.90
136	37.874	5.34

No.	Ep.	Mag.
137	37.730	5.00
138	36.179	3.76
	.905	3.97
	37.812	4.09
	38.004	3.96
	.018	4.17
139	37.730	4.34
	38.018	4.29
140	37.730	4.30
	38.018	4.12
141	37.730	4.66
	.874	4.84
142	35.620	3.62
	36.179	3.38
	.535	3.31
	.541	3.30
	.905	3.01
	37.812	3.28
	38.004	3.45
	.015	3.32
	.018	3.40
	.034	3.23
143	37.732	5.76
144	37.732	5.76
145	37.730	4.71
	.874	4.54
146	38.018	6.08
147	37.874	3.82
148	37.812	4.96
	38.018	4.95
149	37.732	5.71
150	37.812	4.63
	38.018	4.70
151	37.812	5.23
	38.018	5.11
152	38.018	5.69

TABLE II — *Continued.*

No.	Ep.	Mag.
153	38.018	4.90
154	37.874	4.33
155	38.018	3.81
156	36.905	4.45
	37.812	4.39
	38.018	4.48
157	36.179	3.58
	.535	3.52
	.905	3.67
	37.812	3.43
	38.004	3.62
	.015	3.57
	.018	3.62
	.034	3.54
158	36.905	4.81
	37.812	5.14
	38.018	4.92
159	36.869	5.24
160	38.018	4.37
161	38.018	5.69
162	38.018	5.18
163	36.867	1.33
	37.812	1.54
	.957	1.50
	.993	1.16
	38.015	1.38
	.034	1.31
	.237	1.24
	.283	1.47
164	36.869	3.69
	38.018	4.08
165	38.018	4.33
166	36.179	3.41
	.535	3.40
	.905	3.40
	37.812	3.33
	38.004	3.41
	.015	3.52
	.018	3.52

No.	Ep.	Mag.
166	38.026	3.42
	.034	3.47
167	36.869	5.38
	38.018	5.69
168	37.874	4.60
	38.018	4.21
169	38.018	5.69
170	38.018	5.69
171	38.018	4.82
172	38.018	4.77
173	38.018	5.69
174	38.018	5.69
175	36.869	3.99
	38.018	4.25
176	38.015	5.73
177	38.018	6.08
178	38.018	5.69
179	36.869	3.11
180	36.869	4.68
181	37.993	5.24
182	36.869	3.52
183	38.018	5.69
184	38.018	4.85
185	38.018	5.69
186	36.869	3.60
187	36.869	5.00
188	37.993	5.42
	38.015	5.24
189	38.018	5.19

No.	Ep.	Mag.
190	36.869	4.61
	38.018	5.03
191	38.018	5.72
192	38.015	5.99
193	38.018	4.80
194	38.018	6.08
195	36.869	3.48
	37.957	3.50
	38.018	3.57
196	36.867	2.85
	37.812	2.81
	.957	3.22
	38.004	2.75
	.015	3.12
197	38.015	5.99
198	36.869	4.52
	38.018	4.97
199	36.905	4.75
	38.015	4.90
200	37.993	5.56
201	38.015	5.76
202	36.869	4.89
	38.018	4.52
203	36.869	3.56
	37.957	3.62
	38.018	3.62
204	36.869	4.89
	38.018	4.52
205	38.283	0.87
206	36.223	0.99
	.867	1.05
	.869	1.33
	.905	0.96
	37.812	1.17
	.957	0.97
	.990	1.05

TABLE II—*Continued.*

No.	Ep.	Mag.
206	37.993	0.81
	38.004	0.89
	.015	0.96
	.034	0.92
	.149	0.95
	.152	0.94
	.237	0.92
	.283	1.15
207	38.018	5.72
208	36.869	4.10
209	36.869	4.89
	38.018	4.66
210	37.993	5.03
211	36.869	5.55
212	38.018	5.72
213	38.015	5.73
214	38.018	5.72
215	36.867	3.41
	.869	3.56
216	38.034	2.01
	.283	2.17
217	36.223	2.06
	.867	1.68
	.869	1.59
	.905	2.23
	37.957	2.16
	.990	2.15
	.993	1.86
	38.004	2.01
	.015	1.98
	.026	2.08
	.152	1.86
218	38.018	6.08
219	38.015	5.73
220	36.867	2.99
	.869	3.05
	.905	3.25
	37.957	2.93
220	38.004	2.70
	.015	3.02
	.018	2.78
	.026	3.07
	.034	2.92
221	38.015	5.45
222	36.223	2.15
	.867	2.37
	.869	2.49
	.905	2.60
	37.957	2.42
	38.152	2.34
223	38.018	3.95
224	36.867	2.57
	.869	2.70
	.905	2.68
	37.812	2.64
	.957	2.70
	38.004	2.54
	.015	2.77
	.018	2.63
	.026	2.84
	.034	2.73
225	36.869	3.74
226	36.869	4.35
227	36.867	3.13
	.869	3.00
	37.957	3.04
	38.034	3.01
228	36.223	2.02
	.867	1.74
	.869	1.90
	.905	1.84
	37.957	1.64
	.990	1.63
	.993	1.81
	38.004	1.94
	.015	1.80
	.026	1.84
	.034	1.69
	.152	1.86
229	38.018	5.72
230	36.535	3.81
	.905	4.22
	37.812	3.86
	.993	3.34
	38.018	3.72
	.034	4.07
231	36.869	4.44
232	36.223	2.09
	.867	1.95
	.869	1.99
	.905	1.96
	37.957	1.95
	.990	1.98
	.993	1.81
	38.004	2.01
	.015	1.86
	.026	1.96
	.034	1.83
	.152	1.98
233	36.867	2.78
	9.05	2.89
	37.812	2.87
	.957	2.82
	38.004	2.80
	.018	2.71
	.026	2.96
	.034	2.82
	.149	2.79
234	38.018	5.72
235	37.993	5.66
236	36.869	3.88
	37.957	3.56
237	38.018	5.09
238	36.869	3.65
	37.957	3.66
239	36.223	2.20
	.867	2.44
	.869	2.22
	.905	2.47
	37.812	2.36
	.957	2.26
	.990	2.18
	38.004	2.42
	.015	2.42

TABLE II — *Continued.*

No.	Ep.	Mag.
240	38.015	3.96
241	36.179	4.33
	.905	4.50
	37.812	4.50
	.993	4.52
242	36.869	3.94
	37.957	3.82
243	36.869	3.16
	.905	3.36
	37.957	3.16
	38.018	3.06
	.026	3.18
244	37.993	5.06
	38.015	4.86
245	38.015	5.73
246	36.223	1.40
	.867	1.05
	.869	1.33
	.905	1.12
	37.812	0.70
	.812	1.02
	.957	1.35
	.993	1.07
	38.004	1.13
	.015	1.28
	.034	1.22
	.152	1.20
	.237	1.14
	.283	1.27
247	38.018	4.87
248	36.905	4.87
	37.812	4.56
	.933	5.18
249	38.283	2.22
250	36.869	3.83
	37.957	3.77
251	38.018	4.99
252	38.018	5.72
253	38.015	5.73

No.	Ep.	Mag.
254	37.272	4.48
	38.018	4.73
255	36.179	5.07
	37.993	5.28
256	37.272	3.89
	38.018	4.42
257	38.018	5.72
258	38.015	5.42
259	37.272	4.76
	38.018	5.01
260	37.993	5.14
261	37.993	5.80
262	37.993	5.00
263	37.993	5.83
264	37.993	5.75
265	37.272	4.48
266	37.993	5.62
267	36.867	3.20
	.869	3.21
	.905	3.09
	37.272	2.94
	38.018	2.92
	.034	3.17
268	36.223	2.18
	.867	2.16
	.869	2.29
	37.957	2.32
	.990	2.22
	38.004	2.48
	.015	2.30
	.026	2.30
	.034	2.31
	.149	2.30
269	37.272	3.67
270	37.993	5.85

No.	Ep.	Mag.
271	36.114	0.43
	.116	0.60
	.179	0.44
	.415	0.41
	.535	0.50
	.905	0.50
	37.188	0.44
	.812	0.45
	.957	0.44
	990	0.40
	.993	0 38
	38.015	0.40
	.056	0.44
	.149	0.41
	.152	0.41
	.237	0.39
	.283	0.48
272	38.015	5.99
273	37.993	6.03
	.993	6.39
274	37.272	5.00
275	37.993	5.69
276	37.993	6.36
277	37.272	6.19
278	37.993	5.69
279	38.015	5.73
280	37.993	5.98
281	37.993	6.01
	38.015	5.99
282	36.223	2.16
	.867	2.23
	38.015	2.25
	.034	2.27
	.152	2.34
	.286	2.14
283	38.034	5.31
284	38.034	4.26
285	36.179	5.07
	38.015	4.82

TABLE II — *Continued.*

No.	Ep.	Mag.
286	38.034	4.75
287	36.869	3.45
	.905	3.17
	37.270	2.99
	.272	3.36
	38.015	3.37
	.152	3.34
288	38.015	5.73
289	36.116	0.24
	.179	0.05
	.415	0.10
	.905	0.10
	37.957	0.06
	.990	0.10
	.993	0.08
	38.015	0.07
	.056	0.07
	.149	0.10
	.152	0.06
	.237	0.16
	.283	0.17
290	37.272	5.86
291	37.272	5.90
292	37.272	3.62
	38.034	3.83
293	37.272	5.99
294	37.272	6.17
295	36.179	3.17
	.905	2.93
	37.270	3.19
	.272	3.19
	.957	3.28
	38.004	2.97
	.015	3.02
	.152	3.09
296	36.179	3.47
	.535	3.03
	.905	3.05
	37.812	3.33
	38.004	3.70
	.015	3.26
	.018	3.52

No.	Ep.	Mag.
296	38.026	3.42
	.034	3.41
	.152	3.38
297	36.179	5.52
	38.015	4.60
298	38.149	4.29
299	37.272	3.76
	38.034	3.99
300	38.015	5.73
301	36.223	1.88
	.867	1.60
	.869	1.70
	.905	1.77
	37.272	1.88
	.957	1.76
	.990	1.84
	.993	1.81
	38.004	2.01
	.015	1.74
	.018	1.72
	.026	1.73
	.034	1.74
	.149	1.80
	.152	1.95
	.237	1.86
302	37.272	6.14
303	37.272	3.45
	38.034	3.75
304	36.867	3.27
	.869	3.35
	37.272	3.31
	38.026	3.42
	.034	3.41
	.149	3.24
305	36.179	5.52
	38.015	5.73
306	36.867	3.61
	38.034	4.60
	.149	4.39
307	38.015	5.99

No.	Ep.	Mag.
308	37.993	5.72
309	38.015	5.73
310	36.179	5.07
	38.015	5.49
311	38.015	5.73
312	36.179	5.52
313	36.223	2.16
	.867	2.09
	.869	2.07
	.905	2.12
	37.272	2.35
	.957	2.10
	.990	2.10
	.993	2.14
	38.004	2.29
	.015	2.09
	.018	1.90
	.026	2.12
	.034	2.08
	.056	2.16
	.149	2.05
	.152	2.18
	.237	2.14
314	36.179	4.90
	38.015	5.73
315	36.179	4.90
	38.015	4.96
316	36.179	5.07
	38.015	5.47
317	37.272	4.55
318	36.179	4.90
	.869	4.05
	38.015	4.93
319	36.179	3.97
	.905	3.75
	37.812	3.83
	.993	3.89
	38.004	3.83
	.015	4.01
	.018	3.91
	.026	3.68

TABLE II.—*Continued.*

No.	Ep.	Mag.
319	38.034	3.83
	.149	3.86
	.152	3.81
320	37.993	6.41
321	38.115	5.50
322	36.179	5.52
323	36.869	2.82
	37.270	2.75
	.272	2.63
	37.957	2.64
	38.015	2.62
	.026	2.76
	.149	2.69
	.152	2.58
324	36.179	4.33
	.905	4.02
	37.812	4.06
	.993	4.25
	38.152	4.25
325	38.015	5.73
326	36.867	2.65
	.869	2.63
	37.957	2.53
	38.015	2.52
	.026	2.53
	.034	2.54
	.149	2.56
	.152	2.48
327	36.869	2.88
	38.004	3.23
	.015	3.07
	.018	2.98
328	38.015	5.73
329	36.179	5.52
330	36.869	3.42
	37.270	3.43
	.272	3.45
	.957	3.34
	38.015	3.47
	.018	3.46
	.152	3.43

No.	Ep.	Mag.
331	36.223	1.98
	.867	1.82
	38.004	1.70
	.034	2.16
	.286	1.93
332	36.179	5.52
333	37.272	4.69
334	36.179	5.52
	38.015	5.73
335	37.272	4.92
336	36.223	1.17
	.867	1.12
	37.990	1.27
	.993	0.99
	38.004	0.97
	.015	1.08
	.018	0.94
	.034	1.04
	.152	1.08
	.237	1.02
	.283	1.01
337	37.272	5.30
338	36.179	5.76
	38.015	5.99
339	36.869	4.17
	37.272	3.71
	38.149	3.91
340	36.179	5.76
	38.015	5.99
341	38.149	4.88
342	37.272	6.22
343	36.223	1.69
	.867	1.54
	37.993	1.51
	38.004	1.30
	.015	1.54
	.034	1.52
	.149	1.49
	.283	1.56

No.	Ep.	Mag.
344	37.272	5.37
345	37.272	3.99
	38.149	4.12
346	37.272	6.02
347	37.272	3.57
348	36.179	5.52
	37.272	4.84
349	38.015	5.99
350	36.179	4.25
	.905	4.31
	37.812	4.28
	.993	4.10
	38.015	4.21
	.034	4.16
351	36.869	3.30
	37.272	3.41
	38.149	3.18
352	36.179	5.52
353	36.179	4.90
	38.015	4.64
354	37.272	3.80
355	38.015	5.99
356	36.179	4.90
	38.015	5.00
357	36.179	4.90
	38.015	4.77
358	36.179	3.66
	38.015	3.66
	.026	3.60
	.034	3.75
	.149	3.64
	.152	3.57
359	36.179	4.90
	38.015	5.03
360	37.993	6.48

TABLE II — *Continued.*

No.	Ep.	Mag.
361	37.993	6.06
362	36.179	5.07
363	36.116	2.46
	.179	2.40
	.867	2.51
	.869	2.35
	37.270	2.54
	.272	2.54
	.957	2.37
	.990	2.22
	38.015	2.47
	.018	2.48
	.026	2.38
	.034	2.38
	.149	2.45
	.152	2.42
	.237	2.34
364	36.867	2.92
	37.272	3.07
	38.149	2.96
	.152	2.82
365	38.015	5.73
366	38.149	5.16
367	38.015	5.73
368	36.116	1.78
	.179	1.88
	.223	2.11
	.415	1.95
	.867	2.02
	.905	2.12
	37.188	1.76
	.270	1.98
	.270	1.85
	.272	2.06
	.957	2.03
	.990	2.05
	.993	1.92
	38.004	2.08
	.015	1.98
	.026	1.88
	.034	1.93
	.056	1.80
	.149	1.91
	.149	1.97
	.152	2.02

No.	Ep.	Mag.
368	38.237	2.11
369	36.179	5.07
	38.015	5.17
370	37.993	5.10
	38.015	5.20
371	36.179	5.76
372	36.179	4.33
	.905	4.40
	37.812	4.42
	.993	4.57
	38.015	4.30
373	36.179	5.07
	38.015	5.17
374	38.004	3.79
375	36.179	5.76
	37.993	5.10
	38.015	5.20
376	36.179	5.76
	38.015	5.73
377	38.149	4.07
378	36.116	1.90
	.179	1.96
	.223	2.13
	.415	2.02
	.905	2.23
	37.990	2.13
	.993	2.08
	38.015	2.04
	.026	2.04
	.034	1.88
	.056	1.99
	.149	2.00
	.149	2.14
	.152	2.08
	.237	2.08
	.237	2.16
379	36.179	4.90
	37.812	5.51
	.993	4.96
380	36.230	4.32

No.	Ep.	Mag.
380	36.905	4.35
	37.812	4.44
	38.026	4.45
	.152	3.95
381	38.015	5.73
382	37.812	5.58
383	38.015	5.99
384	36.230	4.51
	.905	4.56
	38.026	4.78
	.152	4.61
385	36.179	4.25
	.240	4.10
	.905	3.95
	37.812	4.22
	38.015	4.11
386	36.179	5.52
	38.015	5.99
387	36.179	5.76
	38.015	5.99
388	38.004	4.40
389	36.179	5.76
390	38.004	4.83
391	38.015	5.36
392	36.179	5.52
	38.015	5.38
393	36.179	4.90
	38.015	5.14
394	37.272	4.35
	38.015	4.26
	.034	4.35
	.149	4.77
	.152	4.51
395	36.179	4.33
	38.015	4.52
396	36.179	4.25
	38.015	3.72

TABLE II—*Continued.*

No.	Ep.	Mag.
397	38.004	4.47
398	36.179	5.76
	38.015	5.07
399	38.015	5.99
400	36.179	4.90
401	36.179	5.76
	38.015	5.73
402	37.272	4.03
	38.015	3.86
	.026	4.02
	.149	4.02
	.152	4.10
403	36.179	5.52
404	38.015	5.27
405	36.179	5.76
	38.015	5.52
406	36.179	4.90
	38.015	5.10
407	38.004	3.37
408	38.149	4.94
409	36.116	2.12
	.179	2.28
	.223	2.22
	.415	2.15
	.905	2.33
	37.993	2.20
	38.004	2.22
	.015	2.14
	.018	2.15
	.026	2.19
	.034	2.20
	.149	2.14
	.149	2.18
	.152	2.24
	.237	2.16
410	36.179	4.33
	38.015	4.48
411	36.179	5.07

No.	Ep.	Mag.
411	38.015	5.30
412	38.015	5.99
413	37.272	4.35
	38.149	4.45
	.152	4.83
414	36.179	5.52
	38.015	5.73
415	36.179	5.52
	38.015	5.73
416	38.004	3.02
	.018	3.12
	.152	2.89
	.283	3.17
	.286	3.02
417	37.270	5.21
418	38.015	5.73
419	38.149	5.05
420	36.179	4.33
421	36.179	5.52
	38.015	5.40
422	36.179	5.76
	38.015	5.53
423	38.015	5.73
424	38.015	5.73
425	36.179	5.76
	38.015	5.54
426	38.015	5.99
427	36.179	5.76
	38.015	5.99
428	36.179	4.33
	38.015	4.69
429	36.179	4.25
	.240	4.19
	.905	4.19

No.	Ep.	Mag.
429	37.812	4.02
	38.015	4.34
430	36.179	2.22
	.415	2.21
	37.270	2.30
	.993	2.55
	38.015	2.20
	.018	2.32
	.026	2.26
	.149	2.20
	.149	2.25
	.152	2.29
431	38.026	5.33
432	37.812	5.00
	38.026	5.33
433	36.179	3.89
	38.015	3.81
434	36.179	4.33
435	36.116	1.78
	.179	1.81
	.223	2.04
	.415	1.88
	.905	2.07
	37.188	1.76
	.270	2.05
	.270	2.08
	.272	2.21
	.957	1.86
	.990	2.01
	.993	1.97
	38.004	2.01
	.015	1.92
	.026	2.00
	.034	1.83
	.056	1.90
	.149	1.88
	.152	2.05
	.237	2.04
436	36.179	4.90
	38.015	5.30
437	38.283	3.03
438	36.116	2.34
	.179	2.34

TABLE II—*Continued.*

No.	Ep.	Mag.
438	36.413	2.39
	.415	2.32
	38.015	2.72
	.026	2.65
	.034	2.64
	.149	2.33
	.152	2.36
	.237	2.34
439	38.026	5.81
440	36.179	5.52
441	36.116	2.76
	.179	2.72
	.415	2.72
	38.015	2.57
	.026	2.46
	.034	2.46
	.149	2.72
	.152	2.78
442	36.223	2.22
	.867	1.88
	37.993	2.34
	38.004	1.78
	.015	2.09
	.018	2.07
	.026	2.22
	.034	2.20
	.056	2.12
	.149	2.00
	.149	2.10
	.152	2.15
	.237	2.08
	.237	2.14
	.283	2.03
	.286	2.10
443	38 004	5.07
444	38.004	4.72
445	37.272	4.13
	38.018	3.86
	.026	3.76
	.149	3.97
	.152	3.68
446	38.026	5.81
447	36.179	3.31
447	36.415	3.34
	38.026	3.14
	.149	3.10
448	38.026	5.58
449	36.179	5.07
450	37.272	5.44
451	36.179	4.33
	38.026	5.33
452	37.272	5.07
453	37.272	6.09
454	38.004	4.18
	.015	3.91
	.149	4.02
455	38.018	5.16
	.149	5.10
456	36.179	5.07
457	38.018	5.16
	.149	5.10
458	38.026	5.81
459	38.283	3.07
460	37.272	5.78
461	36.179	3.68
	38.026	3.98
462	36.179	3.28
	.415	3.20
	38.015	2.87
	.026	3.18
	.034	3.11
	.149	3.02
	.149	3.14
	.152	3.18
463	37.272	5.56
464	38.004	4.23
	.149	4.23
465	37.272	5.96
466	37.272	5.14
467	37.272	6.24
468	36.179	3.74
	37.272	4.08
	.993	4.88
469	37.993	6.43
470	38.149	5.21
471	37.993	6.46
472	38.004	4.95
473	37.272	6.07
474	38.149	4.50
475	36.179	1.55
	.223	1.63
	37.993	1.70
	38.026	1.80
	.056	1.67
	.152	1.64
	.237	1.67
	.283	1.64
476	38.026	5.58
477	37.272	5.62
478	38.026	5.33
479	37.272	5.74
480	37.272	5.82
481	37.272	4.29
	38.015	4.56
	.149	4.18
482	38.026	5.81
483	37.272	6.05
484	36.179	3.44
	.415	3.38
	38.015	2.92
	.026	3.52

TABLE II—*Continued.*

No.	Ep.	Mag.
484	38.034	3.61
	.149	3.41
	.149	3.44
485	36.179	3.55
	.415	3.60
	38.026	3.34
	.034	3.29
	.149	3.30
	.149	3.32
486	36.114	2.12
	.223	1.96
	38.026	2.30
	.056	2.06
	.152	2.21
	.237	2.14
	.283	2.08
	.286	2.07
487	38.283	2.99
488	36.179	5.52
489	36.179	5.07
490	36.179	5.76
	38.026	5.58
491	38.004	4.09
	.149	3.97
492	38.149	4.66
493	36.179	4.25
	.230	4.28
	38.026	4.18
494	36.179	5.52
495	36.179	4.33
496	36.179	5.07
	38.026	5.58
497	36.179	3.52
	.415	3.56
	38.026	3.95
498	37.272	5.50
499	38.026	5.81

No.	Ep.	Mag.
500	36.179	5.07
501	37.272	4.24
502	36.179	5.52
503	36.179	4.33
	.240	4.19
	38.026	4.30
	.152	4.96
504	36.179	4.90
505	36.179	3.05
	.415	3.03
	38.026	2.92
	.149	2.75
	.152	2.97
	.237	2.87
506	36.179	5.76
507	36.114	1.75
	.116	1.50
	.179	1.55
	.223	1.48
	.415	1.55
	.867	1.47
	.905	1.54
	37.188	1.48
	.272	1.65
	.957	0.73
	.990	0.65
	.993	0.71
	38.015	0.81
	.018	0.72
	.034	0.80
	.056	0.97
	.149	0.83
	.149	0.94
	.152	0.80
	.237	0.82
	.283	1.38
508	36.179	2.81
	.415	2.45
	37.272	2.80
	38.004	2.88
	.015	2.77
	.026	2.76
	.149	2.79
	.149	2.91

No.	Ep.	Mag.
508	38.149	2.80
	.152	2.82
	.237	2.62
509	36.179	5.52
510	37.188	3.01
	38.004	3.28
	.015	3.42
	.149	3.02
	.152	3.26
	.283	3.12
511	36.179	4.33
	.230	4.46
	38.026	4.33
	.152	4.03
512	38.004	3.75
	.149	3.80
	.152	3.68
513	36.179	4.25
	38.026	3.88
514	38.004	4.28
	.286	4.13
515	38.283	2.44
	.286	2.39
516	38.237	1.93
	.283	1.86
	.286	1.95
517	38.026	5.33
518	38.026	5.58
519	38.026	5.81
520	38.283	2.97
	.286	2.91
521	38.026	4.52
522	38.026	5.81
523	37.270	4.61
	38.004	4.54
	.149	4.56

TABLE II—*Continued.*

No.	Ep.	Mag.
524	36.179	5.07
525	36.223	2.54
	38.283	2.60
	.286	2.48
526	38.026	5.81
527	38.286	3.39
528	38.149	4.61
529	38.283	3.45
530	38.004	3.49
	.149	3.47
	.149	3.50
531	36.179	4.33
532	38.286	3.70
533	38.004	4.14
534	36.179	5.52
	38.026	5.33
535	38.026	5.58
536	37.270	3.71
	38.149	3.58
537	36.179	4.90
538	38.026	5.58
539	38.026	5.58
540	36.179	3.49
	.415	3.29
	38.026	3.45
	.149	3.18
541	38.149	5.00
542	38.283	4.30
543	38.026	5.58
544	38.026	5.81
545	38.026	5.58

No.	Ep.	Mag.
546	37.270	5.08
547	36.179	3.94
	.415	3.90
	38.026	3.83
548	36.179	4.90
	38.026	4.78
549	36.179	5.76
	38.026	5.81
550	36.223	1.92
	38.026	2.38
	.056	2.23
	.237	2.16
	.237	2.23
	.283	2.26
	.286	2.30
551	36.179	5.07
	38.026	4.78
552	37.270	4.88
553	38.026	5.81
554	38.149	3.74
	.283	3.40
	.286	3.59
555	36.179	5.52
	38.026	5 58
556	38.004	3.92
557	38.237	2.34
	.283	2.30
	.286	2.43
558	36.179	5.07
	.230	4.66
	38.026	4.37
	.152	5.23
559	36.179	5.07
	38.026	5.33
560	36.179	5.52
	38.026	5.58
561	36.179	5.76

No.	Ep.	Mag.
562	36.179	4.90
	.240	4.16
	38.026	4.78
563	36.116	2.76
	.179	2.57
	.415	2.67
	.415	2.67
	37.188	2.52
	38.026	2.61
	.149	2.85
	.149	2.84
	.152	2.70
	.237	2.62
564	37.188	3.97
	.270	4.01
	38.004	4.62
565	36.415	2.85
	37.188	3.01
	.270	2.80
	.272	2.88
	38.004	3.06
	.026	2.73
	.149	2.91
	.152	3.01
	.283	2.93
	.286	2.75
566	36.179	4.25
	37.270	4.23
567	36.179	5.76
568	36.179	3.21
	.415	2.99
	37.188	3.43
	.270	3.25
	.993	3.17
	38.004	3.19
	.026	3.26
	.149	3.08
	.149	3.07
	.152	3.26
569	36.223	2.31
	.415	2.58
	37.188	2.42
	.270	2.50
	38.004	2.70
	.149	2.59

TABLE II — *Continued.*

No.	Ep.	Mag.
569	38.152	2.65
	.283	2.60
	.286	2.57
570	36.179	4.90
	38.026	4.78
571	36.179	4.25
	.230	4.60
	38.026	4.41
	.152	4.42
572	38.283	3.20
	.286	3.09
573	36.179	4.90
	.240	4.13
	38.026	4.56
574	36.179	5.52
575	37.270	3.56
	38.149	3.82
	.283	3.55
	.286	3.83
576	37.188	5.41
577	36.179	3.82
	.415	3.78
	37.188	3.77
	.270	3.86
	38.004	4.00
	.026	3.91
	.152	4.33
578	36.179	5.52
	38.026	5.58
579	38.149	4.72
580	38.026	5.58
581	36.114	1.18
	.116	1.38
	.179	1.22
	.223	1.31
	.415	1.22
	.434	1.25
	.867	1.26
	37.188	1.19
	.990	1.47

No.	Ep.	Mag.
581	37.993	1.31
	38.004	1.38
	.015	1.46
	.018	1.14
	.026	1.50
	.034	1.45
	.056	1.36
	.149	1.21
	.149	1.41
	.152	1.31
	.237	1.36
582	38.026	4.78
583	36.179	4.25
	37.270	4.31
	.993	5.32
	38.026	4.10
584	38.004	5.67
585	36.223	2.79
	.415	2.90
	37.188	2.75
	.270	2.66
	38.004	2.92
	.149	3.02
	.152	2.86
	.283	2.91
	.286	2.70
586	36.116	1.66
	.179	1.73
	.223	1.84
	.415	1.81
	37.188	1.94
	.270	1.90
	.990	1.89
	.993	1 75
	38.015	1.68
	.026	1.65
	.034	1.64
	.149	1.68
	.149	1.70
	.152	1.76
	.237	1.77
	.283	2.14
587	38.004 *	5.51
588	36.179	4.25
	.230	4.18

No.	Ep.	Mag.
588	36.415	4.00
	38.026	4.22
	.152	4.33
589	37.188	4.65
	.270	4.79
590	36.223	2.37
	.415	2.63
	37.188	2.52
	.270	2.58
	.272	2.72
	38.004	2.84
	.149	2.69
	.152	2.62
	.283	2.56
	.286	2.61
591	38.004	5.67
592	36.179	2.97
	.415	3.07
	37.272	3.01
	38.004	2.65
	.026	3.03
	.149	2.96
593	36.179	4.90
	.319	5.04
594	36.116	2.34
	.179	2.16
	.223	2.45
	.316	2.26
	.319	2.26
	.322	2.18
	.415	2.35
	.415	2.28
	37.188	2.15
	.270	2.35
	38.026	2.46
	.149	2.40
	.149	2.33
	.152	2.34
	.237	2.23
595	38.004	5.51
596	36.223	2.65
	.415	2.76
	37.188	2.52
	.270	2.66

TABLE II —*Continued.*

No.	Ep.	Mag.
596	37.272	3.01
	38.026	2.17
	.149	2.65
	.283	2.84
	.286	2,65
597	36.179	5.52
598	36.179	3.24
	.415	3.47
	37.272	3.13
	38.004	3.10
	.026	3.49
	.149	3.20
	.152	3.22
599	36.179	5.52
	37.270	5.49
600	36.114	1.94
	.116	1.52
	.179	1.64
	.223	1.75
	.415	1.73
	.601	1.67
	37.188	1.61
	.990	1.70
	.993	1.58
	38.004	1.62
	.015	1.61
	.018	1.54
	.026	1.58
	.034	1.58
	.056	1.67
	.149	1.57
	.152	1.57
	.237	1.57
	.283	1.72
601	37.993	5.49
602	36.179	4.90
603	37.993	4.92
604	36.179	4.25
	.415	3.96
	37.270	4.16
	38.026	4.26
605	36.179	4.90

No.	Ep.	Mag.
606	38.237	1.98
	.283	1.79
	.286	1.90
607	37.270	3.48
	38.283	3.36
	.286	3.48
608	37.270	5.65
609	38.237	2.87
	.283	2.86
	.286	2.70
610	36.179	3.92
	.230	4.23
	.415	3.82
	38.026	4.14
	.152	3.88
611	38.283	2.79
612	37.993	5.46
613	37.993	4.42
614	37.993	4.37
615	37.993	6.28
616	37.993	6.26
617	37.993	6.20
618	36.179	4.90
619	37.993	5.90
620	36.179	5.52
621	36.179	5.07
622	37.993	6.30
623	37.270	5.87
624	36.415	2.94
	38.026	3.30
	.149	3.11
	.152	3.14
	.283	3.15
	.286	3.02

No.	Ep.	Mag.
625	36.116	3.06
	.179	2.89
	.316	2.86
	.319	3.00
	.322	2.87
	.415	2.76
	37.188	3.13
	38.149	2.91
	2.83	2.89
626	36.179	4.90
627	36.179	4.90
628	36.114	1.37
	.116	1.22
	.179	1.34
	.223	1.31
	.415	1.34
	37.188	1.41
	.272	1.82
	.993	1.44
	38.018	1.35
	.056	1.52
	.149	1.47
	.149	1.50
	.237	1.47
629	38.237	2.14
	.283	2.12
	.286	2.18
630	37.993	6.24
631	37.993	4.71
632	37.993	5.92
633	36.319	4.18
	37.993	4.80
634	37.270	3.33
	.272	3.57
	38.283	3.27
	.286	3.23
635	36.116	2.56
	.179	2.51
	.316	2.51
	.319	2.47
	.322	2.47
	.415	2.45

TABLE II — *Continued.*

No.	Ep.	Mag.
635	36.415	2.36
	37.188	2.42
	38.149	2.45
	.152	2.39
	.152	2.55
636	36.179	4.33
	.319	4.49
637	36.179	5.07
	37.993	5.88
638	36.319	5.24
	37.993	6.17
639	37.993	6.22
640	36.179	3.63
	.316	3.41
	.319	3.42
	.322	3.37
	.415	3.74
	37.993	3.34
	38.004	3.58
	.149	3.52
641	36.179	3.60
	.316	3.30
	.319	3.42
	.322	3.37
	.415	3.69
	37.993	3.49
	38.004	3.53
	.149	3.55
642	36.319	4.40
643	38.237	2.16
	.283	1.98
	.286	2.00
644	36.179	5.52
	37.993	6.08
645	36.179	4.33
	.319	4.44
646	37.993	6.10
647	36.179	4.33
	.319	4.87

No.	Ep.	Mag.
648	36.319	5.46
649	37.993	6.10
650	36.116	2.76
	.179	2.66
	.316	2.74
	.319	3.00
	.322	2.87
	.415	2.56
	.415	2.49
	37.188	2.63
	.272	2.38
	38.149	2.55
	.152	2.86
651	36.179	5.52
652	37.272	2.96
	38.283	2.76
653	36.179	3.84
	.316	4.04
	.319	3.99
	.322	4.16
	.415	4.05
	37.993	3.64
	38.149	3.73
654	36.179	4.33
	.316	4.14
	.319	3.92
	.322	4.12
	37.993	4.66
655	36.179	4.25
	.319	4.92
	37.993	5.36
656	36.114	1.00
	.116	1.04
	.179	1.10
	.415	1.09
	.434	1.16
	.535	1.15
	37.188	1.27
	.990	1.37
	.993	1.24
	38.004	1.22
	.004	1.46
	.018	1.25
	.034	1.38

No.	Ep.	Mag.
656	38.056	1.18
	.149	1.39
	.149	1.32
	.152	1.41
657	37.993	6.15
658	36.179	4.33
	37.993	4.75
659	36.415	3.43
	38.149	3.36
660	36.116	2.24
	.179	2.28
	.223	2.17
	.316	2.12
	.319	2.03
	.322	2.33
	.415	2.27
	.415	2.23
	37.188	2.10
	.272	2.15
	38.056	2.12
	.149	2.29
	.283	2.26
661	37.993	6.12
662	36.179	5.52
663	37.993	6.50
664	37.272	4.28
665	37.993	5.59
666	37.272	4.16
667	36.114	0.80
	.179	0.96
	.415	0.80
	37.188	0.78
	.272	0.79
	38.018	0.84
	.056	0.74
	.149	0.80
	.237	0.70
	.283	0.69
668	36.179	3.71
	.316	3.83

TABLE II—*Continued.*

No.	Ep.	Mag.
668	36.322	3.77
	38.149	3.58
669	36.179	4.33
	37.993	4.62
670	38.283	3.50
671	37.272	4.49
672	36.319	5.27
673	36.319	4.54
674	36.179	5.52
	37.993	5.59
675	36.319	4.58
676	36.179	5.52
	.319	5.31
677	36.179	5.52
	.319	4.96
678	36.179	4.90
	.319	4.76
679	38.283	3.05
	.286	2.86
680	36.116	2.76
	.179	2.62
	.316	2.62
	.319	2.68
	.322	2.60
	.415	2.61
	.415	2.54
	37.272	2.69
	38.149	2.62
681	36.319	4.62
	37.993	4.31
682	36.114	0.61
	.116	0.85
	.179	0.65
	.223	0.64
	.415	0.63
	.434	0.75
	.535	0.80
	.601	0.77
682	37.188	0.63
	.990	0.65
	.993	0.61
	38.015	0.63
	.018	0.60
	.034	0.64
	.149	0.63
	.149	0.53
	.152	0.64
	.237	0.55
683	36.179	3.35
	.415	3.47
	37.270	3.40
	38.149	3.26
	.152	3.52
684	36.179	4.33
	.230	4.42
685	36.116	2.56
	.179	2.46
	.316	2.38
	.319	2.47
	.322	2.47
	.415	2.51
	.415	2.41
	37.272	2.46
	38.149	2.48
686	36.319	4.24
687	36.319	4.29
	.322	4.20
688	37.272	4.05
689	38.237	2.62
	.283	2.39
	.286	2.34
690	37.272	3.80
691	36.319	5.08
692	36.415	2.81
	37.272	3.04
	38.283	2.73
693	36.116	2.66
	.179	2.85
	.316	2.97
693	36.319	2.84
	.322	2.74
	.415	3.12
	37.272	3.04
	38.149	2.93
694	37.272	5.45
695	36.116	2.86
	.179	3.09
	.316	3.19
	.319	3.30
	.322	3.25
	.415	3.47
	38.149	3.22
696	38.283	2.14
	.286	2.10
697	37.272	6.11
698	36.316	3.98
	.319	4.12
	.322	4.03
699	37.272	3.36
700	38.283	3.31
701	36.319	4.34
	.322	4.19
	37.272	4.34
702	36.316	3.91
	.322	3.98
	.415	3.86
	37.272	3.25
703	36.179	3.79
	.316	3.67
	.319	3.54
	.322	3.48
	.415	3.64
	38.149	3.65
704	36.319	5.43
705	37.272	4.85
706	36.179	5.52
707	36.179	3.14

TABLE II—*Continued.*

No.	Ep.	Mag.
707	36.415	3.20
	37.270	3.11
	38.149	3.09
	.152	3.05
708	36.179	4.90
709	36.319	3.84
	.322	4.08
710	36.415	2.81
	37.272	3.00
	38.283	2.66
711	38.283	3.27
712	36.316	3.49
	.319	3.42
	.322	3.37
	.415	3.47
	37.272	3.22
	38.149	3.62
713	36.179	4.90
714	36.316	3.76
	.319	3.75
	.322	3.91
715	36.316	3.58
	.319	3.64
	.322	3.58
	.415	3.52
	37.272	3.50
	38.149	3.60
716	36.319	5.00
717	36.179	4.90
	38.152	5.08
718	36.116	2.96
	.179	3.01
	.316	3.08
	.319	3.15
	.322	3.00
	.415	3.16
	37.272	3.12
	38.149	2.98
719	36.319	5.16

No.	Ep.	Mag.
720	37.272	4.10
721	37.272	3.71
722	38.283	2.52
	.286	2.26
723	35.537	4.97
	37.272	3.66
724	36.319	5.12
725	36.319	5.19
726	37.272	2.79
	38.283	2.82
727	37.272	3.42
728	37.272	5.31
729	37.272	3.39
730	36.179	2.93
	.415	3.25
	37.270	3.03
	38.149	3.12
	1.52	3.01
731	37.272	3.85
732	37.272	4.49
733	35.537	4.10
	37.272	4.21
734	37.272	3.99
735	37.272	5.45
736	35.537	2.97
	37.272	3.19
	38.149	2.95
737	36.319	3.75
	.322	3.67
	37.270	3.63
738	35.537	2.63
	36.434	2.42
	37.272	2.46
	38.149	2.52

No.	Ep.	Mag.
739	36.319	5.39
740	37.272	4.49
741	36.319	5.35
742	35.537	2.80
	37.272	2.53
	38.149	2.78
743	36.319	4.80
	37.272	4.95
744	35.537	4.19
745	35.537	4.71
746	36.179	4.90
	38.152	4.33
747	36.179	4.33
748	35.537	5.05
749	35.537	4.28
750	35.537	5.40
751	35.537	5.14
752	37.272	2.88
753	35.537	4.62
754	37.272	3.46
755	37.272	3.76
756	35.537	3.12
	37.270	2.87
	.272	3.19
	38.149	3.34
757	36.179	4.25
	.230	4.37
758	37.272	5.60
759	35.537	4.80
760	37.272	5.19

TABLE II—*Continued.*

No.	Ep.	Mag.
761	35.537	1.49
	36.114	1.56
	.179	1.44
	.434	1.42
	.535	1.41
	.601	1.29
	37.188	0.90
	.272	1.59
	38.149	1.21
762	35.537	4.54
763	35.537	4.37
	37.270	4.46
	.272	4.66
764	35.537	5.22
765	37.272	3.89
766	35.537	4.88
767	37.272	2.92
768	36.179	4.25
769	35.537	3.12
	37.270	2.95
	.272	3.19
	38.149	3.07
770	35.537	4.46
	37.270	4.53
	.272	4.76
771	36.601	2.59
	37.272	2.83
772	36.179	4.90
773	36.116	2.02
	.179	2.09
	.223	2.14
	.415	2.08
	.434	2.00
	.535	1.82
	37.270	2.20
	38.149	2.01
	.152	2.12
774	35.537	2.38
	.540	2.41

No.	Ep.	Mag.
774	38.149	2.41
775	35.537	3.81
	38.149	3.00
776	35.537	3.91
777	35.537	4.01
	38.149	3.69
778	37.272	5.05
779	37.272	3.29
780	36.601	2.52
	37.272	2.64
781	35.537	3.50
	38.149	3.39
782	37.272	3.15
783	35.537	5.32
784	36.601	3.52
785	35.540	3.29
	38.149	2.89
786	35.540	3.50
	38.149	3.41
787	35.540	3.11
	38.149	3.04
788	35.537	2.97
	38.149	3.05
789	35.537	1.85
	36.179	2.03
	.434	1.75
	.535	1.72
	.601	1.84
	38.149	1.82
790	35.537	2.16
	.540	2.08
	36.179	2.22
	.434	2.17
	.535	2.28
	.601	2.00
	37.272	2.01

No.	Ep.	Mag.
790	38.149	2.06
791	37.272	2.27
792	36.601	3.69
793	35.537	2.63
	.540	2.57
	37.272	2.74
	38.149	2.82
794	35.537	3.12
	38.149	3.17
795	35.537	3.38
	.540	3.50
	38.149	3.24
796	36.601	3.01
797	35.540	3.37
798	36.601	4.22
799	35.540	3.50
800	35.540	2.77
	36.541	2.99
	.601	2.72
801	36.601	3.32
802	35.540	2.08
	36.434	1.92
	.535	2.21
	.601	2.08
803	35.540	3.78
804	35.540	2.85
	36.601	2.96
805	36.601	5.17
806	36.541	3.98
807	36.610	3.95
808	36.434	0.91
	.601	0.93

TABLE II—*Continued.*

No.	Ep.	Mag.
809	35.540	3.43
	36.541	3.40
	.601	3.56
810	36.601	4.39
811	35.540	2.41
	36.434	2.26
	.535	2.42
	.601	2.30
	37.590	2.14
812	36.601	5.71
813	36.601	3.78
814	35.540	3.70
	.601	3.64
815	35.540	5.09
816	36.601	4.78
817	35.540	4.55
818	35.540	2.57
	36.541	2.89
	.601	2.84
819	36.601	4.50
820	35.540	3.85
	36.601	4.08
821	35.540	4.11
822	35.540	3.65
	36.601	3.60
823	36.601	3.24
824	36.601	3.40
825	35.540	4.45
826	35.540	4.11
827	35.540	4.11
828	35.540	2.99
	36.541	3.15
	.601	3.12
829	35.540	5.08
830	35.540	3.93
831	35.540	4.67
832	35.540	4.22
	36.601	4.63
833	35.540	4.94
834	35.540	4.02
835	36.601	3.48
836	36.541	3.05
	.601	2.90
	37.590	2.63
837	35.540	5.26
838	36.434	1.50
	.535	1.30
	.601	1.19
	37.590	1.33
839	36.541	3.82
840	36.601	4.01
841	35.540	4.33
842	35.540	4.94
843	36.601	3.89
	37.590	4.02
844	35.540	4.94
845	35.540	4.80
846	35.540	4.94
847	35.540	5.46
848	36.541	3.62
849	36.601	3.40
850	37.590	4.21
851	37.590	3.53
852	37.590	3.10
853	36.415	2.23
	.434	2.09
	.535	2.21
	.601	2.16
	.905	2.02
	37.590	2.34
	.812	1.87
854	36.535	3.20
	.541	3.36
	.905	2.97
	37.590	3.33
855	36.541	3.51
	37.590	3.64
856	36.434	1.84
857	36.541	3.78
858	36.541	4.03
	37.590	3.22
859	37.590	2.72
860	36.541	3.72
861	37.590	3.58
862	35.620	4.43
863	36.535	2.49
	37.590	2.44
864	37.590	2.96
865	35.620	2.60
	36.535	3.13
	.541	3.20
	.905	3.21
	37.590	3.16
	.812	3.43
	.874	2.72
866	37.590	5.00
867	35.620	4.56
868	37.590	2.80

TABLE II—*Continued.*

No.	Ep.	Mag.
869	37.590	4.29
870	35.620	1.74
	36.434	1.75
	.535	1.62
	.601	1.76
	.905	1.62
	37.590	1.76
	.812	1.65
871	37.590	4.79
872	37.590	5.41
873	35.620	5.02
874	35.620	3.20
	36.535	2.94
	.541	2.95
	.905	2.73
	37.590	2.89
	.812	3.18
875	37.590	4.10
876	37.590	5.34
877	37.590	3.78
878	36.541	4.03
	37.812	4.16
879	37.590	4.48
880	35.620	4.16
881	35.620	4.21
882	37.590	3.69
883	37.590	5.28
884	37.590	4.38
885	37.812	5.65
886	37.590	3.93
887	37.590	5.21
888	36.541	3.93

No.	Ep.	Mag.
889	37.590	4.33
890	35.620	2.10
	36.434	2.34
	.535	2.35
	.905	2.18
	37.590	2.01
	.730	1.81
	.812	2.17
	.874	1.88
891	37.874	5.13
892	35.620	5.12
893	35.620	5.08
	37.590	4.14
894	35.620	3.08
	36.541	3.56
	.905	3.61
	37.590	3.48
	.812	3.52
895	37.590	4.06
896	37.590	4.66
897	37.590	3.88
898	37.590	3.43
	.874	2.83
899	37.590	4.73
900	36.434	1.59
	.535	1.53
	.601	1.50
	.905	1.44
	37.590	1.48
	.812	1.42
	.874	1.48
901	35.620	4.07
902	37.590	5.14
903	36.535	2.62
904	36.535	2.54
905	35.620	4.12

No.	Ep.	Mag.
906	37.590	4.93
907	37.590	3.97
	.874	3.44
908	37.590	4.86
909	35.620	3.29
	36.905	3.92
910	36.541	3.88
	.905	3.87
	37.812	3.91
911	37.590	4.18
912	37.590	4.42
913	37.590	4.60
914	35.620	4.47
915	35.620	4.52
916	37.732	4.34
917	35.620	4.90
	37.812	4.74
918	35.620	4.60
	36.905	4.26
	37.812	4.59
919	37.732	4.62
	Unidentified stars	
920	37.874	3.87
921	38.018	6.08
922	36.179	5.76
923	37.993	6.32
924	35.540	5.91

(*To be continued.*)

A METHOD OF DETERMINING THE LUMINOSITY CURVE OF THE SOLAR SPECTRUM.

By D. W. Murphy.

A study of the relations existing between the intensities of the different colors of which the spectrum is composed may, from the nature of the investigation required, be classed as a physical problem. From the nature of the question, however, it may be said to border upon, if not to belong more strictly to, the domain of modern physiology. In a certain sense the luminosity curve may be said to represent the distribution of energy in the spectrum, the energy being expressed in terms of the effects produced upon the special sense organs of sight. This is, however, not a measure of energy according to the physical meaning of the term, and results thus obtained may, and, as we well know, do differ very widely from those obtained by measuring the energy of the spectrum in terms of the thermal effects it produces. The latter may be said to represent the total energy of radiation, and is capable of expression in mechanical units. The accuracy of such measurements depends upon the efficiency of the apparatus designed to receive and record the ether vibrations, and is independent of any peculiarities of vision which the observer may possess.

The results of observations by different observers have shown that the relations between the light-giving powers of the different colors of the spectrum are not the same when judged by different individuals. These results also show that the variations are not great except in cases of abnormal vision.

One of the difficulties, and probably the greatest one in photometric measurements of this kind, is due to the fact that there is no standard of comparison of different colored lights. The somewhat varying results which have been obtained for the visual intensities of the different parts of various spectra have

been reached by more or less modified forms of two quite distinct methods.

One of these methods is to compare the different colors with a source of light, either white or some chosen color, and to give each a value in terms of this as a standard. The other method consists in allowing the different colors of the spectrum to fall upon printed characters or other suitable objects placed upon a screen. All the light except that of the color under consideration being excluded, the intensity of the source is regulated until the characters are just visible to the observer. From the data thus obtained the relative values of the intensities for the different colors are computed.

For the above described methods and the results obtained by them we are indebted to the works of Fraunhofer,[1] Langley,[2] Abney and Festing,[3] Vierordt,[4] and others.

The first of these methods involves the difficult task of comparing photometrically two lights of marked differences in color; while the second measures the intensities of the different colored lights by the amount of each that is reflected from an object whose coefficient of absorption for different wave-lengths is not known.

The following method for determining the luminosity curve of the solar spectrum was suggested to me by the consideration of a series of measurements I had made to determine the errors in the use of the bilateral slit as a means of varying the intensity in spectrophotometric measurements.

The method consists in dividing the spectrum into a large number of separate and adjacent elements, and building up the curve by a comparison of the relative intensities of these elements. Consider the spectrum as being composed of n very narrow equal areas, and let the values of the light-giving powers of each of these areas be denoted by $i_1, i_2, i_3 \ldots . i_n$. If the

[1] FRAUNHOFER, *Gesammelte Schriften*, Münchener Akademie, 1888.

[2] *American Journal of Science*, Vol. 136. "Energy and Vision."

[3] *Phil. Trans.*, 177, 423.

[4] *Annal. Phys. Chem.*, 137, 600.

values $\frac{i_2}{i_1}, \frac{i_3}{i_2}, \frac{i_4}{i_3} \; . \; . \; . \; . \; \frac{i_n}{i_{n-1}}$ be known, the value of the intensity of any area, as the rth, is found in terms of i_1 and is expressed $\left(\frac{i_2}{i_1}\right)\left(\frac{i_3}{i_2}\right)\left(\frac{i_4}{i_3}\right) \; . \; . \; . \; . \left(\frac{i_r}{i_{r-1}}\right) = \frac{i_r}{i_1}$. By giving to r all values from unity to n, this expression gives the values of the ordinates of the curve in terms of some particular one chosen as a standard. The values $\frac{i_2}{i_1}, \frac{i_3}{i_2}, \frac{i_4}{i_3}$, etc., are determined photometrically by comparing the intensities of each pair of adjacent elements. By thus using only small areas of the spectrum, the difference in color of the two lights under consideration in each case is not great and does not interfere with accurate settings of the photometer.

The necessary apparatus for measurements of this kind is some form of spectrophotometer, which allows all the spectrum to be cut off except the narrow area whose intensity is to be measured. The adjustments of the apparatus must also allow an element of one spectrum to be used as a comparison light for the other spectrum, both when the two spectra are exactly superposed, and when an area of the one is superposed upon the corresponding adjacent area of the other. The requirements of the instrument are fulfilled in the Lummer-Brodhun spectrophotometer. The adjustments of the instrument and the methods of taking the observations discussed in this paper were as follows:

The instrument was so placed that the two collimators were equally inclined and pointed toward open sky. No screens of any sort were used before the collimator slit. After various preliminary experiments this was found to be the most constant light source obtainable. The observations were made during the months of June and July, when the sky was free from clouds, and the only variation in intensity was that due to the changing position of the Sun, which during the time required for any set of observations was so small as to be negligible.

The width of the collimator slit used was 0.25 mm; the angle subtended by this width of slit, when viewed through the observing

telescope, was one sixty-third part of the total angle of dispersion of the prism, between the limits within which it was possible to work. The different parts of the spectrum were brought into the field of view by turning the telescope about the axis of the instrument. Collimator *A* (see Fig. 1), which is movable by means of a tangent screw about its axis *O*, was provided with a unilateral slit so arranged that it opened in the direction from the red to the green part of the spectrum; that is, light of a lesser wave-length was brought into the field when the width of slit was increased. Collimator *B* is rigid, and was fitted with a bilateral slit.

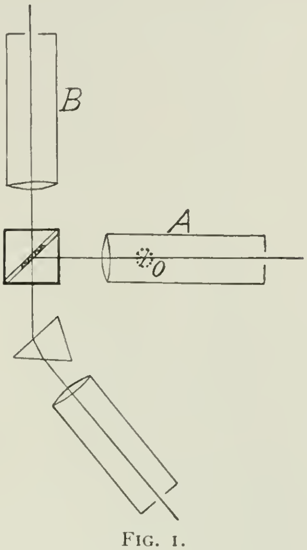


FIG. 1.

The observations were made as follows: first, the collimators were set so that the two spectra were exactly superposed, and the fields of the photometer lighted with lights of the same wave-lengths. The width of slit of *A* being 0.25 mm, slit *B* was changed until the fields showed equal intensities. Collimator *A* was then turned in the direction of the green part of the spectrum through the angular width of the slit, and the fields were again brought to the same intensities by changing the width of slit *A*. The mean values of the series of readings taken in this manner gave the relative intensities for these two elements of the spectrum. The telescope was then turned through the angle subtended by the slit width of 0.25 mm, adjustments similar to those above described were made, and a second series of readings taken. The data thus obtained gave the relative values of the intensities of the second and third areas.

In this manner the entire spectrum, from λ 6880 to λ 4500, was examined. The value of the intensity in the region λ 6880 I have called unity, and have given the values of the intensities

of the other parts of the spectrum in terms of this as a standard.

TABLE I.

Scale reading	Intensity	Scale reading	Intensity	Scale reading	Intensity
525	1.00	630	57.46	735	8.02
30	1.75	35	56.31	40	6.98
35	2.78	40	55.18	45	6.21
40	4.30	45	52.42	50	5.65
45	6.47	50	50.32	55	5.09
50	9.32	55	48.31	60	4.73
55	12.57	60	45.90	65	4.40
60	16.48	65	43.14	70	4.00
65	20.92	70	40.55	75	3.64
70	25.32	75	37.31	80	3.24
75	29.62	80	34.32	85	2.92
80	33.77	85	31.24	90	2.65
85	37.48	90	28.12	95	2.44
90	40.11	95	25.02	800	2.25
95	42.91	700	21.77	05	2.09
600	45.92	05	18.94	10	1.94
05	49.13	10	16.29	15	1.81
10	52.08	15	14.01	20	1.66
15	54.16	20	12.05	25	1.56
20	56.33	25	10.36	30	1.44
25	57.46	30	9.01	35	1.37

TABLE II.

Wave-length	Intensity	Wave-length	Intensity	Wave-length	Intensity
6880	1.00	6000	40.70	5200	31.24
6800	1.75	5900	46.62	5100	18.94
6700	3.08	5800	52.88	5000	12.05
6600	5.17	5700	56.90	4900	6.98
6500	8.75	5600	56.90	4800	4.73
6400	14.24	5500	52.42	4700	3.00
6300	20.92	5400	47.30	4600	1.94
6200	27.92	5300	40.55	4500	1.37
6100	33.77				

The results of the observations are given in Table I, the values of the intensities in the second column, the scale readings of the instrument for the regions of the spectrum having these intensities, directly opposite in the first column. These results are shown graphically by the curve (Fig. 2). This is the form of the luminosity curve of the solar spectrum for the particular prism used.

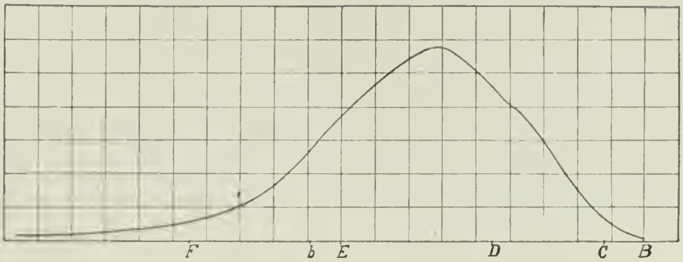


FIG. 2.

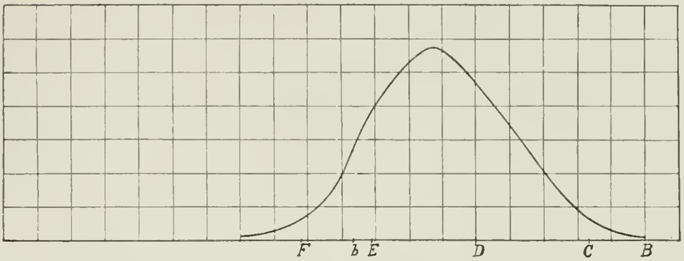


FIG. 3.

From the dispersion of the prism, together with the intensities given above, the results shown in Table II are obtained. The first column represents the wave-lengths, the second the corresponding intensities. The curve obtained from these data is shown in Fig. 3. It shows the form of the luminosity curve for the normal solar spectrum.

STANFORD UNIVERSITY,
January 1900.

SOME SPECTROGRAPHIC RESULTS OBTAINED AT THE INDIAN ECLIPSE BY THE LICK OBSERVATORY-CROCKER EXPEDITION.

By W. W. Campbell.

One of the instruments used by the Lick Observatory-Crocker Eclipse Expedition to India was an objective-grating spectrograph. It was designed for recording, continuously, on a moving plate, the changing spectrum of the Sun's edge at contacts II and III, as the edge was gradually covered and uncovered by the Moon's image; and for recording, during totality, on a fixed plate, the spectrum of the corona in the 1474 *K* region, and especially the monochromatic 1474 *K* ring.

The constants of the instrument were as follows:

A Rowland plane metallic grating, 14438 lines to the inch, ruled surface $2\frac{1}{16} \times 3\frac{3}{16}$ inches, third order, placed about five inches in front of the camera objective.

Camera objective a visual double lens of very ordinary merit, aperture $2\frac{5}{32}$ inches, focal length $20\frac{3}{4}$ inches.

Angle between the central ray incident on the grating, and the diffracted ray passing along the axis of the camera, 40°.

Center of the field at 1474 *K*, at which point, 1 mm of the spectrum equals 7.2 t. m.

Width of spectrum corresponding to the Sun's diameter, 0.20 inch.

A very light plate-holder of brass, containing a Cramer isochromatic plate $2\frac{1}{2} \times 7$ inches, was arranged to move lengthwise in a direction at right angles to the length of the spectrum, in a brass shield or slide of twice the extreme length of the plate-holder. Sensibly uniform motion, at the rate of $\frac{1}{16}$ inch per second, was imparted to the plate-holder by a small common clock-train propelled by a weight. The sensitive plate was mounted as near the front surface of the plate-holder as was

consistent with good construction. Immediately in front of the plate-holder, in contact with it, was a thin slide of brass containing a slit $\frac{1}{32}$ inch wide and $2\frac{1}{2}$ inches long, adjusted to coincidence with the axis of the spectrum. A color-screen of optical glass, stained greenish-yellow, was fixed in front of this slit, to absorb the violet of the overlapping 4th order spectrum. The slit could be removed, in an instant, by withdrawing the slide containing it; in which case no obstruction was offered to the green rays from any part of the coronal image.

The grating, brass camera tube, and plate-holder arrangements, were supported in a wooden mounting, constructed by the Observatory carpenter from my plans. The instrument was mounted on a large polar axis, in such a position that the rulings of the grating were parallel to the Sun's limb at the points of second and third contacts.

Three exposures were made as follows:

A. The plate-holder was set in motion by the clock-train, and an exposure made from 20 seconds before totality to 5 seconds after contact II. The source of light was the central $\frac{1}{32}$ inch length of the uneclipsed crescent.

B. The plate-holder was moved forward $\frac{1}{2}$ inch, and firmly clamped. The brass slide containing the $\frac{1}{32}$ inch slit was removed from in front of the plate, and a 96-second exposure on the corona spectrum was obtained, extending from 12 seconds after II to 12 seconds before III.

C. The plate-holder was unclamped and set in motion, the $\frac{1}{32}$ inch slit was replaced in front of the plate, and an exposure was made from 5 seconds before III to 16 seconds after III. The source of light was the central $\frac{1}{32}$ inch length of the reappearing crescent.

The available portion of the photograph, covering the region λλ 5150—5500, is nearly two inches in length; but as the lens is a very common one, of small field, the definition at the ends of the spectrum is necessarily poor.

The negative, which was fully developed, was found on examination to come up to expectations. The measurements

and the detailed discussion of the plate will be contained in an eclipse report, but its main features will be described below.

At the beginning of exposure *A*, 20 seconds before totality, the negative is strongly overexposed, as was anticipated, but the Fraunhofer lines are satisfactorily recorded, essentially with their usual intensities. The transition from dark lines to bright lines is well shown, and at this point many interesting features appear. Before the continuous spectrum ceased recording, many of the dark lines seem to have disappeared, or at least their intensities relative to the strength of the continuous spectrum have varied greatly. In some cases, *e. g.*, at λ 5430, the interval between a disappearing dark line and its corresponding bright line seems to be strictly continuous spectrum. In numerous other cases the dark lines and their bright counterparts coexist, apparently until the continuous spectrum becomes too weak to record itself as the necessary background for the dark lines. The relative intensities of the dark-line and bright-line spectra show marked anomalies at many points. The 1474 *K* line at the center of the plate, and the *b* group, near the edge of the plate, are the brightest lines recorded. They are of the same order of brightness, with perhaps the *b* lines the stronger, though the serious aberration effects at the edge of the plate do not permit photometric comparisons to be made. The 1474 *K* line is recorded many seconds before the bright *b* group appears at all, whereas the *b* group persists long after the 1474 *K* has disappeared. Without question, the *b* radiations proceed from a higher level in the solar atmosphere than do the 1474 *K* radiations. Similarly, the lines E_1 and E_2 at $\lambda\lambda$ 5271 and 5270 are comparable in brightness to the bright lines at $\lambda\lambda$ 5276 and 5284. The last two begin long before the E lines, but the E lines persist after $\lambda\lambda$ 5276 and 5284 have ceased to record themselves. There are many other cases equally marked. These phenomena of the dark and bright lines can scarcely fail to be of profound significance in the study of conditions existing at the Sun's edge.

About 125 bright lines are recorded in this section, of 350 tenth-meters. They are very closely related to the chromospheric

system of bright lines, as observed without an eclipse by Professor Young, at least so far as their wave-lengths are concerned. The intensities in many cases are quite different; but perhaps the chromospheric bright-line intensities vary widely at different times. It would have been exceedingly valuable, I think, to have had the chromosphere spectrum at the points of second and third contacts observed simultaneously, in the ordinary manner, at stations outside the line of totality, for comparison; and it seems desirable that such observations should be secured at some of the observatories at the time of the 1900 May eclipse. If the observations are undertaken with large telescopes and powerful spectrographs, under favorable atmospheric conditions, it may be found that the chromosphere spectrum as observed at home stations is essentially identical with the bright-line spectrum of the Sun's edge as observed at eclipse stations. Whether the results of the investigations made outside and inside the eclipse path should prove to be identical or quite different, the importance of the home observations, even if only partially successful, would in either case be very great.

The exposure *C* reproduced the above phenomena, in reverse order, at contact III.

The great strength of the calcium spectrum is shown by the fact that the H and K lines in the 4th order spectrum, after passing through the color-screen, recorded themselves faintly, and out of focus, on these photographs.

I wish to call special attention to one feature of this photograph which may be of great interest. In many, but not all, of the cases where dark and bright lines coexist, the bright lines do not have the same wave-lengths as the corresponding dark lines. The dark lines seem to be displaced toward the violet by as much as four or five tenths of a tenth-meter; and, in the case of the *b* and E groups of lines, possibly a little more. This effect is confined almost wholly to exposure *A* at contact II. Except in the case of the *b* group, it does not appear with certainty on exposure *C*, at contact III. I have endeavored, for many months,

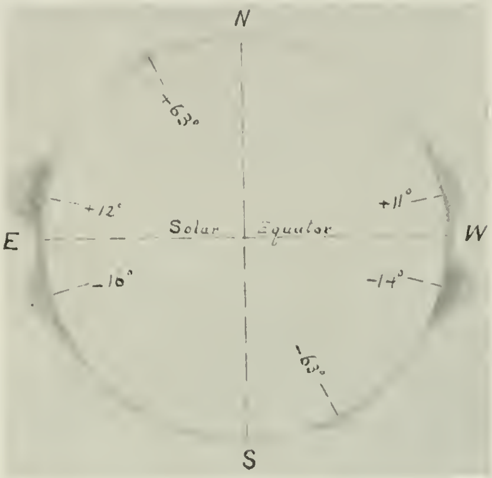
to explain this appearance as an instrumental effect, without success. If it is purely instrumental, should not the effect be essentially identical at the two contacts? If it is due to a faulty placing of the crescents on the slit in front of the plate, so that the spectral lines were not parallel to the direction of motion of the plate, it seems to me that at II the dark lines, if shifted at all, should be shifted toward the red, and at III toward the violet. Further, another photograph obtained on a moving plate, in the same manner, with a collimating spectrograph, slit radial, shows the same effect for the lines $H\gamma$ and $H\delta$, at both II and III. This negative, obtained with a very narrow slit, is underexposed, as was feared might be the case, and shows only the strongest of the bright lines. Its evidence supports the view that the displacement of the dark lines on the objective-grating plate represents a real phenomenon.

Assuming the reality of the displacement, we naturally consider its significance from the point of view of the effect of pressure upon wave-length. If this is a pressure effect, we conclude that the predominating absorption stratum for these lines is above the predominating radiation stratum. Whether this effect is general over the Sun's surface, or is purely local, and confined to certain lines, seems to be immaterial, so far as the interpretation of these particular observations is concerned; provided the pressure in the solar atmosphere increases toward the Sun's center. As to whether the "pressure effect," as a means of investigating the relative sources of absorption and radiation spectra at the Sun's edge, could be utilized without an eclipse, by means of large-scale apparatus, is a question.

The disappearance of the dark-line spectrum before the bright-line spectrum (at contact II) is due in part to the fact that dark lines require a continuous-spectrum background on which to manifest themselves. The continuous spectrum is weakened by the high dispersion necessarily employed, whereas the monochromatic bright-lines are not. This fact must be taken into account, both in designing the observer's instrument, and in discussing the results.

While I believe in the reality of the appearance described above, the necessity for abundant confirmation is conceded, and urged. It is only after long hesitation and delay that this brief and guarded discussion of this feature of the photograph is ventured; and further discussion of the matter, considering it either as a reality, or as an instrumental effect, will be welcomed by me.

Owing to the strong overexposure of one part of the plate, and underexposure of another part, it is unfortunately not suitable for mechanical reproduction.



The exposure *B*, with fixed plate and slit removed, for recording the coronal spectrum in the region λλ 5150–5500, was successful. The green coronal ring at λ 5303 was recorded as an elliptical ring, whose axes are 0.2 and 0.3 inch in length, respectively. This ring, first converted by projection into circular form, is reproduced in the accompanying illustration.

I have already described this ring as "extremely irregular in form" (this JOURNAL, 10, 190). In parts of it, the photographic impression is so feeble as to be scarcely traceable, whereas the inner portions of the three principal masses are fully exposed. The grouping of these masses somewhat symmetrically with reference to the Sun's equator, and near the inner edges of the Sun-spot zones, is of the highest interest. Placing this drawing of the "coronium" ring on the negatives of the inner corona secured with the 40-foot camera, there is found no evidence of any connection of these masses with the prominences, nor with the interesting curved coronal streamers which

form an enclosing hood around the principal prominences, nor with any of the main features of the inner corona. Likewise, if the four coronal extensions be carried back to the Sun's limb, the coronium masses do not seem to lie exactly at their bases. Nevertheless, it seems probable that they must be near the principal seats of coronal activity. It would be interesting to know whether these forms are shown unchanged in the photographs of the coronal rings near λ 423 and λ 399, secured by other parties.

Although the high dispersion employed was calculated to reduced the strength of the continuous-spectrum background, this was still very strong. The source of the continuous spectrum had the *same form* as the monochromatic ring at λ 5303. Each of the prominences in this ring has a strong band of continuous spectrum, of corresponding intensity, passing immediately through it.

The rulings on the grating were parallel to the line drawn from N. 26° W. to S. 26° E. The continuous spectrum along the tangents to the ring at these points was exceedingly strong, for obvious reasons, making it impossible to observe the details of the ring within some 20° of these points.

It is scarcely necessary to say that with prismatic dispersion the image of the ring would have been brighter than with the grating; and no doubt its recorded extent would have been considerably greater.

With the collimating spectrograph, slit radical, referred to above, an exposure of $1^m\ 36^s$ on the coronal spectrum was made on a fixed plate. The continuous spectrum, from λλ 3800 to 4500, was recorded out to a distance of 5½′ on the east limb, and 4½′ on the west limb. There is no evidence whatever of any dark lines, though the conditions for detecting them could scarcely be appreciably better. A bright line of considerable strength at λ 3987.2 and another at λ 4231 seem to be of coronal origin; but the evidence of objective-prism photographs should be of greater weight in deciding this question. The usual hydrogen and calcium lines, of great intensity, were present.

The writer is aware of the incompleteness of the foregoing discussion. Especially is it incomplete in reference to the period of transition from dark lines to bright lines, and the bearing of the evidence upon the question of a reversing layer. But in view of the extreme difficulty of the subject, further observations may with propriety precede the fuller discussion.

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,
March 17, 1900.

Minor Contributions and Notes

SPECTROSCOPIC NOTES.

ABSOLUTE WAVE-LENGTHS, SPECTROSCOPIC DETERMINATIONS OF MOTIONS IN THE LINE OF SIGHT, AND OTHER RELATED SUBJECTS.

In the November number of the ASTROPHYSICAL JOURNAL, Professor Edwin B. Frost has a note upon "Corrections to Determinations of Absolute Wave-length," in which he states that the effect of the rotation of the Earth upon its axis, and the eccentricity of the Earth's orbit, (with the resulting motion of the observer in the line of sight), upon the wave-length of the lines of the solar spectrum, seems never to have been taken into account.

In the paper by Louis Bell on "The Absolute Wave-length of Light,"[1] he concluded that any correction due to these sources would be so small as to be negligible, especially when compared with those arising from other sources of error.

Professor Henry A. Rowland, in his paper on "The Relative Wave-length of the Lines of the Solar Spectrum,"[2] took Bell's value for D_1 as his standard of reference, and in his "New Table of Standard Wave-lengths"[3] took this value, and after applying a slight correction to it compared it with other values, weighted them all and took the mean as his final standard of reference. In a paper by myself on "The Coincidence of Solar and Metallic Lines,"[4] the differences in wave-length of a number of solar and metallic lines were given, but these were not corrected for motion in the line of sight, as sufficient data were not to be obtained from the plates from which the greater part of the measurements were made, although the effect produced was fully recognized. However, in a preliminary paper on "The Rotation Period of the Sun and other Phenomena of the Solar Atmosphere," prepared

[1] LOUIS BELL, "On the Absolute Wave-length of Light," *American Journal of Science*, March 1887.

[2] *Ibid.*

[3] *Astronomy and Astro-Physics*, **12**, 321.

[4] This JOURNAL, February 1896.

for the Harvard Astrophysical Conference in 1898,[1] the effect of motion in the line of sight of the observer and apparatus, caused by the eccentricity of the Earth's orbit and the rotation of the Earth, was carefully allowed for. The values of these factors were calculated for the latitude of Baltimore and the effect of the motion due to both causes was easily detected in my measurements. The paper was a preliminary report on an investigation which had been made in 1896 and 1897 and the reductions of which had been partially completed. For several reasons it has been necessary to postpone the completion of the work, but I am engaged upon it and expect to have it ready for publication shortly. The values due to motion were carefully calculated and diagrams made for applying the reductions graphically.

These diagrams were used in the tables of "The Arc Spectrum of Vanadium" by Professor H. A. Rowland and Caleb B. Harrison[2] and were also used in the tables of "The Arc Spectra of Titanium and Manganese," by the same authors, which are soon to be published in this JOURNAL.

Professor Frost lays rather too much stress upon the absolute wave-lengths. The values of Bell did not pretend to be accurate to such an extent as to make these corrections important, and no other determinations of absolute wave-length compare with Bell's in point of accuracy, if we except Michelson's values for some of the cadmium lines and perhaps a few lines of one or two other elements. The whole of Rowland's "Table of Solar Spectrum Wave-lengths" is based upon the value for the absolute wave-length of D_1 already determined, and it would be a very serious matter to make the change, even if the absolute wave-length of one or more lines were known with a sufficient degree of precision to justify any change.

The correction due to motion in the line of sight could be applied to Bell's measurements, but it is extremely doubtful if it would be so large as the errors arising from other sources, such as irregular changes in temperature during observations and their effect upon the instrument in ways not altogether accounted for; irregular changes in air pressure; unnoticed errors of the gratings used; small irregular errors in parts of the spectrometer; and many other things entering into the problem.

For most purposes the all important thing in a table of wave-lengths is to have the relative wave-lengths determined carefully, and

[1] Sent in but not read. [2] This JOURNAL, 7, 273.

the effect of the Earth's rotation and orbital eccentricity upon this is very slight when compared with other sources of error.

The character of most lines in both solar and metallic spectra also places a definite limit to the accuracy of determination of both their relative and absolute wave-lengths. Careful observation of the lines in solar and metallic spectra has convinced me that both accuracy and resolving power have been pushed well toward the limit already. I am of the opinion that Professor Frost places altogether too high a value upon the determination of wave-lengths from measurements of stellar spectra. In the second spectrum of a grating of 20,000 lines to the inch and 21½ feet focus, an accuracy of a few thousandths of an Angström unit in measurements is difficult to obtain, and an accuracy of a thousandth of a unit may be considered unattainable, except under the very best possible conditions, with the best possible instrumental equipment, and then only with solar or metallic lines of exceptional sharpness. With a dispersion not nearly so great, and definition not to be compared with this, such a degree of accuracy seems to me to be quite illusory.

It is, however, desirable that our reference points should be definitely fixed with the greatest possible accuracy. The relative wave-lengths of the better class of lines in Rowland's Table and Bell's determination of absolute wave-length are quite accurate enough for all such purposes. But for the most accurate work the lines of the solar spectrum are not stable enough for comparison lines, unless the wave-lengths in the table are considered to be the true wave-lengths of the solar lines unaffected by the Earth's motions, and corrections are made for this motion whenever the lines of the solar spectrum are used as standards of reference, and all tables of metallic lines are referred to the solar lines under these conditions.

As a matter of fact, however, the investigations I have made show that many of the solar lines are unsuitable as standards for very accurate work, and these are for the greater part the most prominent solar lines.

Most of the smaller solar lines are produced in the lower portions of the solar atmosphere, and are caused by the absorptive action upon light of matter which is on the average *ascending* over the solar surface at a rate of about 0.35 miles per second. The shaded lines, on the contrary, are produced by matter at various heights in the solar atmosphere. The shaded portions are produced by absorbing gases

at a considerable depth in the atmosphere, and the narrow central absorption line at a considerable elevation, which varies greatly for different lines. Separating these two absorption lines is a bright reversal or emission line, not very prominent, but recognizable in all metallic lines with broad shading, and conspicuous in the case of H and K and a few of the more strongly shaded lines. This emission component must be formed at an elevation somewhere between that of the matter producing the two distinctly different absorption lines. The narrow central component of the shaded lines (which is the portion of the line used in measurements), shows a *descending* motion, over the solar surface, of the absorbing matter producing it, of from about 0.18 miles per second in the case of the D lines to about a mile a second in the case of the H and K lines. The velocity in the case of the H and K lines is decidedly variable, in the cases of other shaded lines much less so.

These narrow components of the shaded lines are probably produced by meteoric matter falling into the solar atmosphere. The bright emission components may possibly be caused by the downrush of this meteoric matter through the denser portions of the chromosphere such that where it meets the uprush of the matter already referred to, the impact of the collisions or the friction caused produces an intense emission of certain lines in this region of the chromosphere. This may be the true seat of the so-called "reversing layer."

Another factor which affects the wave-lengths of solar lines, but to a less extent in general, is pressure. In the case of the central lines of H and K, the pressure is probably not far from zero, or at least small; in the case of the D lines it is about 1½ atmospheres, and in the case of most of the small lines of iron and the other elements it is two or three atmospheres. The shaded component of the shaded lines is probably produced by matter at a pressure as great as or greater than this. The change in wave-length of most solar lines from this cause is slight, but in a few cases, as in the small sodium lines, it is considerable.

Such facts as these rather impair the usefulness of many solar lines for standards of reference where measurements of velocity in the line of sight are in question, and although the smaller solar lines and most of the others probably have a very constant actual wave-length, still this wave-length is not that which these same lines would have if

the matter producing them were at rest and at atmospheric pressure, and if the point on the Earth's surface where the observer may be were free from motion.

Stellar spectra, however, are not compared with solar, but with metallic or gaseous spectra, and the wave-lengths of lines from these sources as usually obtained may be considered to be quite constant, or at least such within reasonable limits. It is desirable, nevertheless, that the wave-lengths of these lines, used as standards, should be determined accurately, and that this determination be made with the observer at rest with respect to the Sun (in a radial direction) by comparison with the lines of the solar spectrum, and that from these lines a careful selection be made.

In my investigation the values thus obtained were determined for a carefully selected list of iron and other lines, and will be published when the work is completed.

The same corrections were in greater part applied to the wave-lengths of the lines in the spectrum of titanium and manganese determined by Rowland and Harrison soon to be published.

What I have said regarding the effect of motion in the line of sight and pressure in the solar atmosphere, upon the wave-length of lines in the solar spectrum, also holds true of stellar spectra; and in some cases the effect may be considerably greater than in the case of the solar spectrum, as in those stellar atmospheres where the disturbances are greater; or the vertical circulation more intense; or the density or extent of the atmosphere greater than with the Sun. As this matter is to be taken up at considerable length in the near future it is not necessary to discuss it to any extent now.

It is, however, well to say that the effect of rotation of the star upon its axis will be merely to broaden the lines in its spectrum, without materially affecting their position, while radial or nearly radial disturbances or atmospheric circulation may considerably affect the wave-length of lines in its spectrum. The effect of a very active vertical circulation will probably be, as seems to be the case with the Sun, to shorten the wave-length of the smaller metallic lines, and of those lines which are produced by absorption in the lower portions of the star's atmosphere, in view of the fact that the gases producing this absorption take part in the uprushes concerned in this vertical circulation. The study of the appearance of the lines in the solar spectrum leads to the conclusion that the uprush is almost wholly

responsible for the absorption producing these lines. The return current or fall of the material concerned in these uprushes seems to be but slightly effective in the production of absorption lines, the material of these return currents being probably too cool to be of influence. This also best fits in with the most approved solar theories.

The more permanent gases which may constitute the bulk of the true solar atmosphere may not indicate by their spectral lines motion in any predominant direction, though I am not in a position at present to state that this is actually the case. However, the lines most prominent in the solar chromosphere are likely to show a change of wave-length which indicates a down-rush of matter, not under very great pressure, but rather attenuated. In the case of the more prominent lines this velocity may be quite variable. This is upon the supposition that the definition of the lines in stellar spectra is such as to admit of the distinguishing of the narrow central components from the shading of these lines. Such is not likely to prove the case, and hence only the uprush will be measured. This downward or falling velocity is probably not directly connected with the vertical atmospheric circulation referred to, but is more likely to be of meteoric origin. Hence the metallic lines affected will for the greater part be those whose intensity is much greater in the spark than in the arc spectrum, the rush of meteoric matter through a star's atmosphere probably giving a spectrum more nearly like that of the spark than the arc, seeing that it is the result of friction or collisions.

These conclusions are certainly indicated by the appearance and wave-lengths of these lines in the solar spectrum and their prominence in the chromosphere. Hence the facts regarding the behavior and appearance of certain lines in the solar spectrum should make us wary about accepting the presence or predominance of arc or spark lines in stellar spectra as an indication of the temperature of the star's atmosphere, since both the extent and density of a star's atmosphere, and the violence or quietness of its atmospheric circulation, and the amount of meteoric matter falling into its atmosphere, may influence the star's spectrum much more than its temperature alone.

The factors mentioned may also to some extent affect our conclusions regarding the real motion of a star in the line of sight, although if the determinations are based upon measurements of the lines of hydrogen or other permanent gases they are probably less likely to be thus affected. This, however, is a matter to be investigated.

The effect of pressure upon the wave-length of lines as a disturbing influence in measurements of motion in the line of sight will probably be slight. As I have stated, the solar lines indicate pressures of from little more than zero to only two or three atmospheres, though the shading of the stronger lines may be produced at a greater pressure. While some of the stars may have atmospheres of much greater extent and under greater pressure than that of our own Sun, it is doubtful if the seat of absorption of measurable lines will be at a pressure much, if any, greater, and as but comparatively few metallic lines have a large pressure shift per atmosphere, the effect of pressure will be of little importance in most cases.

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PHOTOGRAPHIC NOTES.

In astronomical and spectroscopic photography, halation may sometimes prove troublesome. This effect may be gotten rid of by using doubly and triply-coated plates, but in some respects they are troublesome, especially in the matter of thorough fixing, and furthermore are not always at hand when wanted. Many receipts have been given for backing plates to prevent halation. These generally include essential oils for rendering the index of refraction of the medium which contains the dark pigment used, the same as glass. Both the backing of the plates and its removal later is liable to prove troublesome.

I have found water-color lampblack, in moist pans, or better still, in tubes, to answer perfectly as a preventive of halation. It may be that the gum arabic and honey with which the pigment is mixed render the refractive index of the medium nearly the same as glass; at any rate, it has proven effective. To apply and afterwards remove the black is very easy. It is only necessary to squeeze out some of the color, and with a moistened brush render it of about the consistency of paste, and then apply it to the back of the plate. It drys very rapidly and can be readily removed before developing by washing under the top, or by other means. The film side should first be thoroughly wet, so that there may be no danger of the paint getting upon the surface of the film.

Some years ago an extensive series of experiments led me to adopt a developing formula, which has proven to be so satisfactory for all

around work that I have had no occasion to change it. As it may prove useful to other workers in astrophysics, I have thought it might be well to give the formulae I have been using. Some of the essentials of a good developing formula are that it should be convenient, work cleanly, and with nearly absolute uniformity, both when fresh and old; and also that it should readily lend itself to any changes desired in the treatment of different subjects without the expenditure of too much trouble. The *normal* developer which I employ can be used over many times, and, instead of becoming harsh in its action, will give the same gradations, or contrasts in the negatives when old as when fresh, although when old its action is, of course, slower. It can be mixed without a graduate if necessary, and may even be left around in open tumblers for weeks. This, however, is not advisable, and, like all developers, it should be kept from very strong light, and the necks of bottles in which it is contained should be kept clean and no developer allowed to dry upon them.

A small amount of alcohol is used for two reasons: first, the film is more thoroughly wetted at the start, thus rendering "flowing marks" less liable to occur; and second, the developing action begins sooner and proceeds somewhat more rapidly. Potassium ferrocyanide is also used, because it was found in practice that when included the developer kept better and worked with greater clearness and delicacy. It may, however, be left out without serious consequences. The developer is a concentrated one, and, to prevent spoiling the fixing bath, the plate should be washed thoroughly before fixing, or, much better still, some sodium sulphite used with the fixing bath.

In practice I have found it convenient, where it is desired to secure the best possible results, and the exposure is uncertain, to have conveniently at hand a *normal* developer and two correctors for under and overexposures. The developer for underexposures will give a thin negative with much detail, and will become more harsh in its action or give greater contrast after it has been used a few times. The developer for overexposures will give a negative with very great contrast, but this contrast will somewhat decrease with further use of the developer. As already stated, the *normal* developer will remain constant in its action.

When changing from a developer with more potassium bromide to one with less it is important to rinse the plate well before changing. When the change is of the opposite character it is hardly necessary.

Alum should be kept out of the fixing bath, and only used after one has fixed and rinsed the plate well. It is sometimes useful in very hot weather.

FORMULAE.

(H)	Hydrochinone - - - - - - -	1 part
	Sodium sulphite (crystals) - - - -	5 parts
	Water (distilled if obtainable) - - -	25 parts
	Alcohol - - - - - - - -	¼ ± part
(C)	Potassium carbonate - - - - -	1 part
	Potassium ferrocyanide - - - -	1 part
	Water - - - - - - -	12 parts
(b)	Potassium bromide - - - - -	1 part
	Water - - - - - - -	10 parts

NORMAL DEVELOPER.

(N) 25 cc (C) + 75 cc (H) + 10 drops (b) (from pipette).
= 1 oz. (C) + 3 oz. (H) + 10 drops (b) (from pipette).

The amount of bromide for normal developer should be according to class of work. For most spectroscopic work 15 drops will be found best for above amount of developer, but 10 drops is about right for most kinds of photographic work.

UNDEREXPOSED WORK (thin negatives).

(U) 25 cc (C) + 75 cc (H), no bromide.
= 1 oz. (C) + 3 oz. (H), no bromide.

OVEREXPOSED WORK (great contrast).

(O) 20 cc (C) + 10 cc (b) + 70 cc (H).
= 1 oz. (C) + ½ oz. (b) + 3½ oz. (H).

In very hot weather the developer may be diluted to a considerable extent if desired.

If for any reason it is desired to develop more rapidly the amount of accelerator (C) may be increased, the amount of the bromide solution being increased in the same proportion. The contrasts will be about the same as with the normal developer already given, but the grain of the plate may be coarser and some tendency to fogging of the plate may result.

The fixing bath used is about as follows, the exact proportions not being a matter of much consequence providing it is made strong enough.

FIXING BATH.

Sodium sulphite - - - - - - -	1 part
Sodium hyposulphite - - - - -	5 parts
Water - - - - - - - - - -	25 parts

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THE USE OF THE LINES OF TITANIUM FOR COMPARISON SPECTRA AND THEIR PROMINENCE IN THE CHROMOSPHERE.

PROFESSOR FROST's note upon the use of titanium spark lines for comparison spectra is of much interest. This element, though one of our most refractory substances, seems to give a very marked spark spectrum, and is, moreover, in many ways one of the most prominent solar elements. The smaller lines are remarkably prominent in Sun-spot spectra, and the stronger lines, especially those prominent in both spark and arc spectra, form a very large proportion of the strong lines in the chromosphere and extend to a great elevation.

I first discovered the importance of titanium in chromosphere and prominence spectra from some of Professor Hale's photographs of prominence spectra taken several years ago. These showed the titanium lines at λ 3685.339, 3759.447, and 3761.464 to be among the most conspicuous of the chromosphere and prominence lines, and to extend almost if not quite as far out as the lines of hydrogen and helium, although not so far as those of calcium (H and K). In Evershed's eclipse spectra, taken in India in 1898, the ultra-violet part of the spectrum is crowded with the strong titanium lines which are prominent in spark spectra, and of these many are among the strongest of all the chromospheric lines. We find, in fact, that titanium furnishes more strong ultra-violet lines than any other element, there being (together with many weaker lines) about thirty of importance between λ 3340 and λ 4000.

There are, furthermore, several extremely strong manganese and chromium lines prominent in the spark, while the rare element scandium furnishes a remarkably large number of lines, some of which are

also fairly conspicuous. The characteristic spark lines of iron do not play so striking a part in the chromospheric spectrum, and of those which do appear the strongly shaded lines are the most important. The same is true of nickel, cobalt, and magnesium.

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OPPOSITION OF *EROS* IN 1900.[1]

THE opposition of *Eros* during next autumn will afford opportunities for observations of especial interest. The near approach of the planet to the Earth will permit the solar parallax to be determined, while the great variations in phase and distance will give unusual value to photometric observations obtained at this time. The ephemeris of Dr. Millosevich, published in the Berlin *Jahrbuch* for 1902, provides the means for discussing the measures of position of *Eros*. The annexed table, which is based on this ephemeris, furnishes a part of the material required for the investigations mentioned above. The date is given in in the first column. The right ascension and declination for 1855, for Berlin midnight, are given in the second and third columns. This epoch is selected for convenience in identifying *Eros* by comparison with the stars in the *Durchmusterung*. The daily motions in right ascension, expressed in seconds of time, and in minutes of arc when reduced to the equator, are given in the fourth and fifth columns. The daily motion in declination, and the total motion expressed in minutes of arc, are given in the sixth and seventh columns. These quantities are important in planning observations for parallax, especially those made photographically. The logarithm of the distance from the Sun has been kindly furnished by Dr. Millosevich, and is given in the eighth column. The logarithm of the distance from the Earth is given in the ninth column. From this it appears that its distance when nearest the Earth is less than a third of that of the Sun from the Earth. This minimum occurs on December 26, nearly two months after opposition, which takes place on October 30. The phase angle between the Sun and Earth, as seen from *Eros*, is given in the tenth column. There are few asteroids for which this angle much exceeds 30°. In the table, beginning with the value 37°.5 it gradually diminishes to 28°.3 at about the time of opposition, and then gradually

[1] *Harvard College Observatory Circular* No. 49.

EPHEMERIS OF *EROS.*

Date 1900–1901	R. A. 1855	Dec. 1855	Daily Motion R. A.	Daily Motion R. A.	Daily Motion Dec.	Daily Motion Tot.	log r	log Δ	Phase	Mag.	C. Mag.	M. Tr.	Aberr.
	h m		s									h. m.	s.
Sept. 1	2 18.9	+33 27	+ 70	+15	+22	25	0.1830	9.9088	37.5	11.85	12.33	15 39	404
9	27.5	+36 25	+ 60	+12	+22	25	.1761	.8692	36.6	11.62	12.08	15 16	369
17	34.6	+39 26	+ 45	+ 9	+23	25	.1688	.8285	35.4	11.38	11.80	14 52	336
25	39.2	+42 29	+ 25	+ 5	+23	24	.1613	.7871	34.0	11.13	11.51	14 25	305
Oct. 3	40.8	+45 30	− 1	0	+22	22	.1535	.7456	32.5	10.89	11.23	13 55	278
11	38.7	+48 20	− 32	− 5	+20	21	.1455	.7048	31.0	10.64	10.93	13 21	253
19	32.0	+50 52	− 68	−11	+17	20	.1374	.6654	29.6	10.40	10.65	12 43	231
27	20.6	+52 49	−101	−15	+12	19	.1291	.6286	28.6	10.18	10.40	12 0	212
Nov. 4	5.7	+53 55	−121	−18	+ 5	19	.1207	.5954	28.3	9.97	10.18	11 14	196
12	1 49.3	+54 0	−120	−18	− 3	18	.1124	.5666	29.1	9.79	10.02	10 26	184
20	35.0	+53 0	− 92	−14	−11	18	.1041	.5430	30.9	9.63	9.92	9 40	174
28	25.8	+51 2	− 45	− 7	−18	19	.0961	.5248	33.6	9.49	9.86	8 59	167
Dec. 6	23.5	+48 22	+ 9	+ 2	−22	22	.0884	.5117	36.8	9.39	9.85	8 25	162
14	28.2	+45 16	+ 60	+11	−24	26	.0812	.5033	40.2	9.31	9.88	7 58	159
22	39.3	+41 56	+106	+20	−26	33	.0746	.4994	43.7	9.26	9.93	7 38	157
30	56.0	+38 31	+143	+28	−26	38	.0686	.4994	47.0	9.23	10.00	7 23	158
Jan. 7	2 17.1	+35 7	+172	+35	−26	44	.0636	.5029	49.8	9.22	10.07	7 13	159
15	41.6	+31 43	+194	+41	−25	48	.0595	.5101	52.2	9.24	10.17	7 6	161
23	3 8.6	+28 23	+210	+46	−25	52	.0566	.5210	54.3	9.28	10.27	7 1	165
31	37.3	+25 7	+219	+50	−24	55	.0548	.5353	56.1	9.34	10.38	6 58	171

increases, until on January 31, it attains the extraordinary value of 56.1, becoming even greater later. The photometric magnitude, neglecting the phase and assuming that the light is inversely proportional to the squares of the distances of the Earth and the Sun, is given in the eleventh column. It is based on the measures described in *H. C. O. Circular* No. 34, from which it appears that the magnitude would be 11.39 at a distance of unity from the Sun and Earth, and that the photographic magnitude is 0.6 fainter than the photometric. It will be noticed that these last values are nearly 0.8 fainter than those given by Dr. Millosevich, who based his magnitudes on visual observations. As the magnitude 9.5 in the *Durchmusterung* is about 10.5 on the photometric scale, this difference is readily explained. The difference becomes still greater if we apply a correction for phase. This correction, in the case of the asteroids, is about 0.03 p, in which p is the phase angle. If we assume that this law can be applied to *Eros* for angles as great as 56° we obtain the corrected magnitudes given in the twelfth column. The phase angle in the observations described in *Circular* No. 34 is 21°.2. The magnitude at distance unity therefore becomes $11.39 - 0.64 = 10.75$. The approximate mean time of meridian transit is given in the thirteenth column, and the aberration time in the fourteenth.

As an example of the use of this table, let us consider the most favorable conditions for determining the solar parallax. It soon appears that this problem is by no means a simple one. If we select the end of December, when *Eros* is nearest the Earth, we find that meridian transit occurs so early in the evening that *Eros* cannot be photographed far east of the meridian. Moreover, the motion both in right ascension and declination is so great that if the telescope is made to follow the stars, *Eros* will trail so rapidly over the plate that it may not leave any impression on it. If the total diurnal motion is 24′, the motion will be 1″ a minute. If, then, the diameter of the image is 2″, *Eros* cannot be photographed unless an exposure of two minutes is sufficient. In such a case it may be necessary to make the telescope follow on *Eros* and not on a star. All the stars will then appear as short trails which are easily bisected. If the motion of *Eros* is large, its position with relation to the comparison stars will differ greatly when east and when west of the meridian. Moreover, it will be necessary to measure the total motion, and after subtracting the large motion of *Eros*, determine the small remaining parallax. During

the latter part of January *Eros* culminates at nearly the same time on successive nights, and will thus be favorably situated for observations west of the meridian for several weeks. The path of *Eros* has a loop extending over about 13° in right ascension and 20° in declination, and with a center at about R. A. $2^h\ 5^m$, Dec. + 51°. The point of crossing is at R. A. $2^h\ 21^m.9$, Dec. + 34°.25′ (1855). It is therefore not far from the stars + 34°.447, mag. 9.3, and + 34°.448, mag. 8.0. *Eros* will pass through the point of crossing on September 3, 1900, and again on January 8, 1901. Photometric observations, if made on these dates, will have especial value, since the same comparison stars can be used for both.

A photograph of *Eros* was obtained on September 6, 1898, with the 11-inch Draper telescope, whose focal length is 153 inches. Stars of the ninth magnitude are readily photographed with this instrument in 5 seconds. The exposure was 10 minutes, the daily motion 18′, and the computed magnitude 12.1. Allowing for the difference in motion it would be equally difficult to photograph *Eros* on this date and on September 17, 1900.

EDWARD C. PICKERING.

February 14, 1900.

A REMARK ON THE ARTICLES ON THE DENSITY OF THE ALGOL STARS IN THE ASTROPHYSICAL JOURNAL, VOL. 10, NO. 5.

IT has long been known that the mean density of the two components can be deduced from the observations of the light-variation of the Algol stars. This question was discussed thoroughly, and in principle completely, by Mériau in *Comptes rendus*, **122**, 1254. In my article on "Doppelsterne" in Valentiner's *Handwörterbuch der Astronomie* (Bd. I, 694–695), mention is also made of this idea, which naturally suggests itself.

If we retain the symbols employed by Roberts, and further let δ_1 and δ_2 represent the densities of components 1 and 2, δ the *mean* density of the two components, and D that of the Sun, whose radius in astronomical units is R, we shall have

$$\frac{\delta_1}{D}=\frac{R^3}{p^3t^2}\cdot\frac{m_1}{m_1+m_2}\,;\quad\frac{\delta}{D}=\frac{R^3}{q^3t^2}\cdot\frac{m_2}{m_1+m_2}\,;\quad\frac{\delta}{D}=\frac{R^2}{(p^3+q^3)t^2}\,.$$

The last formula alone, which was also derived in my article referred to above, can be computed without any hypothesis. The first two

formulae only were employed by Roberts, who, however, used an incorrect factor. It would appear that the Sun's diameter was taken instead of its radius, for his factor 0.0092 must be replaced by 0.00465 (D being placed equal to unity).

In conclusion, it is by no means intended that these lines shall detract from the interest which is certainly merited by the numerical results of the articles referred to.

H. SEELIGER.

MUNICH, February 1900.

THE SYSTEM OF CAPELLA.

ONE of the most interesting among recent astronomical discoveries is announced in Professor Campbell's note on "The Spectroscopic Binary *Capella*," published in the October 1899 number of this JOURNAL.[1]

I have plotted the data given in Professor Campbell's note, and find that a period of about 105 days will satisfy the six observations recorded. A shorter period, though not absolutely precluded, is clearly improbable. A longer period ($200^d \pm$) cannot be brought into harmony with Vogel's spectrographic observations of 1888–9.[2] The indicated range in radial velocity is about 57 km per second, and the orbit is not far from circular. I have made no attempt to deduce the elements by "least squares," as further observations will doubtless be soon available.

Professor Vogel found no evidence of change in the radial motion of *Capella*. His mean result for this motion (+15.5 English miles per second) should therefore hold good for the center of mass of the system; and we may further assume that the two components are nearly equal in mass, as well as in brightness.[3]

Adopting Elkin's value for the star's parallax, viz., 0".081, I infer that the components will at times be separated by more than 0".04.

[1] The binary nature of this star was independently detected by Mr. H. F. Newall (*Observatory*, December 1899, p. 436).

[2] Reproduced in the *Observatory*, No. 181, p. 374. Vogel's result for the radial velocity of *Capella* considerably exceeds the deduced value for the motion of the center of mass, based on the 200-day period.

[3] The non-detection of changes in the star's spectrum by Professor Vogel is not easily explained; and Professor Campbell's remarks on the dissimilar spectra of the components tend to heighten the difficulty. Evidently from any point of view, the case of *Capella* is of great interest.

Hence Michelson's interference apparatus,[1] fitted to one of our larger telescopes, should furnish reliable measures of the Capella system, thus affording material for a complete knowledge of the orbit and absolute masses, besides yielding a new value for the star's parallax. Since *Capella* is now favorably situated, while suitable apparatus is easily improvised, I trust that no time will be lost in the making of tentative experiments. With the assigned period the components would attain their maximum angular distance about January 1, and again near the end of February 1900.

I have hitherto assumed that the inclination $(90^\circ - i)$ of the orbit plane to the line of sight is not large. This is a natural supposition; but the following deductions lead to a different result, and one which, from our present standpoint, is of great interest.

If m denote the mass of a binary star in units of the Sun's mass, P being the period of revolution in *days*, and $V_0 = V \operatorname{cosec} i$ the *relative* mean orbital velocity in kilometers per second, we shall have:

$$m = [\bar{7}.01645]\, PV^3 \operatorname{cosec}^3 i\,,$$

where the semi-axis major of the Earth's orbit is taken as 149,480,000 kilometers.[2] Assuming equal masses for the components, and putting

$$m_p = [\bar{7}.20386]\, PV^3.$$

$V = 56$, $P = 105$, $i = 90^\circ$, we find $m = 1.9$ for the system of *Capella*.

Now this value of the mass is notably less than that which we should be led to assign from a consideration of the star's actual brightness. *Capella* is intrinsically about five times brighter than *Sirius*, twenty-one times brighter than *Procyon*, and eighty-seven times as bright as *α Centauri*.[3] And the masses of *Sirius*, *Procyon*, and *α Centauri* are respectively equal to 2.4, 3, and 2 Sun-masses, according to the reliable data supplied by Auwers, Gill, Elkin, and See. Taking

[1] This is, in brief, a cap fitting over the telescope objective and provided with two parallel slits whose breadth and distance apart can be varied at will. A simple cap with fixed slits and a graduated band for the measurement of position-angles will serve the purpose.

[2] In the deduction of approximate or limiting values for the masses of spectroscopic binaries, this formula will be found very convenient. In general, the *most probable* value (m_p) of the mass will be obtained by putting $i = 60^\circ$. The corresponding formula is:

[3] These comparisons are based on the Harvard photometric magnitudes of the four stars. The adopted parallaxes are those of Gill for *Sirius* and *α Centauri*, and of Elkin for *Procyon* and *Capella*.

these and other related facts into consideration, we may assume with much confidence that $m > 6$, in which case $a > 0''.064$, and $i < 43°$, for the system of *Capella*.

If the foregoing deductions are sound — and they certainly possess a high measure of probability — it will be possible to study this interesting system by interference methods at almost any epoch, with the large refractors of the Yerkes and Lick Observatories.[1]

J. MILLER BARR.

ST. CATHERINES, ONTARIO, CANADA.
January 4, 1900.

Addendum. — Since the foregoing paper was written, Professor Campbell has announced his preliminary results relative to the orbit of the Capella system.[2] These results are in remarkably close agreement with those deduced by the writer. Using Professor Campbell's data ($P = 104^d.1$, $V = 57$ km per second), I find:

$$a = 0''.044 \text{ cosec } i,$$
$$m = 2.00 \text{ cosec}^3 i,$$

the orbit being supposed circular. Putting $m = 6$, as already assumed, the maximum and minimum distances of components will exceed $0''.064$ and $0''.046$, respectively. The actual value of a will probably be found to lie between $0''.1$ and $0''.25$. Hence this interesting system should come within the range of direct observation in our larger telescopes — a suitable absorbent screen being used to reduce the apparent dimensions of the diffraction-disks.

J. M. B.

[1] It may be worth while to directly examine *Capella* with high powers, under exceptionally favorable conditions.

[2] *Observatory*, February 1900, p. 92. See also Mr. Newall's interesting letter in the same number.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XI MAY, 1900 NUMBER 4

ON THE ESCAPE OF GASES FROM PLANETARY ATMOSPHERES ACCORDING TO THE KINETIC THEORY.

By G. Johnstone Stoney.

PART I.—GENERAL CONSIDERATIONS.

Mr. S. R. Cook, in his paper under the above title in the January number of the Astrophysical Journal, points out that the present writer, when investigating the escape of gases from atmospheres, does not base his argument upon "the determination by the Kinetic Theory of the relative number of molecules which would have a velocity sufficient to enable them to escape from the Earth or planet."

This is so: and the reason is that no such determination exists except that arrived at in the paper criticised by Mr. Cook, where data drawn from outside the Kinetic Theory are employed to supplement what the Kinetic Theory teaches. These auxiliary data are: (1) that the Moon has not retained an atmosphere; and (2) that the Earth and *Venus* do retain the vapor of water in their atmospheres.

When I first attacked the problem of the escape of gases from atmospheres, which was, I think, in 1867 or 1868, I, like Mr. Cook, hoped that Maxwell's law for the distribution of the speeds of the molecules in gases under normal conditions would render aid; but when I came to consider what the true physical meaning of that law is, and within what limits it holds good, I found that it fails us just where we want its aid, *viz.*, in that outer region of an atmosphere from which alone the entire of the escape of molecules takes place.

My first attempt was to discover a more comprehensive law, of the same kind as Maxwell's, which should give the average proportion of molecules having a given velocity, using the word velocity to include both speed and direction. Such a formula can, without much difficulty, be obtained in a form which covers some states of gas which lie outside the grasp of Maxwell's law. But, after long thought over the subject, I was unable to include under it, or any other formula that I could discover, the unique conditions that prevail near the confines of an atmosphere. All that I could discover, applicable to this case, was some general inferences from general dynamical considerations. These, so far as they need be referred to here, went to show that the breaking down of Maxwell's law as the boundary of an atmosphere is approached is accompanied by an augmentation in the proportion of molecules moving with what may be called outlying speeds, *i. e.*, speeds which deviate much either above or below the speed which Maxwell calls "the velocity of mean square."

On finding that the attempt to base the inquiry upon Maxwell's law could only lead to illusive results, it appeared necessary to cast about for some other way of approaching the problem; and, finally, I adopted the line of argument which is developed in my paper (see this JOURNAL for January 1898, or the *Scientific Transactions of the Royal Dublin Society*, Vol. VI, Part 13).

Maxwell's formula for the distribution of the speeds of molecules in their free paths has reference to what happens in

the case of quiescent gas within an enclosure of which the walls are perfectly elastic, and where the gas is not acted on by any other external force. Let the space within the enclosure be divided up into equal cubes of such a size, ΔV, that each shall contain a sufficient number of molecules, N, for a probability law such as Maxwell's to be apparent in the distribution of the N speeds with which these N molecules are actually moving at any assumed instant of time. This is the point of view from which Maxwell discusses the problem. It is, however, more convenient to secure a sufficient number of molecular speeds in another way. We may, in fact, employ smaller cubes containing fewer molecules, if we compensate for this by extending our survey to all the free paths which these molecules pursue within a sufficient duration, Δt. This change in the point of view is legitimate, since Maxwell's proof of his law applies without alteration to either case; and the change is of advantage, inasmuch as it appears, from investigations which have been made since Maxwell's paper was published, that the smallest volume which we can safely attribute to ΔV, if we adopt Maxwell's point of view, is inconveniently large for some purposes; and since, if we avail ourselves of the new point of view, it becomes possible to gauge by experiment the number of free paths which require to be included in order that the distribution of the speeds of the molecules in them may, in a satisfactory degree, conform to Maxwell's law (see the second part of this paper). If (adopting the new point of view) N be the number of molecules within the space ΔV, and if n be the average number of encounters met with by each molecule in the time Δt, then will Nn be approximately the number of free-paths described within the volume ΔV in the time Δt.

Now Maxwell's law is a probability law, and is, therefore, a limiting law: that is, it is never more than approximately fulfilled; it is widely departed from within the space ΔV if Nn is too small a number, and it more nearly represents the actual state of things the larger the number Nn is, provided that some necessary conditions are complied with. What these conditions

are may be collected from an examination of the successive steps of the proof which Maxwell gives of the law. They are all, as will be seen, fulfilled in a degree which is sufficient for practical purposes, in the air we find about us at the bottom of the atmosphere and in the gases we have to deal with in most laboratory experiments. So far as these conditions need be referred to, they are as follows:

1. It is assumed in the proof of Maxwell's Law that the gas is uniformly distributed throughout the space considered, and that it is isotropic, *i. e.*, the condition of the gas is supposed to be the same in all directions. Now, though gravity increases the density of the lower strata of gas confined within a room, or in the apparatus of a laboratory, it does so in so slight a degree that the first condition is sufficiently fulfilled for practical purposes. This, however, is by no means the case in that outermost stratum of the atmosphere from which the escape of molecules takes place, when we give to the cube ΔV the size which is there necessary to include within it a sufficient number of free paths, and where, besides, the free paths are no longer straight.

2. The gas must also be quiescent or else traveling with uniform speed in some fixed direction. When thus traveling, air is a wind blowing with a velocity that does not vary either in speed or direction; and what Maxwell's law gives when applied to such a wind, is the distribution of the speeds of the velocities that remain after geometrically subtracting the velocity of the wind from the several velocities of the individual molecules.

3. The portion of space ΔV to which we apply Maxwell's law, must be not too narrow in any direction. We have secured compliance with this condition wherever ΔV is of sufficient size to fulfill the other requirements, by selecting a cube as the form of ΔV.

4. The volume ΔV must be large enough and the time Δt long enough, to make the free paths described within the space ΔV in the time Δt, sufficiently numerous to warrant our using

Maxwell's law as representing approximately the actual distribution of speeds among these molecular journeys.

In the present state of our knowledge we are unable to determine by *a priori* considerations what is necessary for the fulfillment of this last condition. We must have recourse to experiment and look out for cases in which there is an appreciable breaking down of Maxwell's law.

The law does not, for example, hold good of the gas within the bulb of a Crookes' radiometer; where, when the instrument is exposed to radiant heat, the molecules thrown back with augmented speed from the darkened side of the vane do not meet with a sufficiency of encounters with other molecules in front of that side of the vane to be able to keep them back in the degree which would equalize the pressure on the front and back of the vane.

Nor does Maxwell's law hold good in ordinary air when we are able to scrutinize effects due to what happens within very small volumes of the air. Thus, if tobacco smoke, confined between two thin slips of glass, is examined with a microscope, the minute flakes of smoke will be seen to dance in a surprisingly lively manner. They are, in fact, being hammered about by coöperating movements of small groups of molecules which now and then form themselves in the air about them, thus betraying to us the deviations from Maxwell's law which must arise whenever we have to deal with swarms of molecules, in which the individual molecules are too few to provide a sufficient number of encounters within the shortest time that our senses can appreciate from changes in the motions of the flakes of smoke. The impacts of single molecules upon those flakes do not, of course, produce appreciable effect, but molecular events succeed one another with such astonishing rapidity—as will appear when we come to deal with details in the second part of this paper—that there is time enough within the hundredth of one second of time for little rushes of small bodies of molecules to have now and then arisen. Such being the case, Maxwell's law cannot assert itself until we contemplate a larger volume or a longer

time or both, until, in fact, so many of such irregularities are embraced in the survey that they are able to balance one another.

What happens here may be illustrated by what takes place in a more familiar case. If 5000 shots were fired at the center of a target from a given distance, out of equally good guns and by equally skilled artillerists, the distribution of the shots round the center of the target would approximately conform to a probability law of the same class as Maxwell's law. But if instead of being directed against one target, the 5000 shots are fired at 1000 targets, five shots at each, the divergence of the effects on the several targets will become conspicuous. On some targets the five shots will be more scattered than on others; on some they will preponderate towards one side; and so on. On scarcely any of them will it be possible to trace even a vestige of the probability law. These are phenomena very much akin to those little rushes in various directions of little swarms of gaseous molecules, which we find now and then occur when we can succeed in scrutinizing what takes place at sufficiently close quarters in ordinary air, and which cause minute bodies suspended in the air to dance. A similar phenomenon, affecting the agitation of the molecules of liquids, is made known to us by the so-called Brownian movements which are familiar to microscopists.

Other instructive instances might be adduced where Maxwell's law ceases to be a safe guide, as, for example, in the distribution of the speeds of the molecules in the stratum of air adjoining a hot surface, and in other cases where special conditions prevail under which a gas may find itself; but we may pass at once to that remarkable exception to Maxwell's law which is forced upon our attention when we endeavor to understand the circumstances under which gases escape from an atmosphere. The entire of the escape takes place from that outer stratum of the atmosphere throughout which the molecules are within practical striking distance of the void space beyond, and out of which they do pass into that void space whenever the circumstances favor their doing so. This stratum of the atmosphere is of great depth, partly on account of the tenuity of the atmosphere in its upper regions, and partly because of the excessively

small ratio which the space occupied by the molecules in those regions holds to the unoccupied space, which greatly lengthens the free paths. The molecules which pursue their long parabolic trajectories within this outermost layer, sometimes making excursions into the void space beyond, find themselves under very different circumstances in different parts of it, and these various circumstances when taken together, as they must be in order that the cubes ΔV may be of sufficient size, constitute a condition of gas complex and altogether remote from that which is essential to the proof of Maxwell's law. Accordingly, throughout this deep outer layer, throughout the neighboring parts of the void space beyond, and throughout such underlying strata of the atmosphere as are sensibly affected by it—in fact throughout the whole of the upper region of the atmosphere—Maxwell's law fails to represent the actual distribution of molecular speeds. At the same time no escape of gas from an atmosphere takes place except what is owing to the events which happen in these very regions. All the numerical results, therefore, which have been reached by employing Maxwell's formula, we must unfortunately expect to be wide of the truth.

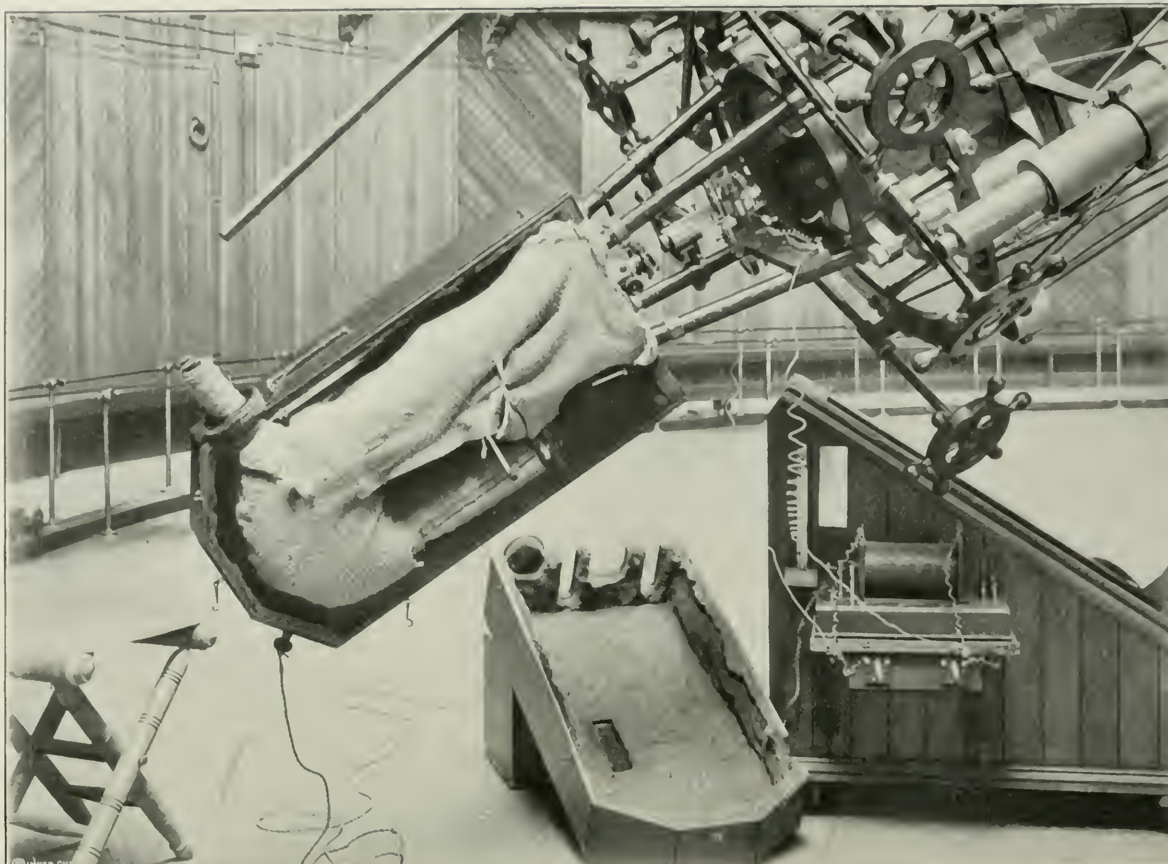
We can, nevertheless, make a certain amount of use of Mr. Cook's results. Maxwell's law may be looked at from another point of view. The physical events with which it deals are these: when an abnormal speed develops itself in any molecule (and we shall find, in the second part of this paper, that such speeds develop themselves several times in each second in every molecule of the air at the bottom of the Earth's atmosphere), then if the crowding of the molecules, the size of the target which each presents to the rest, and the duration of the time Δt, are sufficient to make it probable that, in a sufficiently great majority of cases, the molecules so affected shall subsequently meet with encounters so numerous and of such a kind as will knock the abnormality out of them before the expiration of the time Δt—*in such cases*, and if the gas be gas of uniform density, the outstanding deviations of speed above and below that speed which Maxwell calls the velocity of mean square, will be nearly those which are represented by Maxwell's law.

On the other hand, where, as near the boundary of an atmosphere, an opportunity is afforded to molecules in which an abnormal speed springs up, to place themselves beyond the reach of those subsequent encounters which would tone it down; or in those cases where the volume of ΔV is too small for a sufficiency of such encounters, as happens in the small swarms whose behavior we study in the tobacco-smoke observation; or where from any other cause there is an insufficiency of those encounters which eliminate anomalies: in all cases of any of these kinds the larger deviations of speed will not have been sufficiently toned down for Maxwell's law, but will grow to be more numerous, of larger amount, occasionally more concurrent, and so on, than that law would tolerate.

This is manifested at one extreme by the consequent effect upon suspended flakes of smoke, and at the other extreme it occasions an outflow of molecules from an atmosphere greater than that which would prevail if Maxwell's law held good in respect to the part of the atmosphere which emits these molecules. Thus the numbers obtained by Mr. Cook's method of investigation are of use by furnishing a rate of escape of gas which we know that the real rate of escape must exceed. This in itself is valuable information, and it would be very important information if we had no better way of investigating the problem.

In another respect, not yet taken into account, the constituent gases of an atmosphere are exposed in its upper regions to conditions which further facilitate the escape of gaseous molecules from planets and satellites; namely, owing to interchanges of energy between the internal events which go on within the molecules, and their motions of translation amongst one another. But this branch of the subject is postponed, as it involves the consideration of details which are to form the subject-matter of the second part of this paper, and which it is hoped will throw much light upon several of the matters already partly discussed in the preceding pages.

PLATE V



TEMPERATURE CONTROL OF THE MILLS SPECTROGRAPH

THE TEMPERATURE CONTROL OF THE MILLS SPECTROGRAPH.

By W. W. CAMPBELL.

THE great importance of the temperature factor in spectrographic investigations is recognized by all observers. Variations in the temperature of the apparatus, and especially of the dense prisms, during the progress of an exposure, affect both the definition and the deviation. As previously stated in this JOURNAL (8, 140), the Lick Observatory is comparatively free from rapid changes of temperature. It is seldom, after 9 o'clock, that the thermometer readings in the large dome change a half degree Centigrade during exposures of an hour. In order to protect the Mills spectrograph, to a considerable extent, from irregularities in the atmospheric temperature, and from the heat of the observer's breath and body, the entire instrument was covered with two thicknesses of heavy gray woolen blanket, and the prism-box with four thicknesses. A large-scale thermometer was mounted with its bulb inside and near the middle of the prism-box. In a general way, the temperature of the air in the prism-box followed the temperature outside of the blankets ; but the irregularities in the latter were rounded off in the former. The rate of change in the glass prisms themselves was probably even more uniform.

For more than a year past the spectograph has been still further protected against temperature changes by completely inclosing it, save for the slit and guiding telescope, in a box constructed of slowly-conducting material. The arrangement is shown in Plate V. One half of the box is in place on the instrument; and the other half, shown near the floor, fits against and into this by "tongued and grooved" joints. The two halves are held together by five large hooks-and-eyes. The box is of $\frac{7}{16}$ inch Spanish cedar, lined throughout with $\frac{1}{2}$ inch hair felt.

The whole weight, thirty-five pounds, is supported on the upper section of the steel framework. It does not touch the spectrograph at any point. The free spaces between the guiding telescope and the box, and around the steel rods at the upper end, are closed by means of felt pads. A thermometer mounted on the inner surface of the box, and the thermometer in the prism-box can both be read through glass windows, to $\frac{1}{25}^\circ$ Centigrade. A hinged door gives access to the plate-holder, and to the sliding diaphragm which controls the exposures on the brightest comparison lines. The woolen coverings remain in use as before.

German silver wire — B. & S. No. 29, 16½ feet in length in each half of the box — is mounted near the surface of the felt. This wire is readily connected with a 10-volt storage battery, and furnishes the means of controlling a falling temperature.

Care is taken to ventilate the dome and spectrograph, during the day and early evening, so that at dark the readings of the thermometers in the prism-box, in the protecting box, and in the open dome, generally show a range of about $0^\circ.5$; and the reading in the dome will usually vary less than 2° during the night. After the adjustments have been made, the spectrograph is inclosed in the blankets and in the box, and allowed to stand from fifteen to thirty minutes, until the temperatures in the prism-box and in the wooden case are partially equalized, before beginning an exposure. The equalizing process is apparent from the fact that, while the dome temperature is usually stationary or slowly falling, the temperature in the wooden case invariably rises to meet that of the prism-box.

The observer notes the reading of the thermometer in the wooden case — shown just above the guiding telescope — and endeavors to keep it constant by means of the electric current, which he controls with a switch near at hand. The temperatures in the case respond slowly to those outside, and rather rapidly to the influence of the current. Some irregularities necessarily exist in these temperatures; but their effects are rounded off before reaching the prism-box, and are scarcely appreciable.

The efficiency of the apparatus is illustrated by the following extract from the observing books, on a night when the temperature changes in the dome were unusually irregular. The readings were made at the beginning and ending of the exposures:

Plate number	Limits of exposure time. Mt. Hamilton M.T. 1899, Aug. 29	Temp. in dome	Temp. in wooden box	Temp. in prism-box
1433 A	$8^h\ 28^m$	+17°.9 C.	+18°.4 C.	+18°.84 C.
	10 02		18.9	18.80
1434 B	10 24	18.5	18.8	18.78
	11 00		19.0	18.80
1435 C	11 31		19.0	18.80
	13 07	17.2	19.0	18.80
1436 D	13 38		18.7	18.70
	15 15		18.8	18.70
1437 A	15 32		18.8	18.74
	16 09	19.0	19.0	18.74

An automatic control of the current could be arranged, but the need of it has not been seriously felt. Such a control should be independent of the variable component of gravity in the different positions of the instrument.

No provision has been made for reducing a rising temperature which sometimes occurs. It is possible that a spray of some volatile liquid within the wooden case would be efficient and practicable.

The use of this box and the controlling current has contributed appreciably to the accuracy of our line-of-sight determinations.

Acknowledgments are due to Mr. Wright for many of the ideas utilized; and to Professor Cory, of this University, who verified the computations for the thermal system and supplied the constants of the wire.

Lick Observatory,
University of California,
March 1900.

ON THE ORIGIN OF CERTAIN UNKNOWN LINES IN THE SPECTRA OF STARS OF THE β *CRUCIS* TYPE, AND ON THE SPECTRUM OF SILICON.[1]

By Joseph Lunt.

In a recent paper "On the presence of Oxygen in the Atmospheres of certain Fixed Stars,"[2] Dr. Gill calls attention to three unknown lines in the spectra of β *Crucis*, ϵ *Canis Majoris*, and stars of their type, *viz.*, wave-lengths 4552.79, 4567.09, 4574.68.

Mr. McClean had previously also recorded these lines in his measures of the spectrum of β *Crucis*[3] as wave-lengths 4552.6, 4567.5, 4574.5, but beyond pointing out the approximate coincidence of the first of these with lines due to barium or titanium, he assigns no origin to them.

Sir Norman Lockyer frequently records them as unknown lines. In his recent paper "On the appearance of the Clèveite and other New Gas Lines in the Hottest Stars" (June 1897),[4] he records all three lines as unknown. The first occurs in a map of the spectrum of *Bellatrix* as a line in a probably new series found by Dr. W. J. S. Lockyer. The second and third lines, given as 4566.8 and 4574.8, occur in a Table of Lines which Sir Norman regards as belonging, with high probability, to gaseous substances which have yet to be discovered.

It will be noticed that the connection existing between the three lines is not there recognized.

In none of the Tables of Wave-Lengths available for reference at the Cape could any satisfactory clue be obtained as to the origin of the lines.

During some experiments made with a view to securing the best elementary line spectrum of oxygen as a comparison

[1] *Proc. R. Soc.*, **66**, 44–50, December 1899.

[2] *Proc. R. Soc.*, **65**, 205; this Journal, **10**, 272, 1899.

[3] *Spectra of Southern Stars* (Stanford, 1898), p. 13.

[4] *Proc. R. Soc.*, **62**, 60.

spectrum for stars of the *β Crucis* type, I found that a tube of carbon dioxide gave the best results, being freer from impurities, and giving stronger oxygen lines than any of the oxygen tubes at my disposal. By the use of a jar and air gap in the secondary circuit of the coil the gas was dissociated, and gave the spectra of carbon and oxygen. During use, the carbon dioxide tubes became more vacuous, and, with a view to obtaining a brighter discharge and shorter exposure, I passed the induced current from an 18-inch Apps' coil, using four large jars and an air gap.

Whilst using these electrical conditions, I happened to expose an argon tube marked 2 mm (pressure), and on developing the photograph was much surprised to find that it, too, gave the well-recognized lines of oxygen. Stronger than these I at once noticed two lines at the green end of the spectrum, which recalled the lines in *β Crucis*, which were unknown terrestrially, whilst the expected argon spectrum was almost entirely absent.

On comparing the negative, film to film, with one of *β Crucis*, and allowing for the difference of temperature conditions under which the two negatives were taken, the identity of the three unknown lines in *β Crucis* with three lines on the argon negative was at once apparent, and a subsequent photograph of the spectrum of *ε Canis Majoris*, in which the argon tube was used, as stated, as a comparison spectrum, established their absolute identity both as regards position and relative intensity.

It was, therefore, evident that a terrestrial source of the three unknown lines had been discovered, and with the behavior of the carbon dioxide tube fresh in mind, and the replacement of the argon spectrum by unknown lines and those of oxygen by use of a highly disruptive spark, it is not surprising that an obvious startling explanation as to the nature of the element thus found terrestrially should have suggested itself.

It was at first assumed, erroneously as it afterwards proved, that the origin of the unknown lines lay in the gaseous contents of the argon tube. Four argon tubes in succession gave precisely the same results, *viz.*, the argon spectrum with an ordinary

discharge and the unknown lines and oxygen, together with the disappearance of the argon spectrum, as a result of using the jars and air gap. On communicating these results to Dr. Gill, he at once interested himself in the matter, and gave every facility for a further prosecution of the inquiry. He remembered that Professor Ramsay had furnished him with a specimen tube of pure argon, and this tube had not been examined. On trying this tube under the same conditions as the others, it was found to give the argon spectrum under all conditions. Neither the unknown lines nor oxygen made their appearance, even when the most intense disruptive spark available was employed.

The first four tubes had aluminium electrodes, whilst Professor Ramsay's tube had platinum electrodes, and was more vacuous and much shorter.

A pair of aluminium electrodes was then taken from a vacuum tube, and a spark between the metal terminals in air was next examined, with the result that the unknown lines were not found. A line appeared very approximately in the same position as the strongest of the three lines, but this was only one of the numerous air lines, and was due to nitrogen (4552.6 Neovius). Therefore, the electrodes of the argon tubes did not account for the unknown lines.

On a further examination of the negatives, the H and K lines of calcium were recognized in the spectra of the argon tubes subjected to the highly disruptive spark, pointing to the fact that the lime of the glass was being volatilized.

This fact alone might account for the presence of the oxygen lines in the spectra, and the materials of the glass were then suspected as being the origin of the lines under consideration.

A tube of pure helium, kindly furnished to Dr. Gill by Professor Ramsay, was next examined, and, with much surprise, this was found also to behave exactly as the first argon tubes had done.

With an ordinary discharge it gave the pure helium spectrum, but with the highly disruptive discharge the helium spectrum *vanished entirely*, and was replaced by the unknown lines and the

spectrum of oxygen. The helium spectrum could be obtained at will by reverting to the ordinary discharge.

This helium tube had platinum electrodes, and these last observations finally banished any idea that the gaseous contents of the tubes or the metallic electrodes could be the origin of the substance searched for, and the conclusion that the glass of the tubes contained the substance sought was now irresistible. Yet in some of the spectra from the helium tube, the H and K lines of calcium were absent when those of oxygen were present, showing that the lime of the glass did not necessarily account for the presence of oxygen.

After various fruitless experiments, sparks were taken between the platinum terminals of a broken up vacuum tube on which still adhered some of the blue fusible glass, commonly used in sealing in platinum wire in glass. The spectrum of this spark in air showed the unknown lines.

Beads of glass made from ordinary soda glass tubing, were then fused on platinum wires, and the spark from these was examined. The unknown lines again appeared. The substance sought was now strongly suspected to be the element silicon. The siliceous diatomaceous earth "kieselguhr" was next used as the most convenient source of silica, and beads of sodium silicate were made by fusing this material with sodium carbonate on platinum wire. The result of the examination of the spark was that the unknown lines were again found. The next step was to replace the kieselguhr by pure rock crystal obtained from the South African Museum by Dr. Gill. Sodium silicate made from the pure rock crystal also furnished the unknown lines, whilst the sodium carbonate alone failed to give them.

These experiments left little room for doubt that the element sought was silicon. Nevertheless, it was very desirable to confirm the result in another way, by examining the spectrum of a gaseous siliceous compound.

Platinum wires were sealed into the ends of a piece of wide glass tubing, $\frac{5}{8}$ inch internal diameter, the ends of the wires leaving a gap of only $\frac{5}{8}$ inch for the passage of the spark. The tube

was also furnished with an inlet and outlet tube for the gas. No capillary tube was used in order to avoid the hot spark coming into direct contact with glass. The tube was then filled with silicon tetrafluoride, and after the gas had been passing for some time, it was sealed off at atmospheric pressure.

An ordinary discharge passed through the gas without jars or air gap gave a banded spectrum of the compound itself.

The disruptive discharge obtained by using four jars and an air gap, at once gave the unknown lines, which were thus proved to be undoubtedly due to silicon.

This silicon spectrum was not accompanied by that of oxygen, thus proving that it could not be due to any dissociation of the silica of the glass, and that in this case, the gaseous contents of the tube and not the tube itself furnished the lines under consideration.

Sir Norman Lockyer's papers were then consulted for any reference to the presence of silicon in stars, and it is necessary to refer in some detail to his observations. It is evident that he has used similar powerful disruptive discharges with vacuum tubes, and obtained partial decomposition of the glass, for he says:[1] "The use of the spark with large jars in vacuum tubes results in the partial fusion of the glass, and the appearance of lines which have been traced to silicium."

Unfortunately he does not give the wave-lengths of the lines thus traced to silicon, and from his statement alone, one would surmise that the origin of the three lines was recognized by Lockyer.

There is evidence, however, *in the same paper* that he cannot have traced the lines in question to silicon notwithstanding the above statement, because, as previously pointed out, Sir Norman regards two of the lines as belonging to gases yet undiscovered, and includes them in a table of wave-lengths of lines due to unknown *gases*.

The other line he also includes as an unknown line in *Bellatrix*, and Dr. W. J. S. Lockyer places this as a member of a probably rhythmic series due to an unknown substance.

[1] *Proc. R. Soc.*, **62**, 65, 1897.

It is a curious fact that Hartley and Adeney, and Eder and Valenta, who alone give us any extended list of lines due to silicon, appear not to have examined the spectrum of this element in the region of the three lines here considered. Their published wave-lengths show only lines in the extreme ultra-violet, and the majority of them are quite outside the region which can be examined by the McClean star spectroscope.

Watts's *Index of Spectra* (Appendix E, p. 21) records a line at 4566 (Salet), but no lines appear corresponding to 4552.79 and 4574.68.

Sir Norman Lockyer[1] regards two lines at 4128.6 and 4131.4 as the most conspicuous enhanced lines of silicon; indeed these two lines are the only silicon lines he labels *Si* in his published photographs. Eder and Valenta give 4131.5 and 4126.5 as the least refrangible on their list, and although there is a rather excessive discrepancy in the wave-lengths of one of the lines, they are probably the same pair of lines. They are shown in Lockyer's photographs of the spectra of *α Cygni* and *Sirius*[2] and also of *α Cygni* and *Rigel*.[3]

It is a remarkable fact that these three stars, which may be considered as amongst the best examples of silicon stars in the light of the spectrum of silicon hitherto known, *do not show* the three silicon lines which are so prominent in *β Crucis*, *ε Canis Majoris*, etc. Scheiner has measured the spectra of all three stars[4] in this region, but does not record the lines in his table of wave-lengths.

Their absence from the spectra of these stars (as well as the presence of Lockyer's enhanced silicon lines) is fully confirmed by photographs taken here with the special object of searching for the new silicon lines in the best known silicon stars.

This can be readily understood in the light of the experiments with the tube of silicon tetrafluoride.

With the highest disruptive spark, Lockyer's silicon lines 4128.6 and 4131.4 are much enhanced as compared with the

[1] *Proc. R. Soc.*, **61**, 443, 1897.

[2] *Proc. R. Soc.*, **65**, 191.

[3] *Phil. Trans.*, A (1893), plate 2.

[4] SCHEINER'S *Astronomical Spectroscopy*.

lines 4552.79, 4567.09, 4574.68, and it was found possible by suitable exposure to obtain the two enhanced lines without the presence of the other three lines becoming evident.

The latter lines would be much more rapidly obliterated in the absorption spectra of stars, than in the bright line spectrum from the tube, and therefore their absence from certain stars in which the enhanced lines are strong need not occasion much surprise.

In other stars, however, all five lines are present. Lockyer has recorded them in *Bellatrix* and their presence has been confirmed by photographs of the spectrum of this star taken here.

Mr. McClean has measured all five lines in β *Crucis*, where Lockyer's enhanced silicon lines are certainly not so conspicuous as the lines 4552.79 and 4567.09.

The same may be said of ϵ *Canis Majoris*, in which star the new silicon lines are very prominent, whilst the enhanced lines are very faint.

In the silicon spectrum from the argon and helium vacuum tubes, the enhanced lines noted by Lockyer are by no means so prominent as they are in the silicon spectrum, obtained from silicon tetrafluoride with the intense disruptive spark. It is evident, therefore, that great variations in the relative intensities of the silicon lines occur in stellar spectra, and that such variations can be produced to a certain extent in the laboratory, and these require further investigation.

The behavior of the silicon lines will give us valuable data for the elucidation of the problem of relative stellar temperatures.

It is clear that if we regard, with Lockyer, the lines 4128.6 and 4131.4 to be the enhanced lines of silicon, and their presence, enhanced, to be a criterion of a higher temperature than occurs in stars where these lines are *not* enhanced, it must follow that such stars as α *Cygni*, *Rigel*, and *Sirius* are hotter than *Bellatrix*, β *Crucis*, and ϵ *Canis Majoris*. Whereas Lockyer[1] in his most recent paper "On the Chemical Classification of the Stars"

[1] *Proc. R. Soc.*, **65**, 189.

(April 1899), regards the so-called "Crucian" stars as at a higher temperature than the "Rigelian" and "Cygnian," and indeed he regards *Bellatrix* "as a type of the hottest stars, exception being made of ζ *Puppis*."

Of the other lines recorded by Eder and Valenta[1] as due to silicon, 3905.4, 3862.5, and 3855.7 are present both in the spectra of the dissociated glass and in the high temperature spectrum of silicon obtained from the silicon tetrafluoride tube.

They are enhanced lines in the latter case, occurring together with Lockyer's enhanced lines in the absence of the three new silicon lines, but they lie outside the region measured by Scheiner in *α Cygni*, *Sirius* and *Rigel*.

In the Harvard "Spectra of Bright Stars"[2] the two latter lines are, however, specially noted in *Rigel* as 3863.2 and 3856.2 as "conspicuously strong in the ultra-violet," whilst all three are recorded (3905.6, 3863.2, 3856.2) in stars of Groups VI to VIII (Harvard), comprising *α Cygni*, *Sirius*, and *Rigel*. They would thus appear in these stars to accompany the enhanced silicon lines, especially noted by Lockyer, *viz.*, 4128.6 and 4131.4.

The lines 3834.4 and 3836.7 recorded by Eder and Valenta are not present in any of the photographs of silicon spectra, and may possibly be due to impurities.

The lines 3795.9 and 3791.1 recorded by Eder and Valenta are present in all the silicon photographs, but do not become enhanced at high temperatures. There is, however, a third line, approximately λ 3807, not recorded by them, but which appears in all the photographs of silicon spectra. It is stronger than 3795.9 and 3791.1, and does not become enhanced with high temperature. All three lines accompany the three new silicon lines in ε *Canis Majoris*.

[1] WATTS, *Index of Spectra*.

[2] *Harvard Annals*, **28**, Part I, Table 7, p. 23.

ON THE MAGNITUDES OF 919 FIXED STARS DETERMINED FROM SEQUENCES OBSERVED BY SIR JOHN HERSCHEL DURING THE YEARS 1835 TO 1838.

II.

By W. Doberck.

TABLE III.

Catalogue of mean magnitudes of stars observed by Herschel, and reduced by aid of the *Uranometria Argentina.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m	° ′							
1	α *Andromedae*....	0 1.9	+28 24	2.14	1	—	—	—	—	—
2	κ^1 *Sculptoris*......	0 3.0	—28 41	5.82	1	5.0	5.5	—	—	—
3	ε *Phoenicis*.......	0 3.1	—46 26	3.89	3	4.1	3.8	4.0	4.0	3.96
4	κ^2 *Sculptoris*......	0 5.2	—28 30	5.82	1	4.5	5.4	—	—	—
5	γ *Pegasi*.........	0 6.8	+24 29	2.78	1	—	—	—	—	—
6	ζ *Toucani*.......	0 13.5	—65 37	4.34	3	4.5	4.1	4.5	4.1	4.31
7	ι *Sculptoris*.......	0 15.2	—29 40	5.84	1	6.4	5.3	5.6	—	—
8	ξ *Toucani*.......	0 18.5	—72 47	4.53	1	—	4.5	—	4.1	cum.
9	β *Hydri*.........	0 19.1	—77 57	2.90	11	3.1	2.7	2.9	3.2	—
10	κ *Phoenicis*.......	0 20.0	—44 22	4.06	3	4.1	3.9	4.1	4.0	4.03
11	α *Phoenicis*.......	0 20.1	—42 59	2.54	5	2.3	2.4	2.5	2.7	2.49
12	*Lac. 106*.........	0 24.1	—24 29	5.57	1	5.4	5.2	—	—	—
13	λ^1 *Phoenicis*......	0 25.4	—49 30	4.78	2	5.0	4.6	5.1	4.8	4.86
14	β *Toucani*.......	0 25.8	—63 39	3.65	4	3.8	3.7	3.9	4.0	—
15	*Lac. 125*.........	0 27.5	—30 15	5.83	1	6.4	5.8	6.0	—	6.01
16	μ *Phoenicis*	0 35.4	—46 46	4.95	2	5.0	4.7	4.9	4.9	4.89
17	ρ *Toucani*........	0 37.1	—66 9	5.33	1	5.8	5.7	5.8	—	—
18	β *Ceti*...........	0 37.3	—18 40	2.22	6	—	2.3	—	—	2.26
19	η *Phoenicis*	0 37.7	—58 9	4.34	2	5.4	4.5	4.7	—	var.?
20	ϕ^1 *Ceti*..........	0 37.9	—11 17	4.88	1	—	5.1	—	—	—
21	*Lac. 193*.........	0 38.6	—22 42	5.74	1	5.4	5.3	—	—	—
22	*18 Ceti*..........	0 39.2	—13 33	5.70	1	—	6.3	—	—	—
23	*Lac. 203*.........	0 40.0	—23 12	5.78	1	6.4	5.8	—	—	—
24	*Ll. 1349*.........	0 43.2	—14 14	5.90	1	—	5.9	—	—	—
25	ϕ^2 *Ceti*..........	0 43.9	—11 19	5.66	1	—	5.5	—	—	—
26	ρ *Phoenicis*.......	0 45.0	—51 40	5.19	1	4.5	5.6	5.4	—	—
27	ϕ^3 *Ceti*....	0 49.7	—11 57	5.88	1	—	5.7	—	—	—
28	ϕ^4 *Ceti*..........	0 52.5	—12 3	5.88	1	—	5.9	—	—	—
29	α *Sculptoris*......	0 52.6	—30 2	4.02	1	4.1	4.2	4.7	—	—
30	*27 Ceti*..........	0 59.3	—10 39	5.79	1	—	6.5	—	—	—
31	*28 Ceti*..........	0 59.8	—10 31	5.88	1	—	5.7	—	—	—
32	β *Phoenicis*......	1 0.5	—47 23	3.43	3	3.1	3.3	3.4	3.4	3.33
33	*30 Ceti*..........	1 1.5	—10 27	5.89	1	—	5.8	—	—	—
34	η *Ceti*...........	1 2.3	—10 51	3.25	3	—	3.5	—	—	—
35	ζ *Phoenicis*.......	1 3.1	—55 55	3.94	2	4.1	4.2	4.2	—	4.11

TABLE III—*Continued.*

No.	Name.	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
36	*37 Ceti*	1 8.1	— 8 36	5.15	1	—	5.3	—	—	—
37	κ *Toucani*.......	1 11.5	−69 32	5.11	1	5.0	5.1	5.2	4.8	5.04
38	*Polaris*..........	1 13.0	+88 39	2.18	2	—	—	—	—	—
39	θ *Ceti*...........	1 17.8	— 8 50	3.60	2	—	3.2	—	—	—
40	*46 Ceti*.....	1 19.5	—15 15	5.35	2	—	5.1	—	—	—
41	γ *Phoenicis*......	1 22.9	—43 57	3.51	3	3.1	3.4	3.4	3.5	3.38
42	*48 Ceti*..........	1 23.6	—22 17	5.22	1	5.4	5.3	—	—	—
43	*Br. 214*.........	1 25.7	—30 38	5.87	1	6.4	6.8	—	—	—
44	*Br. 215*.........	1 25.9	—30 56	5.62	1	6.4	6.0	6.1	—	—
45	δ *Phoenicis*......	1 26.0	—49 43	3.95	3	4.1	4.0	4.1	4.0	4.03
46	*Ll. 2848*.........	1 27.4	— 7 40	5.91	1	—	5.9	—	—	—
47	τ *Sculptoris*......	1 30.4	—30 33	5.45	1	5.4	5.9	6.2	—	var.?
48	α *Eridani*.......	1 33.0	—57 52	1.07	15	1.0	1.0	1.0	1.1	1.03
49	*Ll. 3137*.........	1 35.6	—11 56	5.92	1	—	5.8	—	—	—
50	τ *Ceti*...........	1 38.3	—16 36	3.45	2	—	3.4	—	—	—
51	ε *Sculptoris*......	1 39.8	—25 41	5.41	1	5.0	5.4	—	—	—
52	χ *Ceti*...........	1 43.4	—11 18	4.96	3	—	4.8	—	—	—
53	ζ *Ceti*...........	1 45.3	—10 57	4.45	1	—	3.5	—	—	—
54	*Ll. 3479*.........	1 46.9	—17 33	5.80	1	—	5.8	—	—	—
55	β *Arietis*.........	1 47.7	+20 11	3.07	1	—	—	—	—	—
56	ψ *Phoenicis*......	1 48.6	—46 55	4.90	1	5.0	4.8	4.3	5.0	4.88[1]
57	τ^2 *Hydri*	1 49.0	—80 48	5.07[2]	1	6.4	6.1	6.5	—	—
58	φ *Phoenicis*......	1 49.2	—43 7	4.73	1	5.4	5.5	5.3	5.0	—
59	*56 Ceti*..........	1 50.8	—23 8	5.31	2	5.8	5.0	—	—	—
60	χ *Eridani*.......	1 51.1	—52 14	3.73	3	4.1	3.9	3.7	3.8	3.85
61	η^2 *Hydri*........	1 51.8	—68 16	5.10	2	4.5	4.9	4.9	4.9	4.86
62	*Lac. 585*........	1 52.2	—48 0	4.93	1	5.4	5.1	4.9	5.1	5.09
63	*57 Ceti*..........	1 53.9	—21 26	5.60	1	6.4	5.7	—	—	—
64	υ *Ceti*...........	1 54.1	—21 41	3.95	2	3.8	3.9	—	—	—
65	α *Hydri*.........	1 54.8	—62 11	3.06	8	3.1	2.9	3.0	2.8	2.97
66	α *Piscium*.......	1 55.6	+ 2 10	3.53	2	—	3.8	—	—	—
67	π *Fornacis*.......	1 55.7	—30 36	5.10	1	5.8	5.5	5.6	—	—
68	ν *Fornacis*.......	1 58.9	—29 54	4.44	1	5.0	4.9	5.1	—	—
69	α *Arietis*	2 0.1	+22 52	2.06	4	—	—	—	—	—
70	μ *Fornacis*.......	2 7.4	—31 19	5.03	1	5.4	5.4	5.6	—	—
71	*67 Ceti*..........	2 10.7	— 7 0	5.48	1	—	5.7	—	—	—
72	φ *Eridani*......	2 12.0	—52 5	3.56	5	3.5	3.5	3.9	4.0	3.69
73	ο *Ceti*.	2 13.0	— 3 33	5.35	1	—	var.	—	—	var.
74	*Ll. 4396*.........	2 15.9	—11 21	5.28	1	—	5.4	—	—	—
75	*Ll. 4466*.........	2 18.2	—16 49	5.76	1	—	6.4	—	—	—
76	δ *Hydri*.........	2 19.5	—69 14	4.04	4	3.8	4.1	4.4	4.0	4.07
77	ρ *Ceti*...........	2 19.9	— 2 51	4.61	1	—	4.6	—	—	—
78	ξ^2 *Ceti*	2 21.5	+ 7 54	4.12	1	—	4.4	—	—	—
79	κ *Eridani*.......	2 22.4	—48 17	4.06	5	4.1	4.2	4.5	4.4	var.?
80	*Ll. 4681*.........	2 25.0	+ 1 43	5.81	1	—	5.7	—	—	—
81	*75 Ceti*..........	2 25.8	— 1 35	5.57	2	—	5.9	—	—	—
82	σ *Ceti*...........	2 26.2	—15 48	4.86	1	—	4.8	—	—	—
83	ω *Fornacis*.......	2 28.4	—28 47	4.71	1	5.0	4.9	—	—	—
84	δ *Ceti*...........	2 33.1	— 0 13	4.20	2	—	4.0	—	—	—
85	ε *Ceti*...........	2 33.5	—12 24	4.55	1	—	4.6	—	—	—
86	s *Eridani*.......	2 35.0	—43 26	4.80	1	5.0	5.0	5.0	4.7	4.90
87	ι *Eridani*.......	2 35.7	—40 23	4.13	1	4.5	4.2	4.1	4.2	4.23

[1] For S.M.P. the visual mag. 4.7 was adopted. [2] Perhaps for *Hydri* (Gould.)

TABLE III — *Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m	° ′							
88	γ *Ceti*	2 36.8	+ 2 42	3.49	3	—	3.2	—	—	—
89	ε *Hydri*	2 37.7	−68 48	4.02	4	3.8	4.2	4.4	4.2	4.12
90	μ *Ceti*	2 38.2	+ 9 35	4.44	1	—	4.3	—	—	—
91	π *Ceti*	2 38.2	−14 23	4.06	2	—	4.1	—	—	—
92	τ¹ *Eridani*	2 39.3	−19 6	4.45	2	—	4.5	—	—	—
93	*Lac. 896*	2 42.8	−64 14	5.72	1	5.8	6.1	6.0	—	—
94	ζ *Hydri*	2 43.6	−68 8	4.90	4	4.5	5.2	5.1	5.0	—
95	τ² *Eridani*	2 45.4	−21 31	4.96	2	5.0	4.9	—	—	—
96	η *Eridani*	2 50.3	− 9 24	3.90	3	—	3.7	—	—	—
97	ν *Hydri*	2 51.3	−75 35	5.28	1	4.5	5.1	5.0	5.1	5.00
98	θ *Eridani*	2 53.5	−40 48	3.22	4	3.1	2.6	3.1	3.4	3.08
99	ρ¹ *Eridani*	2 55.0	− 8 9	5.90	1	—	6.1	—	—	—
100	α *Ceti*	2 55.7	+ 3 36	2.67	4	—	2.4	—	—	—
101	β *Horologii*	2 56.4	−64 34	5.44	3	5.0	5.2	5.3	—	5.23
102	ρ² *Eridani*	2 56.6	− 8 11	5.77	1	—	5.6	—	—	—
103	τ³ *Eridani*	2 56.9	−24 7	4.16	2	3.8	4.1	—	—	—
104	*Br. 465*	2 57.3	−64 7	5.39	1	—	—	—	—	var.?
105	ρ³ *Eridani*	2 58.1	− 8 6	5.68	1	—	5.3	—	—	—
106	α *Fornacis*	3 6.7	−29 29	3.93	1	—	3.6	—	—	—
107	ζ *Eridani*	3 9.8	− 9 17	4.85	2	—	4.9	—	—	—
108	*Lac. 1051*	3 12.8	−22 58	4.81	1	5.4	5.3	—	—	—
109	*Ll. 6161*	3 13.0	−19 1	5.72	1	—	5.8	—	—	—
110	τ⁴ *Eridani*	3 13.9	−22 13	3.69	2	3.8	3.4	—	—	—
111	*e Eridani*	3 14.9	−43 33	4.56	1	5.0	4.4	4.5	4.4	4.57
112	ζ¹ and ζ² *Reticuli*.	3 15.2	−63 1	4.97	3	5.0	5.1	[5.5]	5.2	5.07
113	ο *Tauri*	3 18.1	+ 8 35	3.56	1	—	3.4	—	—	—
114	ξ *Tauri*	3 20.4	+ 9 18	3.66	1	—	3.5	—	—	—
115	*v Eridani*	3 24.4	− 5 30	4.91	1	—	4.7	—	—	—
116	*Ll. 6535*	3 25.7	−13 10	5.82	1	—	—	—	—	—
117	ε *Eridani*	3 27.0	− 9 53	3.74	3	—	3.6	—	—	—
118	κ *Reticuli*	3 27.2	−63 23	4.91	3	4.5	5.0	5.1	5.0	4.90
119	τ⁵ *Eridani*	3 28.3	−22 3	4.22	2	4.1	4.5	—	—	—
120	*Ll. 6661*	3 30.0	−11 37	5.85	1	—	5.8	—	—	—
121	*10 Tauri*	3 30.5	+ 0 1	3.96	1	—	4.5	—	—	—
122	*20 Eridani*	3 30.6	−17 53	5.31	2	—	5.3	—	—	—
123	*y Eridani*	3 32.6	−40 41	4.63	1	5.0	4.8	4.7	4.6	4.75
124	γ *Eridani*	3 35.2	−13 52	3.13	2	—	2.8	—	—	—
125	δ *Eridani*	3 37.3	−10 11	3.46	4	—	3.3	—	—	—
126	*17 Tauri*	3 37.4	+23 43	4.42	1	—	—	—	—	—
127	*Ll. 6914*	3 37.6	−10 53	5.84	1	—	5.9	—	—	—
128	*19 Tauri*	3 37.8	+24 4	5.09	1	—	—	—	—	—
129	*h Eridani*	3 38.2	−37 42	4.71	3	5.4	4.8	4.7	4.7	4.86
130	*20 Tauri*	3 38.4	+23 58	4.79	1	—	—	—	—	—
131	*23 Tauri*	3 38.9	+23 33	4.79	1	—	—	—	—	—
132	η *Tauri*	3 40.1	+23 43	3.13	1	—	—	—	—	—
133	π *Eridani*	3 40.2	−12 30	4.79	2	—	4.7	—	—	—
134	τ⁶ *Eridani*	3 41.5	−23 37	4.29	2	3.8	3.9	—	—	—
135	*27 Tauri*	3 41.7	+23 40	3.90	1	—	—	—	—	—
136	*28 Tauri*	3 41.7	+23 45	5.34	1	—	—	—	—	—
137	τ⁷ *Eridani*	3 42.3	−24 16	5.00	1	4.5	5.5	—	—	—
138	β *Reticuli*	3 42.6	−65 12	3.99	5	4.1	3.9	3.9	4.0	3.98
139	*f Eridani*	3 44.0	−38 0	4.32	2	5.0	4.3	4.8	4.3	var.?

TABLE III — *Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U. A.	S.M.P.	W.	Mean
		h m	° ′							
140	*g Eridani*	3 44.8	—36 35	4.21	2	5.0	4.1	4.3	4.1	—
141	τ^8 *Eridani*	3 48.4	—24 59	4.75	2	3.8	4.4	5.1	—	var.?
142	γ *Hydri*........	3 49.2	—74 37	3.33	10	3.1	3.2	3.1	3.2	3.19
143	*Ll. 7273*	3 49.4	—12 27	5.76	1	—	6.1	—	—	—
144	*Ll. 7422*	3 53.6	—12 56	5.76	1	—	6.1	—	—	—
145	τ^9 *Eridani*	3 54.6	—24 22	4.63	2	3.8	4.4	—	—	—
146	*Lac. 1316*	3 55.7	—30 51	6.08	1	6.4	5.9	6.5	—	—
147	ν *Tauri*.........	3 56.5	+ 5 38	3.82	1	—	3.9	—	—	—
148	δ *Reticuli*	3 56.8	— 61 45	4.96	2	4.5	4.7	4.5	4.8	4.69
149	*Ll. 7600*	3 58.5	—13 8	5.71	1	—	6.0	—	—	—
150	γ *Reticuli*	3 59.1	—62 30	4.67	2	4.5	4.7	4.7	4.7	4.65
151	ι *Reticuli*	3 59.3	—61 26	5.17	2	5.4	5.1	5.1	5.2	5.19
152	*Lac. 1344*	4 0.5	—28 0	5.69	1	5.8	5.8	—	—	—
153.	δ *Horologii*	4 6.6	—42 19	4.90	1	5.8	5 3	5.1	4.9	—
154	o^2 *Eridani*	4 9.5	— 7 48	4.33	1	—	4.4	—	—	—
155	α *Horologii*	4 9.9	—42 36	3.81	1	5.0	3.8	3.9	4.1	—
156	γ *Doradûs*.......	4 12.7	—51 48	4.44	3	5.0	4.4	4.5	—	—
157	α *Reticuli*........	4 12.8	—62 47	3.57	8	3.1	3.3	3.4	3.2	3.31
158	ϵ *Reticuli*........	4 14.3	—59 36	4.96	3	4.5	4.6	4.6	4.8	4.69
159	ξ *Eridani*........	4 17.4	— 4 2	5.24	1	—	5.6	—	—	—
160	*d Eridani*	4 19.3	—34 18	4.37	1	3.8	4.0	4.1	3.7	—
161	*Ll. 8588*	4 26.4	— 3 29	5.69	1	—	6.0	—	—	—
162	v^1 *Eridani*	4 28.6	—30 1	5.18	1	3.8	4.7	4.8	4.6	var.?
163	α *Tauri*.........	4 28.7	+16 15	1.37	8	—	—	—	—	—
164	ν *Eridani*	4 30.1	— 3 37	3.89	2	—	3.8	—	—	—
165	v^2 *Eridani*	4 30.7	—30 49	4.33	1	3.5	3.7	3.9	4.0	—
166	α *Doradûs*.......	4 31.3	—55 18	3.43	9	3.5	3.1	3.6	3.6	3.45
167	*51 Eridani*	4 31.3	— 2 40	5.54	2	—	5.8	—	—	—
168	*l Eridani*........	4 32.5	—14 33	4.41	2	—	4.1	—	—	—
169	*Ll. 8809*.........	4 33.6	—14 36	5.69	1	—	5.6	—	—	—
170	*Lac. 1544*........	4 34.9	—24 44	5.69	1	5 8	6.0	5.8	—	5.82
171	*54 Eridani*.......	4 35.0	—19 55	4.82	1	5.8	4.6	—	—	—
172	α *Coeli*...........	4 36.5	—42 6	4.77	1	5.0	4.6	4.8	4.6	4.75
173	β *Coeli*	4 37.6	—37 23	5.69	1	4.5	5.1	5.5	5.3	—
174	*Ll. 8951*.........	4 38.6	—18 54	5.69	1	—	5.7	—	—	—
175	μ *Eridani*.......	4 39.2	— 3 29	4.12	2	—	4.0	—	—	—
176	*Lac. 1585*........	4 39.6	—50 43	5.73	1	5.4	5.5	5.7	—	—
177	*Ll. 8996*	4 39.7	—21 31	6.08	1	—	6.1	—	—	—
178	*58 Eridani*.......	4 42.0	—17 10	5.69	1	—	5.7	—	—	—
179	π^3 *Orionis*	4 43.0	+ 6 44	3.11	1	—	3.1	—	—	—
180	π^2 *Orionis*	4 43.8	— 8 41	4.68	1	—	4.7	—	—	—
181	μ *Mensae*........	4 44.3	—71 10	5.24	1	5.0	5.6	6.2	5.7	var.?
182	π^4 *Orionis*	4 44.5	+ 5 23	3.52	1	—	3.7	—	—	—
183	*60 Eridani*	4 44.6	—16 26	5.69	1	—	5.0	—	—	—
184	ω *Eridani*.......	4 46.7	— 5 40	4.85	1	—	4.7	—	—	—
185	*Lac. 1628*........	4 46.9	—35 7	5.69	1	5.8	6.2	6.4	6.2	var.?
186	π^5 *Orionis*	4 47.7	+ 2 14	3.60	1	—	3.7	—	—	—
187	π^1 *Orionis*	4 48.0	— 9 57	5.00	1	—	5.0	—	—	—
188	ι *Pictoris*	4 48.1	—53 41	5.33	2	5.4	5.4	[6.1]	—	—
189	*R Leporis.*	4 53.9	—15 0	5.19	1	—	var.	—	—	var.
190	ψ *Eridani*.......	4 55.4	— 7 21	4.82	2	—	5.3	—	—	—
191	*Lac. 1686*........	4 57.1	—26 27	5.72	1	5.0	5.4	—	—	—

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P	W.	Mean
		h m								
192	η¹ *Pictoris*	4 59.5	—49 20	5.99	1	5.4	5.5	5.8	5.6	—
193	γ *Caeli*	4 59.9	—35 39	4.80	1	5.4	4.7	4.9	4.7	—
194	*Lac. 1710*	5 0.2	26 19	6.08	1	5.8	6.1	—	—	—
195	ε *Leporis*	5 0.2	—22 32	3.52	3	3.1	3.1	—	—	var. ?
196	β *Eridani*	5 1.7	— 5 15	2.95	5	—	2.8	—	—	—
197	η² *Pictoris*	5 1.7	—49 45	5.99	1	5.4	5.3	5.3	5.4	—
198	λ *Eridani*	5 3.2	— 8 55	4.75	2	—	4.6	—	—	—
199	ζ *Doradûs*	5 3.4	—57 39	4.83	2	5.0	4.8	5.0	5.0	4.93
200	β *Mensae*	5 4.3	—71 29	5.56	1	5.4	5.7	5.6	5.8	5.61
201	*Lac. 1751*	5 5.1	—55 9	5.76	1	—	—	(7.6)	—	—
202	ι *Leporis*...	5 6.5	—12 1	4.71	2	—	4.4	—	—	—
203	μ *Leporis*	5 7.3	—16 21	3.60	3	—	3.4	—	—	—
204	κ *Leporis*	5 7.5	—13 5	4.71	2	—	4.2	—	—	—
205	α *Aurigae*	5 7.5	+45 52	0.87	1	—	1.0	—	—	—
206	β *Orionis*	5 8.6	— 8 21	1.00	15	—	1.0	—	—	—
207	*Lac. 1773*	5 10.1	—36 7	5.72	1	6.4	5.8	[6.3]	6.5	—
208	τ *Orionis*	5 11.5	— 6 59	4.10	1	—	3.9	—	—	—
209	λ *Leporis*	5 13.8	—13 18	4.78	2	—	4.1	—	—	—
210	θ *Doradûs*	5 13.9	—67 20	5.03	1	5.0	5.1	5.1	5.0	5.05
211	ν *Leporis*	5 14.2	—12 27	5.55	1	—	5.7	—	—	—
212	*Ll. 10063*	5 15.1	—21 22	5.72	1	5.0	4.9	—	—	—
213	*Lac. 1825*	5 16.3	—50 45	5.73	1	5.9	5.8	6.1	5.8	5.87
214	δ *Leporis*	5 17.8	—14 3	5.72	1	—	5.7	—	—	—
215	η *Orionis*	5 18.2	— 2 31	3.49	2	—	3.4	—	—	—
216	γ *Orionis*.	5 18.4	+ 6 14	1.98	11	—	1.7	—	—	—
217	β *Tauri*	5 18.4	+28 30	2.09	2	—	—	—	—	—
218	*Ll. 10254*	5 20.6	—19 48	6.08	1	—	6.2	—	—	—
219	θ *Pictoris*	5 21.9	—52 26	5.73	1	5.9	6.0	[6.7]	5.8	—
220	β *Leporis*	5 22.9	—20 52	2.97	9	2.9	2.9	—	—	—
221	λ *Doradûs*	5 24.5	—59 1	5.45	1	5.4	5.6	5.4	5.6	5.49
222	δ *Orionis*	5 25.6	— 0 24	2.40	6	—	2.3	—	—	—
223	ε *Columbae*	5 26.8	—35 34	3.95	1	3.8	4.1	4.0	4.1	3.99
224	α *Leporis*	5 27.2	—17 55	2.68	10	—	2.7	—	—	—
225	λ *Orionis*	5 28.2	+ 9 51	3.74	1	—	3.5	—	—	—
226	θ *Orionis*	5 29.1	— 5 28	4.35	1	—	4.8	—	—	—
227	ι *Orionis*	5 29.3	— 5 59	3.05	4	—	2.9	—	—	—
228	ε *Orionis*	5 29.9	— 1 17	1.81	12	—	1.8	—	—	—
229	*Lac. 1904*	5 30.9	—33 21	5.72	1	5.9	6.8	(6.7)	—	var.?
230	β *Doradûs*	5 32.5	—62 34	3.84	6	3.8	3.9	3.8	4.2	—
231	σ *Orionis*	5 32.5	— 2 40	4.44	1	—	4.0	—	—	—
232	ζ *Orionis*	5 34.4	— 2 1	1.95	12	—	1.8	—	—	—
233	α *Columbae*	5 35.1	—34 8	2.83	9	2.3	2.5	2.8	2.6	—
234	*Lac. 1936*	5 35.2	—32 42	5.72	1	5.9	5.9	5.8	6.0	5.86
235	γ *Mensae*	5 36.8	—76 26	5.66	1	5.9	5.6	5.3	5.6	5.61
236	γ *Leporis*.	5 39.2	—22 29	3.72	2	3.5	3.5	—	—	—
237	μ *Columbae*	5 41.3	—32 21	5.09	1	5.4	5.4	5.8	5.4	5.42
238	ζ *Leporis*	5 41.3	—14 52	3.66	2	—	3.6	—	—	—
239	κ *Orionis*	5 41.8	— 9 43	2.33	9	—	2.3	—	—	—
240	β *Pictoris*	5 44.3	—51 7	3.96	1	5.0	3.9	4.1	4.0	—
241	δ *Doradûs*	5 44.5	—65 47	4.46	4	4.5	4.5	4.8	4.8	4.61
242	δ *Leporis*	5 45.9	—20 53	3.88	2	3.8	3.7	—	—	—
243	β *Columbae*.	5 46.5	—35 49	3.18	5	2.9	2.9	3.1	3.0	3.02

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P	W.	Mean
		h m								
244	γ *Pictoris*	5 47.5	−56 12	4.96	2	5.0	4.7	4.5	4.6	—
245	*Lac. 2052.*	5 48.1	−52 8	5.73	1	5.4	5.6	5.1	5.4	5.45
246	α *Orionis*........	5 48.4	+ 7 23	1.16	14	—	1.2	—	—	—
247	λ *Columbae*	5 48.6	−33 50	4.87	1	4.5	5.2	5.4	5.3	—
248	ϵ *Doradûs*.......	5 50.0	−66 56	4.87	3	5.0	5.1	5.3	5.4	var.?
249	β *Aurigae*... .	5 50.4	+44 56	2.22	1	—	—	—	—	—
250	η *Leporis*........	5 50.7	−14 12	3.80	2	—	3.8	—	—	—
251	ξ *Columbae*	5 51.2	−37 8	4.99	1	5.4	5.4	5.3	5.6	—
252	σ *Columbae*	5 51.6	−31 24	5.72	1	6.4	5.6	6.0	6.0	5.94
253	*Lac. 2087*	5 52.1	−52 40	5.73	1	5.4	5.8	5.6	5.4	5.59
254	γ *Columbae*......	5 53.1	−35 18	4.61	2	4.5	4.5	4 7	4.6	4.58
255	*Lac. 2106*	5 53.1	−63 8	5.18	2	5.0	4.9	4.7	5.1	4.98
256	η *Columbae*......	5 55.3	−42 49	4.16	2	4.1	4.0	4.2	4.3	4.15
257	*Lac. 2115*	5 58.2	−26 17	5.72	1	6.4	5.5	—	—	—
258	*Lac. 2137*	6 0.9	−45 2	5.42	1	5.0	5.8	[6.3]	5.8	—
259	θ *Columbae*.......	6 3.2	−37 14	4.89	2	5.0	5.3	5.8	5.3	var.?
260	*Lac. 2174*...... .	6 4.9	−44 20	5.14	1	6.4	6.6	—	6.5	—
261	η^1 *Doradûs*	6 6.0	−66 1	5.80	1	5.9	6.0	6.3	5.0	var.?
262	ν *Doradûs*	6 9.5	−68 49	5.00	1	4.5	5.2	5.6	5.5	—
263	η^2 *Doradûs*	6 11.0	−65 34	5.83	1	5.4	5.5	5.1	6.2	—
264	*Lac. 2224*........	6 11.2	−61 26	5.75	1	—	6.6	—	—	—
265	κ *Columbae*.......	6 12.1	−35 6	4.48	1	4.5	4.8	4.8	4.6	4.64
266	α *Mensae*........	6 14.0	−74 43	5.62	1	5.0	5.3	5.6	5.5	5.40
267	ζ *Canis Maj.*.....	6 15.5	−30 1	3.09	6	2.9	3.2	3.3	—	—
268	β *Canis Maj*.....	6 17.2	−17 54	2.29	10	—	2.2	—	—	—
269	δ *Columbae*.......	6 17.5	−33 22	3.67	1	4.1	3.9	4.0	4.1	—
270	ν *Pictoris*........	6 20.7	−56 18	5.85	1	5.9	6.0	6.2	5.8	5.95
271	α *Argûs*.........	6 21.2	−52 38	0.44	17	1.0	0.4	(−1.0)	(−0.8)	—
272	*G Puppis*........	6 22.4	−48 6	5.99	1	5.9	5.9	6.5	6.0	6.06
273	*Lac. 2311*........	6 22.7	−60 13	6.21	2	5.9	6.1	6.5	5.9	6.12
274	λ *Canis Maj*	6 23.5	−32 30	5.00	1	4.5	4.7	4.9	5.0	4.82
275	π^1 *Doradûs*	6 23.8	−69 55	5.69	1	5.4	6.1	6.0	5.8	5.80
276	*Lac. 2328*........	6 25.1	−57 55	6.36	1	5.9	6.3	6.2	6.3	6.21
277	*Lac. 2309*........	6 25.8	−27 41	6.19	1	—	6.1	—	—	—
278	π^2 *Doradûs*	6 26.5	−69 37	5.69	1	5.4	5.9	5.8	5.8	—
279	*Lac. 2333*........	6 26.7	−50 9	5.73	1	5.4	5.5	5 7	5.8	5.63
280	*Lac. 2343*........	6 27.3	−56 46	5.98	1	5.9	6.9	5.6	5.5	var.?
281	μ *Pictoris*........	6 30.1	−58 40	6.00	2	5.9	6.0	6.3	5.8	6.00
282	γ *Geminorum*....	6 30.5	+16 30	2.23	6	—	—	—	—	—
283	ν^1 *Canis Maj* ..	6 30.9	−18 34	5.31	1	—	6.4	—	—	—
284	ν^2 *Canis Maj*	6 31.2	−19 9	4.26	1	—	4.1	—	—	—
285	*N Carinae*.......	6 32.2	−52 52	4.95	2	5.0	4.8	4.7	4.7	—
286	ν^3 *Canis Maj*	6 32.4	−18 8	4.75	1	—	4.9	—	—	—
287	ν *Puppis*.......	6 33.9	−43 5	3.28	6	3.5	3.5	3.3	3.2	3.36
288	*Lac. 2402*........	6 35.3	−48 7	5.73	1	5.4	5.3	5.4	5.6	5.49
289	α *Canis Maj.*.....	6 39.6	−16 33	0.10	13	—	0.1	—	—	—
290	*10 Canis Maj.*....	6 39.7	−30 57	5.86	1	5.4	5.7	5.4	5.6	5.59
291	*Lac. 2438*........	6 40.7	−30 49	5.90	1	5.9	6.3	6.5	—	—
292	κ *Canis Maj.*.....	6 45.2	−32 22	3.73	2	4.1	4.0	4.0	4.2	var.?
293	*Lac. 2479*........	6 45.7	−31 34	5.99	1	5.9	6.3	6.2	6.0	6.08
294	*Lac. 2486*........	6 46.3	−34 13	6.17	1	5.4	5.4	5.3	5.4	—
295	τ *Puppis*.........	6 46.8	−50 28	3.11	8	3.5	3.2	2.8	3.0	3.12

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
296	α *Pictoris*	6 46.9	−61 48	3.36	10	3.5	3.5	3.3	3.4	3.41
297	*A Carinae*	6 47.1	−53 29	5.06	2	5.0	4.8	4.6	4.8	—
298	θ *Canis Maj.*	6 48.4	−11 53	4.29	1	—	4.4	—	—	—
299	o¹ *Canis Maj.*	6 48.9	−24 2	3.88	2	4.1	3.9	—	—	—
300	*Lac. 2557*	6 52.9	−48 33	5.73	1	5.4	5.5	5.3	5.7	5.53
301	ε *Canis Maj.*	6 53.7	−28 48	1.80	16	1.7	1.8	—	—	—
302	*t Puppis*	6 53.8	−33 57	6.14	1	5.4	5.4	5.6	5.5	—
303	σ *Canis Maj.*	6 56.7	−27 45	3.60	2	4.1	3.6	—	—	—
304	o² *Canis Maj.*	6 57.8	−23 39	3.33	6	3.8	3.4	—	—	—
305	*Lac. 2601*	6 57.8	−51 14	5.63	2	5.9	5.8	5.4	—	5.68
306	γ *Canis Maj.*	6 58.1	−15 27	4.20	3	—	4.5	—	—	var.?
307	*Lac. 2621*	6 59.1	−58 46	5.99	1	—	6.4	—	—	—
308	*Lac. 2646*	7 0.0	−67 45	5.72	1	5.4	5.7	5.4	5.8	5.60
309	*Lac. 2608*	7 0.1	−43 26	5.73	1	5.4	5.8	5.9	6.1	—
310	*Lac. 2624*	7 0.6	−49 24	5.28	2	5.4	5.3	5.6	5.5	—
311	*Lac. 2640*	7 1.3	−59 0	5.73	1	6.4	6.0	6.3	—	—
312	*Lac. 2642*	7 2.0	−56 34	5.52	1	6.4	5.7	5.7	5.6	5.78
313	δ *Canis Maj.*	7 3.3	−26 12	2.13	17	2.0	2.1	—	—	2.08
314	*Lac. 2673*	7 7.4	−48 44	5.32	2	5.4	5.6	5.5	5.7	—
315	*I Puppis*	7 9.0	−46 33	4.93	2	5.0	4.8	4.8	5.0	4.91
316	*L¹ Puppis*	7 9.5	−44 58	5.27	2	5.0	5.3	5.5	5.5	var.?
317	ω *Canis Maj.*	7 9.7	−26 33	4.55	1	5.0	4.2	—	—	—
318	*L² Puppis*	7 9.7	−44 26	var.	3	5.0	var.	var.	—	var.
319	γ *Volantis*	7 9.8	−70 18	3.85	11	4.1	3.8	3.7	3.8	3.85
320	*Lac. 2735*	7 10.8	−62 59	6.41	1	5.9	6.3	6.5	—	—
321	*Lac. 2711*	7 11.2	−48 3	5.50	1	5.0	5.0	5.2	5.3	—
322	*Lac. 2732*	7 12.6	−46 33	5.52	1	6.4	6.1	6.1	—	—
323	π *Puppis*	7 12.7	−36 52	2.69	8	2.7	2.7	2.5	2.5	2.62
324	δ *Volantis*	7 16.9	−67 44	4.18	5	4.5	4.1	4.1	4.2	4.22
325	*Lac. 2783*	7 17.6	−51 51	5.73	1	—	5.9	6.0	—	—
326	η *Canis Maj.*	7 19.1	−29 4	2.56	8	2.7	2.9	—	—	—
327	β *Canis Min.*	7 20.4	+ 8 32	3.04	4	—	3.0	—	—	—
328	*Lac. 2827*	7 22.0	−58 15	5.73	1	—	6.8	—	—	—
329	*Lac. 2829*	7 23.2	−50 47	5.52	1	5.9	5.7	5.4	—	5.63
330	σ *Puppis*	7 25.3	−43 3	3.43	7	3.5	3.5	3.0	3.4	3.37
331	α *Geminorum*	7 26.6	+32 10	1.92	5	—	—	—	—	—
332	*Ll. 14810*	7 28.7	−22 2	5.52	1	5.4	5.2	—	—	—
333	*p Puppis*	7 30.4	−28 6	4.69	1	5.0	5.3	—	—	—
334	*Q Carinae*	7 32.6	−52 15	5.63	2	5.4	5.5	5.3	—	—
335	*f Puppis*	7 32.7	−34 41	4.92	1	5.0	4.8	5.0	—	4.93
336	α *Canis Min.*	7 32.8	+ 5 33	1.06	11	—	1.2	—	—	—
337	*m Puppis*	7 33.1	−25 5	5.30	1	5.4	5.4	5.0	—	—
338	*Lac. 2904*	7 33.2	−48 33	5.88	2	5.9	6.1	6.4	—	—
339	*k Puppis*	7 33.7	−26 31	3.93	3	4.1	4.0	—	—	—
340	*Lac. 2918*	7 34.8	−48 19	5.88	2	5.9	6.0	6.1	—	—
341	α *Monocerotis*	7 35.3	− 0 16	4.88	1	—	4.0	—	—	—
342	*Lac. 2923*	7 37.6	−26 3	6.22	1	6.4	6.5	—	—	—
343	β *Geminorum*	7 37.6	+28 20	1.51	8	—	—	—	—	—
344	1 *Puppis*	7 38.5	−28 7	5.37	1	5.0	5.0	—	—	—
345	*l Puppis*	7 38.8	−28 40	4.06	2	5.0	4.2	—	—	—
346	*Lac. 2940*	7 39.3	−24 23	6.02	1	6.4	6.4	—	—	—
347	*c Puppis*	7 40.8	−37 40	3.57	1	3.8	3.6	3.5	3.6	3.61

TABLE VIII — *Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
348	o *Puppis*	7 42.9	−25 38	5.18	2	5.0	5.3	—	—	—
349	*Lac. 3011*	7 43.0	−56 25	5.99	1	6.4	6.4	—	—	—
350	ζ *Volantis*	7 43.3	−72 18	4.22	6	4.5	4.3	4.0	4.1	4.22
351	ξ *Puppis*	7 44.0	−24 33	3.30	3	3.8	3.5	—	—	—
352	*Q Puppis*	7 44.6	−46 46	5.52	1	5.0	5.1	5.0	5.1	—
353	*P Puppis*	7 45.4	−46 4	4.77	2	5.0	4.3	4.3	4.3	—
354	*a Puppis*	7 47.9	−40 15	3.80	1	4.1	4.0	3.8	3.8	3.90
355	*Lac. 3074*	7 49.5	−54 3	5.99	1	5.9	6.1	6.5	6.2	—
356	*Lac. 3069*	7 49.5	−49 17	4.95	2	5.4	5.0	5.1	5.0	5.09
357	*J Puppis*	7 49.6	−47 47	4.84	2	5.0	4.5	4.5	4.5	—
358	χ *Carinae*	7 53.6	−52 39	3.65	6	4.1	3.7	3.6	3.8	3.77
359	*V Puppis*	7 54.6	−48 54	4.97	2	5.0	5.0	4.8	var.	var.
360	*Br. 1855*	7 55.6	−62 58	6.48	1	6.4	6.5	—	—	—
361	*Lac. 3134*	7 56.7	−60 29	6.06	1	—	5.0	—	—	cum.
362	*D Carinae*	7 58.7	−63 13	5.07	1	5.0	5.2	5.3	5.4	—
363	ζ *Puppis*	7 59.2	−39 39	2.42	15	2.7	2.5	2.4	2.3	2.46
364	ρ *Puppis*	8 2.2	−23 57	2.94	4	2.9	3.2	—	—	—
365	*Lac. 3163*	8 2.7	−44 54	5.73	1	5.9	5.7	5.1	—	—
366	*19 Puppis*	8 5.4	−12 33	5.16	1	—	5.2	—	—	—
367	*Lac. 3180*	8 5.5	−43 45	5.73	1	5.9	5.9	5.6	—	5.78
368	γ *Velorum*	8 5.7	−46 58	1.96	22	2.3	2.9	1.9	1.9	var.?
369	h^1 *Puppis*	8 6.9	−39 15	5.12	2	5.4	4.8	4.5	4.8	—
370	*B Carinae*	8 6.9	−60 55	5.15	2	5.4	5.3	5.1	5.1	5.21
371	*Lac. 3197*	8 7.2	−42 37	5.76	1	5.4	5.3	5.2	4.8	var.?
372	ε *Volantis*	8 7.5	−68 15	4.40	5	4.5	4.5	4.7	4.8	—
373	h^2 *Puppis*	8 9.6	−39 58	5.12	2	5.4	4.8	4.5	4.9	—
374	β *Cancri*	8 9.7	+ 9 34	3.79	1	—	3.5	—	—	—
375	*C Carinae*	8 13.4	−62 32	5.35	3	5.4	5.7	5.7	5.5	—
376	*B Velorum*	8 18.7	−48 5	5.75	2	5.4	5.4	5.2	5.3	—
377	*C Hydrae*	8 19.4	− 3 30	4.07	1	—	3.8	—	—	—
378	ε *Carinae*	8 20.0	−59 6	2.05	16	2.0	2.1	1.9	1.8	1.97
379	κ^1 *and* κ^2 *Volantis*	8 20.2	−71 7	5.12	3	4.5	4.7	5.2	—	—
380	α *Chamaeleontis*	8 21.7	−76 31	4.30	5	4.1	4.2	4.2	4.3	4.22
381	*Lac. 3345*	8 22.8	−49 5	5.73	1	—	—	—	—	—
382	η *Volantis*	8 23.2	−73 0	5.58	1	5.0	5.7	5.8	5.7	5.56
383	*F Velorum*	8 24.2	−52 41	5.99	1	5.9	5.7	—	—	—
384	θ *Chamaeleontis*	8 24.3	−77 5	4.62	4	4.1	4.7	4.4	4.6	4.48
385	β *Volantis*	8 24.4	−65 43	4.13	5	4.1	3.9	3.7	3.9	—
386	*A Velorum*	8 25.1	−47 31	5.76	2	5.9	6.0	5.9	5.8	5.87
387	*C Velorum*	8 30.9	−49 31	5.88	2	5.9	5.6	5.3	5.6	—
388	δ *Hydrae*	8 31.0	+ 6 8	4.40	1	—	4.2	—	—	—
389	*Lac. 3443*	8 32.1	−50 32	5.76	1	6.4	6.4	—	6.2	—
390	σ *Hydrae*	8 32.2	+ 3 47	4.83	1	—	4.9	—	—	—
391	e^1 *Carinae*	8 32.4	−57 48	5.36	1	6.4	5.9	5.8	5.8	—
392	e^2 *Carinae*	8 32.4	−57 35	5.45	2	5.9	5.4	5.1	5.5	5.47
393	*c Velorum*	8 33.2	−42 33	5.02	2	4.5	4.6	4.3	4.6	—
394	β *Pyxidis*	8 35.2	−34 52	4.45	5	5.0	4.4	4.1	4.5	4.49
395	*b Velorum*	8 36.5	−46 12	4.43	2	4.1	4.1	3.8	4.0	—
396	o *Velorum*	8 36.7	−52 29	3.99	2	4.5	4.0	3.7	3.8	4.00
397	η *Hydrae*	8 36.7	+ 3 51	4.47	1	—	4.6	—	—	—
398	*n Velorum*	8 37.1	−46 52	5.42	2	5.4	5.2	5.1	—	—
399	*Lac. 3483*	8 37.1	−48 29	5.99	1	6.4	6.6	—	—	—

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
400	*l Carinae*	8 37.8	—50 19	4.90	1	4.5	4.7	4.6	4.8	4.70
401	*Lac. 3492*........	8 38.2	—47 39	5.75	2	—	5.9	5.9	—	—
402	α *Pyxidis*........	8 38.6	—32 44	4.01	5	4.1	3.8	3.8	3.8	3.90
403	θ *Volantis*	8 38.6	—69 50	5.52	1	5.4	5.6	5.6	5.6	5.54
404	*Lac. 3505*.	8 38.7	—52 39	5.27	1	5.4	5.7	[6.1]	—	—
405	*D Velorum*	8 39.8	—49 22	5.64	2	5.9	5.8	5.6	—	5.74
406	*d Velorum*	8 39.9	—42 11	5.00	2	4.5	4.4	4.1	4.4	—
407	ε *Hydrae*.......	8 40.1	+ 6 53	3.37	1	—	3.3	—	—	—
408	*D Hydrae*.	8 40.5	—13 6	4.94	1	—	4.4	—	—	—
409	δ *Velorum*.......	8 41.2	—54 15	2.19	15	2.3	2.2	2.1	2.0	2.16
410	*i Velorum*......	8 41.8	—45 35	4.41	2	4.1	4.1	4.2	4.0	—
411	*f Carinae*........	8 43.5	—56 19	5.19	2	5.4	5.1	4.9	5.0	5.12
412	*Lac. 3556*........	8 45.0	—39 51	5.99	1	6.4	6.2	5.9	—	—
413	γ *Pyxidis*........	8 45.2	—27 15	4.54	3	5.4	4.4	—	—	—
414	*z Velorum*.......	8 45.5	—44 51	5.63	2	5.9	5.7	5.3	5.5	—
415	*f Velorum*	8 46.3	—46 4	5.63	2	5.9	5.6	5.6	—	5.68
416	ζ *Hydrae*	8 48.8	+ 6 25	3.04	5	—	3.1	—	—	—
417	*17 Hydrae*.......	8 49.4	— 7 30	5.21	1	—	6.5	—	—	—
418	*Lac. 3597*	8 49.6	—47 3	5.73	1	5.9	5.8	5.7	—	5.78
419	δ *Pyxidis*	8 50.2	—27 12	5.05	1	5.9	5.4	—	—	—
420	*c Carinae*	8 52.2	—60 10	4.33	1	4.5	4.0	4.1	4.3	4.25
421	*H Velorum*	8 52.6	—52 15	5.46	2	5.4	5.4	5.0	—	—
422	*b¹ Carinae*	8 53.9	—58 45	5.65	2	5.4	5.4	5.5	5.0	—
423	*Lac. 3635*	8 54.6	—46 45	5.73	1	5.9	5.9	5.6	—	—
424	*w Velorum*.......	8 55.4	—40 46	5.73	1	5.4	5.2	4.6	—	—
425	*b² Carinae*.......	8 56.3	—58 36	5.65	2	5.9	5.7	5.6	5.3	—
426	*Lac. 3651*	8 56.7	—41 23	5.90	1	6.4	6.2	6.1	—	—
427	*Lac. 3667*	8 57.9	—51 42	5.88	2	6.4	5.9	5.8	—	—
428	*c Velorum*	8 59.8	—46 36	4.51	2	4.5	4.6	3.9	3.9	—
429	α *Volantis*	9 0.5	—65 54	4.20	5	4.5	4.2	4.3	4.2	4.28
430	λ *Velorum*..	9 3.4	—42 56	2.28	10	2.0	2.5	2.2	2.2	2.24
431	*E Carinae*	9 4.6	—70 2	5.33	1	5.4	5.2	5.1	4.8	—
432	*G Carinae*	9 4.8	—72 6	5.17	2	4.5	4.8	4.8	4.7	—
433	*a Carinae*	9 7.7	—58 27	3.85	2	4.1	3.8	3.6	3.8	3.83
434	*i Carinae*	9 8.4	—61 48	4.33	1	4.1	4.3	4.4	4.7	—
435	β *Carinae*	9 11.8	—69 12	1.96	20	1.7	2.0	1.9	1.7	1.85
436	*g Carinae*	9 12.7	—57 1	5.10	2	4.5	4.8	4.4	4.8	—
437	α *Lyncis*	9 13.4	+34 55	3.03	1	—	—	—	—	—
438	ι *Carinae*	9 13.7	—58 45	2.44	10	2.7	2.5	2.3	2.2	—
439	*Lac. 3846*	9 17.6	—74 22	5.81	1	5.0	5.7	5.8	5.5	—
440	*k Carinae*	9 18.0	—61 52	5.52	1	5.9	5.5	5.1	5.3	—
441	κ *Velorum*.......	9 18.2	—54 29	2.65	8	2.9	2.7	2.6	2.4	2.65
442	α *Hydrae*	9 21.4	— 8 7	2.10	16	—	2.1	—	—	—
443	τ^1 *Hydrae*	9 22.8	— 2 13	5.07	1	—	4.8	—	—	—
444	τ^2 *Hydrae*	9 25.6	— 0 38	4.72	1	—	4.8	—	—	—
445	ψ *Velorum*	9 25.8	—39 55	3.88	5	3.8	3.7	3.6	3.7	3.74
446	*Lac. 3914*	9 25.9	—71 4	5.81	1	6.4	6.0	5.8	5.9	5.98
447	*N Velorum*	9 27.4	—56 29	3.22	4	3.5	3.2	3.0	var.	var ?
448	ι *Chamaeleontis*...	9 28.2	—80 15	5.58	1	5.0	5.8	5.7	6.2	—
449	*R Carinae*.......	9 29.1	—62 14	5.07	1	—	var.	var.	var.	var.
450	*Lac. 3917*........	9 29.3	—48 27	5.44	1	5.4	5.6	5.7	5.6	—
451	*h Carinae*	9 30.8	—58 40	4.83	2	4.5	4.9	4.4	4.4	—

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean.
		h m	° ′							
452	*M Velorum*	9 32.3	−48 48	5.07	1	5.0	4.9	4.7	4.8	4.89
453	*y Velorum*	9 33.1	−42 38	6.09	1	6.4	6.0	5.8	6.3	6.12
454	*ι Hydrae*	9 33.5	− 0 34	4.04	3	—	3.9	—	—	—
455	*I Hydrae*........	9 35.6	−23 1	5.13	2	5.0	5.2	—	—	—
456	*m Carinae*.....	9 35.9	−60 46	5.07	1	5.0	5.1	4.8	4.6	—
457	*Ll. 19093*	9 36.6	−23 22	5.13	2	5.0	5.4	—	—	—
458	*ζ Chamaeleontis*...	9 37.5	−80 23	5.81	1	5.0	5.5	5.6	5.4	—
459	*ε Leonis*	9 38.7	+24 21	3.07	1	—	—		—	—
460	*Lac. 4022*	9 41.6	−44 11	5.78	1	5.9	6.0	6.2	6.0	—
461	*l Carinae*........	9 41.8	−61 56	3.83	2	4.5	4.4	var.	var.	var.
462	*υ Carinae*	9 44.0	−64 29	3.12	8	—	3.3	3.0	2.8	3.05
463	*u Velorum*	9 45.1	−45 9	5.56	1	5.9	5.6	5.7	—	5.69
464	*υ¹ Hydrae*	9 45.5	−14 16	4.23	2	—	4.0	—	—	—
465	*Lac. 4055*	9 46.5	−45 37	5.96	1	6.4	6.2	6.1	—	—
466	*m Velorum*.......	9 46.8	−45 58	5.14	1	5.4	4.8	4.7	—	—
467	*Lac. 4068*	9 49.4	−44 42	6.24	1	6.4	6.1	6.3	—	—
468	*φ Velorum*	9 52.5	−53 58	3.91	2	4.1	3.9	3.8	3.6	—
469	*Lac. 4094*	9 52.6	−52 3	6.43	1	6.4	6.5	--	—	—
470	*η Antliae*.......	9 53.5	−35 18	5.21	1	5.9	5.6	5.7	5.8	—
471	*Lac. 4123*	9 57.1	−52 46	6.46	1	6.4	6.5	—	—	—
472	*υ² Hydrae*	9 59.0	−12 28	4.95	1	—	4.6	—	—	—
473	*G. 1186*	10 1.2	−46 46	6.07	1	6.4	5.7	5.6	—	—
474	*α Sextantis*	10 1.5	+ 0 14	4.50	1	—	4.9	—	—	—
475	*α Leonis*	10 1.7	+12 35	1.66	8	—	—	—	—	—
476	*μ Chamaeleontis*..	10 4.0	−81 37	5.58	1	5.0	6.0	6.0	6.0	—
477	*Q Velorum*....	10 4.2	−51 12	5.62	1	5.4	5.3	5.5	5.4	5.44
478	*Lac. 4246*	10 6.4	−80 57	5.33	1	6.4	7.0	7.0	6.6	var.?
479	*Lac. 4206*	10 8.6	−50 37	5.74	1	5.9	5.8	6.0	5.6	5.81
480	*Lac. 4208*	10 8.7	−51 8	5.82	1	6.4	6.2	6.5	—	—
481	*q Velorum*.......	10 9.5	−41 30	4.34	3	3.8	4.0	4.1	4.1	4.07
482	*M Carinae*	10 10.0	−65 45	5.81	1	6.4	5.7	5.8	5.7	5.88
483	*Lac. 4222*	10 10.3	−42 29	6.05	1	5.9	6.2	6.1	—	6.06
484	*ω Carinae*.......	10 10.8	−69 25	3.39	7	3.5	3.6	3.7	3.2	3.48
485	*q Carinae*	10 12.9	−60 42	3.40	6	3.8	3.3	3.5	3.8	3.56
486	*γ Leonis*	10 13.1	+20 28	2.12	8	—	—	—	—	—
487	*μ Ursae Maj*.....	10 14.8	+42 8	2.99	1	—	—	—	—	—
488	*Lac. 4263*	10 14.9	−54 24	5.52	1	5.4	5.4	4.7	5.4	—
489	*J Velorum*.......	10 16.3	−55 25	5.07	1	4.5	5.0	4.7	5.3	var.?
490	*L Carinae*.......	10 19.3	−66 16	5.67	2	5.9	5.4	5.8	5.7	5.69
491	*μ Hydrae*	10 20.0	−16 12	4.03	2	—	4.0	—	—	—
492	*α Antliae*........	10 21.4	−30 26	4.66	1	4.5	4.4	4.4	4.0	var.?
493	*I Carinae*	10 21.9	−73 24	4.24	3	5.0	4.3	[4.2]	3.7	var.?
494	*Lac. 4310*	10 22.7	−57 0	5.52	1	5.4	5.4	5.3	5.2	5.36
495	*s Carinae*	10 23.3	−58 6	4.33	1	4.5	4.6	4.2	4.2	4.37
496	*K Carinae*	10 27.2	71 21	5.33	2	5.4	5.0	5.3	5.0	5.21
497	*p Carinae*	10 27.6	−61 3	3.68	3	3.8	3.6	3.7	3.5	3.66
498	*Lac. 4344*	10 27.7	−46 22	5.50	1	5.9	5.6	5.5	5.4	5.58
499	*Lac. 4367*	10 28.1	−72 35	5.81	1	5.9	5.6	5.1	5.4	—
500	*Lac. 4375*	10 31.1	−57 35	5.07	1	—	5.7	—	—	—
501	*p Velorum*.......	10 32.0	−47 35	4.24	1	4.5	4.1	4.2	3.9	—
502	*t² Carinae*.......	10 34.0	−58 32	5.52	1	5.4	5.2	5.0	5.4	5.30
503	*γ Chamaeleontis*..	10 34.0	−77 58	4.47	4	5.0	4.4	4.3	4.3	4.49

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m	° ′							
504	*x Velorum*	10 34.3	−54 57	4.90	1	4.5	4.8	4.6	4.4	—
505	*θ Carinae*	10 38.5	−63 44	2.93	6	3.1	2.9	3.0	2.6	2.91
506	*Lac. 4455*	10 39.6	−63 18	5.76	1	6.4	5.6	5.5	—	—
507	*η Argûs*	10 40.2	−59 2	var.	21	var.	var.	var.	—	var.
508	*μ Velorum*	10 41.4	−48 46	2.76	11	3.1	2.9	2.8	2.6	2.83
509	*Lac. 4473*	10 41.9	−63 36	5.52	1	—	5.8	5.9	—	—
510	*ν Hydrae*	10 43.4	−15 32	3.19	6	—	3.0	—	—	—
511	*δ Chamaeleontis*	10 44.6	−79 53	4.29	4	5.0	4.5	[4.9]	4.9	—
512	*λ Hydrae*	10 45.0	−11 44	3.74	3	—	3.4	—	—	—
513	*u Carinae*	10 48.4	−58 11	4.07	2	4.1	4.0	4.0	4.3	4.09
514	*α Crateris*	10 53.7	−17 38	4.21	2	—	4.4	—	—	—
515	*β Ursae Maj*	10 54.3	+57 3	2.42	2	—	—	—	—	—
516	*α Ursae Maj*	10 56.0	+62 25	1.91	3	—	—	—	—	—
517	*Lac. 4604*	11 1.2	−58 0	5.33	1	—	6.5	6.5	—	—
518	*z Carinae*	11 1.4	−61 45	5.58	1	5.4	5.3	5.1	—	—
519	*Lac. 4613*	11 1.4	−64 10	5.81	1	—	7.0	—	—	—
520	*ψ Ursae Maj*	11 2.6	+45 11	2.94	2	—	—	—	—	—
521	*x Carinae*	11 3.3	−58 18	4.52	1	4.5	4.6	4.2	—	4.46
522	*Lac. 4629*	11 3.4	−61 16	5.81	1	5.9	6.0	5.9	—	5.90
523	*β Crateris*	11 5.5	−22 9	4.57	3	3.8	4.6	—	—	—
524	*y Carinae*	11 7.2	−59 38	5.07	1	5.4	5.2	5.0	—	5.17
525	*δ Leonis*	11 7.4	+21 12	2.54	3	—	—	—	—	—
526	*Lac. 4657*	11 7.6	−63 29	5.81	1	5.9	5.7	6.0	5.9	5.86
527	*θ Leonis*	11 7.7	+16 3	3.39	1	—	—	—	—	—
528	*φ Leonis*	11 10.3	− 2 58	4.61	1	—	4.2	—	—	—
529	*ξ Ursae Maj*	11 11.5	+32 14	3.45	1	—	—	—	—	—
530	*δ Crateris*	11 13.1	−14 6	3.49	3	—	3.8	—	—	—
531	*π Centauri*	11 15.3	−53 43	4.33	1	4.5	4.3	4.6	4.5	4.45
532	*ι Leonis*	11 17.4	+11 13	3.70	1	—	—	—	—	—
533	*γ Crateris*	11 18.6	−17 0	4.14	1	—	4.2	—	—	—
534	o¹ *and* o² *Centauri*	11 26.0	−58 45	5.43	2	5.9	5.2	5.3	5.8	—
535	*Br. 3634*	11 26.1	−59 55	5.58	1	—	—	[7.5]	—	—
536	*ξ Hydrae*	11 26.9	−31 10	3.65	2	3.5	3.7	3.7	3.1	var.?
537	*A Centauri*	11 28.8	−53 34	4.90	1	5.0	5.2	5.1	5.4	—
538	*Lac. 4798*	11 29.5	−60 12	5.58	1	—	—	[6.7]	—	—
539	*Lac. 4801*	11 29.9	−60 36	5.58	1	—	—	—	—	—
540	*λ Centauri*	11 30.0	−62 20	3.35	4	3.8	3.4	3.4	3.4	3.47
541	*θ Crateris*	11 30.3	− 9 7	5.00	1	—	5.0	—	—	—
542	*υ Leonis*	11 30.5	− 0 8	4.30	1	—	4.4	—	—	—
543	*Br. 3659*	11 32.3	−61 8	5.58	1	—	6.0	5.8	6.1	—
544	*Lac. 4843*	11 33.7	−64 42	5.81	1	5.4	5.8	5.5	6.0	5.70
545	*Lac. 4868*	11 37.6	−61 48	5.58	1	5.9	5.8	5.6	5.7	5.72
546	*ζ Crateris*	11 38.4	−17 39	5.08	1	—	5.2	—	—	—
547	*λ Muscae*	11 39.7	−66 2	3.89	3	4.1	3.8	3.9	4.2	3.98
548	*Lac. 4885*	11 40.5	−60 29	4.84	2	4.5	4.7	4.5	4.6	4.63
549	*Lac. 4899*	11 42.2	−66 7	5.79	2	5.4	5.3	5.0	5.5	—
550	*β Leonis*	11 42.7	+15 16	2.21	7	—	—	—	—	—
551	*Lac. 4903*	11 43.6	−63 5	4.93	2	4.5	4.9	4.9	4.9	4.83
552	*Ll. 22339*	11 44.0	−15 10	4.88	1	—	6.5	—	—	—
553	*Lac. 4907*	11 44.0	−69 32	5.81	1	5.9	5.6	5.3	5.8	—
554	*β Virginis*	11 44.2	+ 2 28	3.58	3	—	3.7	—	—	—
555	*Lac. 4920*	11 45.7	−64 31	5.55	2	5.4	5.4	5.5	5.4	5.45

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m	° ′							
556	*β Hydrae*	11 46.6	−33 13	3.92	1	4.5	4.5	4.7	4.1	—
557	*γ Ursae Maj.*	11 47.2	+54 23	2.36	3	—	—	—	—	—
558	*ε Chamaeleontis*...	11 53.5	−77 31	4.83	4	5.4	4.9	[5.4]	5.5	—
559	*θ¹ Crucis*.	11 56.7	−62 37	5.20	2	5.0	4.7	4.7	4.8	—
560	*θ² Crucis*.........	11 57.9	−62 28	5.55	2	5.4	5.3	5.3	5.1	—
561	*κ Chamaeleontis*...	11 58.3	−75 50	5.76	1	5.9	5.6	5.3	5.8	5.67
562	*η Crucis*.........	12 0.4	−63 55	4.61	3	4.1	4.7	4.5	4.5	4.48
563	*δ Centauri*.......	12 1.9	−50 2	2.68	10	2.9	2.8	2.8	2.4	2.72
564	*α Corvi*..........	12 2.0	−24 2	4.20	3	4.1	4.2	—	—	—
565	*ε Corvi*..........	12 3.7	−21 55	2.89	10	2.9	3.3	—	—	—
566	*ρ Centauri*.......	12 5.1	−51 40	4.24	2	4.1	4.5	4.3	4.5	—
567	*Lac. 5072*	12 7.6	−65 51	5.76	1	—	6.8	—	—	—
568	*δ Crucis*.........	12 8.5	−58 3	3.19	10	3.1	3.4	3.1	3.0	3.16
569	*γ Corvi*..........	12 9.4	−16 51	2.56	9	—	2.5	—	—	—
570	*ε Muscae*	12 10.8	−67 16	4.84	2	4.5	4.7	4.3	4.8	—
571	*β Chamaeleontis* ..	12 11.0	−78 37	4.42	4	5.0	4.6	4.6	4.8	4.68
572	*δ Ursae Maj.*.....	12 11.6	+57 28	3.15	2	—	—	—	—	—
573	*ζ Crucis*.........	12 11.7	−63 18	4.53	3	4.5	4.6	4.5	4.3	4.49
574	*F Centauri*	12 12.3	−54 27	5.52	1	6.4	5.8	5.3	5.9	—
575	*η Virginis*.......	12 13.5	+ 0 2	3.69	4	—	4.0	—	—	—
576	*ζ Corvi*..........	12 14.1	−21 31	5.41	1	5.4	5.3	—	—	—
577	*ε Crucis*.........	12 14.6	−59 43	3.92	7	4.1	4.0	3.6	3.7	—
578	*ζ² Muscae*........	12 15.2	−66 50	5.55	2	5.4	5.8	5.7	5.6	5.61
579	*x¹ Centauri*......	12 17.0	−34 43	4.72	1	5.9	5.5	5.8	5.7	—
580	Unknown........	12 18.	−53 41	5.58	1	—	—	—	—	—
581	*α Crucis*.........	12 19.7	−62 24	1.32	20	1.0	1.3	1.3	1.3	1.24
582	*G Centauri*.......	12 19.8	−50 45	4.78	1	5.4	5.7	5.4	5.3	—
583	*σ Centauri*.......	12 21.3	−49 32	4.50	4	5.0	4.3	4.3	4.1	—
584	*Lac. 5172*........	12 23.3	−60 18	5.67	1	—	—	[6.9]	—	—
585	*δ Corvi*..........	12 23.4	−15 49	2.83	9	—	3.0	—	—	—
586	*γ Crucis*.........	12 24.2	−56 25	1.78	16	1.7	2.0	1.7	1.7	1.78
587	*Lac. 5185*........	12 24.7	−58 44	5.51	1	6.4	6.4	—	6.3	—
588	*γ Muscae*.........	12 25.0	−71 26	4.20	5	4.1	4.0	4.2	3.9	4.08
589	*η Corvi*..........	12 25.7	−15 30	4.72	2	—	4.5	—	—	—
590	*β Corvi*..........	12 27.8	−22 42	2.61	10	2.3	2.6	—	—	—
591	*Lac. 5208*........	12 28.4	−61 3	5.67	1	6.4	6.2	—	—	—
592	*α Muscae*..... .	[illegible] 29.7	−68 27	2.95	6	3.1	2.9	2.9	3.1	2.99
593	*τ Centauri*.......	12 30.9	−47 51	4.97	2	4.5	4.4	4.2	4.1	—
594	*γ Centauri*.......	12 34.6	−48 16	2.30	15	2.9	2.4	2.4	2.2	—
595	*Lac. 5241*........	12 34.8	−59 0	5.51	1	5.9	6.0	5.4	5.8	—
596	*γ Virginis*.......	12 35.3	− 0 46	2.72	9	—	3.1	—	—	—
597	*ι Crucis*	12 38.3	−60 18	5.52	1	5.9	5.7	5.0	5.3	—
598	*β Muscae*.........	12 38.6	−67 25	3.26	7	3.8	3.4	3.3	3.2	—
599	*Lac. 5273*........	12 39.2	−55 48	5.51	2	5.4	5.4	5.3	5.2	—
600	*β Crucis*.........	12 40.4	−59 0	1.64	19	1.7	1.7	1.7	1.6	1.67
601	*Lac. 5305*........	12 45.8	−54 16	5.49	1	—	6.0	—	—	—
602	*κ Crucis (Cl.)*.....	12 46.4	−59 42	4.90	1	5.4	5.6	[6.3]	5.1	—
603	*n Centauri*.......	12 46.5	−39 30	4.92	1	5.0	4.4	4.5	4.6	—
604	*μ Crucis*.........	12 47.3	−56 30	4.16	4	4.1	4.1	[4.5]	4.1	4.12
605	*λ Crucis*.........	12 47.3	−58 28	4.90	1	5.4	5.6	5.2	5.1	—
606	*ε Ursae Maj.*.....	12 48.6	+56 38	1.89	3	—	—	—	—	—
607	*δ Virginis*.......	12 49.3	+ 4 5	3.44	3	—	3.5	—	—	—

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
608	*H Centauri*	12 49.9	−50 31	5.65	1	5.9	5.8	5.6	5.6	5.71
609	*α Canum Venat.*..	12 50.2	+39 0	2.81	3	—	—	—	—	—
610	*δ Muscae*	12 53.7	−70 52	4.00	5	4.1	3.6	[3.7]	3.7	—
611	*ε Virginis*........	12 55.9	+11 38	2.79	1	—	—	—	—	—
612	*ξ¹ Centauri*	12 56.3	48 51	5.46	1	5.9	5.8	5.4	5.3	—
613	*f Centauri*.......	12 59.0	−47 47	4.42	1	5.4	5.3	5.3	5.3	—
614	*ξ² Centauri*	12 59.6	−49 14	4.37	1	5.4	4.8	4.6	4.5	—
615	Unknown........	13 3.	−43 44	6.28	1	—	—	—	—	—
616	*Lac. 5420*........	13 4.0	−41 34	6.26	1	6.4	6.1	6.3	—	6.27
617	*Lac. 5422*	13 4.2	−42 42	6.20	1	6.4	5.7	5.5	6.4	—
618	*Lac. 5418*........	13 4.5	−59 15	4.90	1	5.4	5.4	5.0	5.2	—
619	*Lac. 5429*........	13 5.1	−37 8	5.90	1	5.9	5.3	5.1	5.6	—
620	*Lac. 5437*	13 6.5	−58 26	5.52	1	5.9	5.9	5.4	5.4	—
621	*η Muscae*	13 6.8	−67 14	5.07	1	5.4	5.3	5.3	5.1	5.23
622	*Lac. 5460*...... ..	13 9.3	−44 3	6.30	1	—	—	—	—	—
623	*r Centauri*.......	13 9.9	−30 51	5.87	1	5.9	5.7	5.5	6.0	—
624	*γ Hydrae*........	13 12.1	−22 31	3.11	6	3.5	3.2	—	—	—
625	*ι Centauri*.......	13 13.6	−36 3	2.93	9	3.1	3.0	3.0	3.0	3.01
626	*J Centauri*.......	13 14.6	−60 20	4.90	1	5.4	5.2	4.8	5.0	5.06
627	*m Centauri*	13 15.6	−63 53	4.90	1	5.0	5.2	4.7	5.6	—
628	*α Virginis*.......	13 18.6	−10 30	1.43	13	—	1.5	—	—	—
629	*ζ Ursae Maj*......	13 18.9	+55 35	2.15	3	—	—	—	—	—
630	*Lac. 5531*........	13 18.9	−39 6	6.24	1	6.4	5.9	5.5	5.9	—
631	*ω Centauri*.......	13 19.3	−46 50	4.71	1	—	4.	—	—	—
632	*κ Centauri*.......	13 21.8	50 31	5.92	1	5.9	5.8	5.7	5.8	5.82
633	*d Centauri*.......	13 23.8	−38 46	4.49	2	4.1	4.5	4.1	4.6	4.36
634	*ζ Virginis*.......	13 28.3	+ 0 3	3.35	4	—	3.6	—	—	—
635	*ε Centauri*.......	13 32.0	52 50	2.47	11	2.9	2.6	2.6	2.6	2.63
636	*i Centauri*.......	13 38.6	−32 25	4.41	2	4.5	4.5	4.5	4.8	—
637	*M Centauri*......	13 38.7	−50 48	5.48	2	5.4	5.2	4.9	5.3	—
638	*z Centauri*.......	13 39.7	35 38	5.71	2	5.4	5.8	5.6	5.6	5.62
639	*Lac. 5674*........	13 39.8	49 42	6.22	1	—	6.0	6.0	—	—
640	*ν Centauri*.......	13 42.0	41 4	3.50	8	4.1	3.7	3.6	3.6	3.70
641	*μ Centauri*.......	13 42.1	−41 51	3.49	8	3.8	3.4	3.4	3.3	3.48
642	*g Centauri*.......	13 42.2	−33 49	4.40	1	4.5	4.6	4.5	4.8	—
643	*η Ursae Maj*.....	13 42.6	+49 56	2.05	3	—	—	—	—	—
644	*N Centauri*......	13 44.0	−52 11	5.80	2	6.4	5.9	5.8	5.6	5.90
645	*k Centauri*.......	13 44.6	−32 22	4.39	2	5.0	4.7	4.6	5.0	—
646	*Lac. 5711*........	13 45.6	−52 45	6.10	1	5.9	6.5	6.5	6.1	6.22
647	*h Centauri*.......	13 46.0	−31 19	4.60	2	5.4	5.2	5.2	5.4	—
648	*y Centauri*.......	13 46.2	−35 3	5.46	1	6.4	6.0	6.2	6.3	—
649	*Lac. 5727*......	13 47.1	−51 33	6.10	1	6.4	6.1	6.3	5.7	—
650	*ζ Centauri*.......	13 47.7	−46 40	2.68	11	2.9	2.7	2.8	2.8	2.78
651	*Lac. 5733*........	13 48.6	−63 4	5.52	1	5.4	5.7	5.1	5.9	—
652	*η Bootis*.... ...	13 48.7	+19 2	2.86	1	—	—	—	—	—
653	*φ Centauri.*	13 50.7	−41 29	3.92	7	4.5	4.1	4.2	4.1	4.16
654	*υ¹ Centauri*	13 51.0	−44 11	4.23	5	4.5	4.2	4.2	4.0	4.23
655	*υ² Centauri*	13 53.9	−45 0	4.84	3	5.0	5.0	4.7	4.4	—
656	*β Centauri*.......	13 55.0	−59 46	1.24	17	1.0	1.2	1.2	1.2	1.17
657	*Lac. 5787*........	13 55.3	−51 27	6.15	1	—	—	—	—	—
658	*χ Centauri.*	13 58.4	40 35	4.54	2	5.0	4.8	4.9	5.1	—
659	*π Hydrae*........	13 59.2	−26 5	3.40	2	3.8	3.6	—	—	—

TABLE III—*Continued.*

No.	Name	R. A.	Dec.	h.	n.	B.	U. A.	S.M.P.	W.	Mean
		h m								
660	θ *Centauri*.......	13 59.3	−35 45	2.20	13	2.7	2.2	2.2	2.3	2.32
661	*Lac. 5825*........	14 1.4	−50 55	6.12	1	6.4	6.3	—	—	—
662	*Lac. 5827*........	14 1.6	−52 51	5.52	1	—	5.4	5.2	—	—
663	*Lac. 5840*........	14 4.9	−53 5	6.50	1	6.4	6.1	5.9	—	—
664	κ *Virginis*.......	14 6.2	− 9 41	4.28	1	—	4.2	—	—	—
665	*Lac. 5850*........	14 6.3	−56 30	5.59	1	5.9	5.6	5.6	5.8	5.70
666	ι *Virginis*........	14 9.5	− 5 24	4.16	1	—	4.1	—	—	—
667	α *Bootis*.........	14 10.0	+19 50	0.79	10	—	—	—	—	—
668	ι *Lupi*...........	14 11.4	−45 29	3.72	4	4.5	3.8	4.1	3.6	—
669	*v Centauri*.......	14 11.6	−55 48	4.48	2	5.4	5.0	4.6	4.8	—
670	λ *Bootis*.........	14 11.6	+46 40	3.50	1	—	—	—	—	—
671	λ *Virginis*.......	14 12.3	−12 48	4.49	1	—	5.0	—	—	—
672	*Lac. 5891*........	14 12.7	−44 36	5.27	1	5.9	5.7	5.1	5.5	—
673	ψ *Centauri*......	14 13.0	−37 18	4.54	1	4.5	4.4	4.4	5.0	—
674	*Lac. 5893*........	14 13.7	−57 53	5.56	2	5.4	5.6	5.3	5.6	5.49
675	*a Centauri*	14 15.3	−38 56	4.58	1	5.0	4.9	4.9	5.5	var.?
676	τ^1 *Lupi*	14 18.1	−44 39	5.42	2	5.4	5.3	4.9	5.1	—
677	τ^2 *Lupi*	14 18.1	−44 49	5.24	2	5.4	4.9	4.6	4.9	—
678	σ *Lupi*..........	14 24.2	−49 54	4.83	2	5.0	5.2	4.9	4.8	4.95
679	γ *Bootis*.........	14 27.1	+38 51	2.96	2	—	—	—	—	—
680	η *Centauri*	14 27.6	−41 36	2.64	9	2.9	2.5	2.6	2.7	2.67
681	ρ *Lupi*..........	14 29.5	−48 53	4.47	2	5.0	4.5	4.2	4.6	—
682	α *Centauri*	14 31.1	−60 19	0.66	18	1.0	0.7	(0.20)	(0.55)	—
683	α *Circini*........	14 32.4	−64 26	3.40	5	3.5	3.5	3.4	3.7	3.50
684	α *Apodis*	14 32.4	−78 31	4.38	2	4.1	4.0	3.9	3.7	—
685	α *Lupi*..........	14 33.6	−46 51	2.47	9	2.7	2.6	2.5	2.5	2.55
686	*b Centauri*.......	14 34.2	−37 15	4.24	1	4.5	4.2	4.4	5.3	—
687	c^1 *Centauri*	14 36.0	−34 38	4.25	2	4.5	4.3	3.9	4.5	—
688	μ *Virginis*.......	14 36.5	− 5 7	4.05	1	—	4.0	—	—	—
689	ε *Bootis*.........	14 39.5	+27 36	2.45	3	—	—	—	—	—
690	*109 Virginis*.....	14 39.9	+ 2 25	3.80	1	—	4.0	—	—	—
691	ο *Lupi*..........	14 43.5	−43 3	5.08	1	5.0	5.0	4.7	5.0	4.96
692	α *Librae*.........	14 43.8	−15 29	2.86	3	—	3.0	—	—	—
693	β *Lupi*..........	14 50.3	−42 38	2.89	8	3.1	2.8	2.8	2.7	2.86
694	*16 Librae*	14 50.7	− 3 50	5.45	1	—	4.8	—	—	—
695	κ *Centauri*	14 51.0	−41 36	3.20	7	3.8	3.3	3.4	3.2	—
696	β *Ursae Min*.....	14 51.1	+74 40	2.12	2	—	—	—	—	—
697	δ *Librae*.........	14 54.3	− 8 1	var.	1	—	var.	—	—	var.
698	π *Lupi*.	14 56.6	−46 34	4.04	3	5.0	4.3	4.0	4.5	—
699	*20 Librae*........	14 56.7	−24 47	3.36	1	3.5	3.5	3.2	—	3.39
700	β *Bootis*.........	14 57.2	+40 53	3.31	1	—	—	—	—	—
701	λ *Lupi*..........	15 0.4	−44 48	4.29	3	5.0	4.8	4.6	4.6	—
702	κ *Lupi*..........	15 3.2	−48 16	3.75	4	4.5	4.1	[4.4]	4.1	—
703	ζ *Lupi*..........	15 3.3	−51 37	3.63	6	3.8	3.6	3.5	3.4	3.59
704	*e Lupi*	15 4.4	−44 2	5.43	1	6.4	3.5	5.3	5.5	—
705	ι *Librae*.........	15 5.1	−19 19	4.85	1	—	5.0	—	—	—
706	δ *Circini*	15 6.9	−60 29	5.52	1	5.4	5.4	[5.6]	5.1	—
707	γ *Trianguli*	15 7.3	−68 13	3.12	5	3.1	3.1	3.0	3.3	3.12
708	β *Circini*........	15 7.7	−58 20	4.90	1	4.5	4.7	4.2	4.5	—
709	μ *Lupi*..........	15 9.9	−47 25	3.96	2	5.0	4.8	4.4	4.9	—
710	β *Librae*.........	15 10.3	− 8 55	2.82	3	—	3.1	—	—	—
711	δ *Bootis*	15 10.5	+33 47	3.27	1	—	—	—	—	—

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
712	δ *Lupi*........	15 13.2	−40 12	3.43	6	4.1	3.6	3.4	3.5	—
713	γ *Circini*........	15 13.4	−58 52	4.90	1	5.0	5.2	4.7	5.0	4.96
714	ϕ^1 *Lupi*.........	15 13.9	−35 48	3.81	3	4.5	3.6	3.3	3.8	—
715	ε *Lupi*..........	15 14.2	−44 14	3.57	6	4.1	3.7	3.6	3.7	3.73
716	ϕ^2 *Lupi*.........	15 15.2	−36 25	5.00	1	5.9	5.1	5.0	5.5	—
717	ε *Trianguli*	15 25.3	−65 54	4.99	2	5.0	4.6	4.2	4.7	—
718	γ *Lupi*..........	15 26.8	−40 45	3.06	8	3.5	3.2	3.0	2.9	—
719	*d Lupi*..........	15 27.3	−44 32	5.16	1	5.4	5.1	5.1	5.4	5.23
720	γ *Librae*.........	15 28.5	−14 22	4.10	1	—	4.4	—	—	—
721	δ *Serpentis*.......	15 28.8	+10 57	3.71	1	—	—	—	—	—
722	α *Coronae Bor* ...	15 29.4	+27 8	2.39	2	—	—	—	—	—
723	υ *Librae*.........	15 29.4	−27 43	4.32	2	4.5	3.9	—	—	—
724	ω *Lupi*..........	15 29.6	−42 9	5.12	1	5.4	4.7	4.2	5.3	—
725	ξ *Lupi*..........	15 32.6	−44 15	5.19	1	5.4	5.2	4.8	5.4	—
726	α *Serpentis*	15 38.1	+ 6 49	2.81	2	—	2.6	—	—	—
727	β *Serpentis*.......	15 40.4	+15 49	3.42	1	—	—	—	—	—
728	*Ll.* 28745........	15 40.4	+ 7 45	5.31	1	—	4.8	—	—	—
729	μ *Serpentis*.......	15 43.1	− 3 3	3.39	1	—	3.3	—	—	—
730	β *Trianguli*	15 44.1	−63 2	3.07	5	3.1	3.1	3.1	3.3	3.13
731	ε *Serpentis*.......	15 44.6	+ 4 51	3.85	1	—	3.7	—	—	—
732	θ *Librae*	15 46.7	−16 22	4.49	1	—	4.8	—	—	—
733	ρ *Scorpii*.	15 49.2	−28 51	4.16	2	4.5	4.5	—	—	—
734	γ *Serpentis*.......	15 50.7	+16 4	3.99	1	—	3.9	—	—	—
735	*f* 18 *Librae*	15 51.2	−13 55	5.45	1	—	5.4	—	—	—
736	π *Scorpii*........	15 51.3	−25 45	3.04	3	2.9	3.4	—	—	—
737	η *Lupi*..........	15 51.8	−38 2	3.68	3	5.0	3.7	3.9	3.7	—
738	δ *Scorpii*...	15 52.9	−22 16	2.51	4	2.3	2.4	—	—	—
739	η *Normae*........	15 54.0	−48 53	5.39	1	6.4	5.2	5.0	5.6	—
740	ξ *Scorpii*.........	15 57.5	−11 2	4.49	1	—	4.6	—	—	—
741	δ *Normae*.	15 57.7	−44 50	5.35	1	5.9	4.9	5.1	5.7	—
742	β *Scorpii*	15 58.2	−19 28	2.70	3	—	2.5	—	—	—
743	θ *Lupi*	15 58.4	36 28	4.88	2	5.9	4.9	4.7	4.8	—
744	ω^1 *Scorpii*........	15 59.5	−20 20	4.19	1	4.5	4.4	—	—	—
745	ω^1 *Scorpii*........	16 0.1	−20 32	4.71	1	5.0	4.6	—	—	—
746	δ *Apodis*.........	16 1.9	78 22	4.62	2	5.0	4.6	4.5	4.6	4.66
747	δ *Trianguli*......	16 4.1	−63 22	4.33	1	4.1	4.3	4.2	4.0	4.19
748	ι^2 *Scorpii*........	16 4.6	−27 36	5.05	1	5.4	5.3	—	—	—
749	ν *Scorpii*.........	16 4.7	−19 8	4.28	1	—	4.3	—	—	—
750	ψ *Scorpii*	16 5.2	9 44	5.40	1	—	5.2	—	—	—
751	χ *Scorpii*	16 6.9	11 31	5.14	1	—	5.6	—	—	—
752	δ *Ophiuchi*.......	16 7.8	− 3 22	2.88	1	—	2.7	—	—	—
753	"Cluster"	16 9.6	−22 40	4.62	1	—	—	—	—	—
754	ε *Ophiuchi*.......	16 11.7	− 4 23	3.46	1	—	3.3	—	—	—
755	ο *Scorpii*.........	16 13.1	−23 52	3.76	1	5.4	5.1	—	—	—
756	σ *Scorpii*	16 13.6	−25 17	3.13	4	3.1	3.4	3.0	—	3.16
757	γ *Apodis*	16 14.3	−78 37	4.31	2	4.5	3.9	4.1	4.0	4.16
758	σ *Serpentis*.......	16 15.7	+ 1 10	5.60	1	—	5.4	—	—	—
759	ρ *Ophiuchi*.......	16 18.1	−23 9	4.80	1	5.0	4.8	—	—	—
760	υ *Ophiuchi*.......	16 21.0	− 8 5	5.19	1	—	5.2	—	—	—
761	α *Scorpii*.	16 21.7	26 9	1.37	9	1.3	1.4	—	—	—
762	*i Scorpii*........	16 22.6	−24 50	4.54	1	5.9	5.3	5.3	—	—
763	*N Scorpii*........	16 23.2	−34 26	4.50	3	5.0	4.6	4.7	4.8	—

TABLE III — *Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m	° ′							
764	φ *Ophiuchi*	16 24.0	−16 20	5.22	1	—	4.6	—	—	—
765	λ *Ophiuchi*	16 24.6	+ 2 16	3.89	1	—	3.8	—	—	—
766	ω *Ophiuchi*	16 24.7	−21 12	4.88	1	5.0	4.7	—	—	—
767	β *Herculis*	16 24.8	+21 46	2.92	1	—	—	—	—	—
768	β *Apodis*	16 25.3	−77 15	4.25	1	4.5	4.5	4.4	4.4	4.41
769	τ *Scorpii*	16 28.1	−27 57	3.08	4	3.1	3.2	—	—	—
770	*H Scorpii*	16 28.1	−35 0	4.58	3	5.4	4.4	4.3	4.6	var.?
771	ζ *Ophiuchi*	16 30.3	−10 19	2.71	2	—	2.6	—	—	—
772	*Lac. 6906*	16 34.1	−66 52	4.90	1	5.9	5.6	5.7	5.4	—
773	α *Trianguli*	16 35.4	−68 48	2.05	9	2.3	2.2	2.0	2.1	2.13
774	ε *Scorpii*	16 42.1	−34 4	2.40	3	2.9	2.3	2.3	2.4	2.46
775	μ^1 *Scorpii*	16 43.4	−37 50	3.41	2	3.8	3.6	3.3	3.2	3.46
776	μ^2 *Scorpii*	16 43.9	−37 48	3.91	1	4.5	3.9	3.9	3.5	—
777	ζ^2 *Scorpii*	16 45.8	−42 9	3.85	2	4.5	3.6	3.6	3.5	—
778	ι *Opiuchi*	16 48.1	+10 22	5.05	1	—	—	—	—	—
779	κ *Ophiuchi*	16 51.7	+ 9 34	3.29	1	—	3.4	—	—	—
780	η *Ophiuchi*	17 3.2	−15 34	2.58	2	—	2.4	—	—	—
781	η *Scorpii*	17 3.2	−43 4	3.45	2	3.8	3.6	3.4	3.7	3.59
782	α *Herculis*	17 8.9	+14 32	3.15	1	—	—	—	—	—
783	ο *Ophiuchi*	17 10.4	−24 9	5.32	1	6.4	5.5	—	—	—
784	θ *Opihuchi*	17 14.3	−24 52	3.52	1	3.5	3.6	3.5	—	3.53
785	β *Arae*	17 14.9	−55 24	3.09	2	2.9	2.8	2.7	2.8	2.86
786	γ *Arae*	17 14.9	−56 15	3.46	2	3.5	3.6	3.5	3.4	3.49
787	α *Arae*	17 22.2	−49 46	3.08	2	2.9	2.9	2.9	3.1	2.98
788	υ *Scorpii*	17 22.3	−37 12	3.01	2	2.9	3.2	2.9	3.0	3.00
789	λ *Scorpii*	17 25.1	−37 1	1.84	6	2.0	2.0	1.9	2.2	1.99
790	θ *Scorpii*	17 28.3	−42 55	2.12	8	2.3	2.1	2.1	2.2	2.16
791	α *Ophiuchi*	17 29.1	+12 39	2.27	1	—	2.1	—	—	—
792	ξ *Serpentis*	17 30.4	−15 19	3.69	1	—	3.7	—	—	—
793	κ *Scorpii*	17 33.8	−38 58	2.69	4	2.7	2.6	2.6	2.7	2.66
794	ι^1 *Scorpii*	17 38.8	−40 4	3.15	2	3.1	3.3	3.1	3.2	3.17
795	*G Scorpii*	17 41.3	−37 0	3.37	3	3.1	3.4	3.2	3.5	3.31
796	ν *Ophiuchi*	17 52.2	− 9 45	3.01	1	—	3.5	—	—	—
797	γ *Sagittarii*	17 57.8	−30 25	3.37	1	3.1	2.8	3.0	—	—
798	μ *Sagittarii*	18 6.3	−21 5	4.22	1	3.8	4.3	—	—	—
799	η *Sagittarii*	18 9.2	−36 48	3.50	1	3.1	3.3	3.0	3.4	—
800	δ *Sagittarii*	18 13.0	−29 53	2.83	3	2.9	2.8	2.7	—	2.81
801	η *Serpentis*	18 14.8	− 2 56	3.32	1	—	3.5	—	—	—
802	ε *Sagittarii*	18 15.9	−34 26	2.07	4	2.3	2.2	2.0	2.4	2.19
803	α *Telescopii*	18 17.7	−46 2	3.78	1	4.1	3.5	3.8	3.8	3.80
804	λ *Sagittarii*	18 20.2	−25 29	2.91	2	2.9	2.7	—	—	—
805	*Ll. 34356*	18 28.1	−11 4	5.17	1	—	5.7	—	—	—
806	ζ *Pavonis*	18 28.4	−71 32	3.98	1	4.1	4.2	4.2	4.8	—
807	α *Scuti*	18 28.4	− 8 20	3.95	1	—	3.6	—	—	—
808	α *Lyrae*	18 32.7	+38 40	0.92	2	—	—	—	—	—
809	φ *Sagittarii*	18 37.8	−27 7	3.48	3	3.5	3.7	—	—	—
810	β *Scuti*	18 40.5	− 4 53	4.39	1	—	4.5	—	—	—
811	σ *Sagittarii*	18 47.5	−26 27	2.31	5	2.3	2.4	—	—	—
812	ξ^1 *Sagittarii*	18 49.9	−20 49	5.71	1	—	5.7	—	—	—
813	θ *Serpentis*	18 50.0	+ 4 3	3.78	1	—	4.2	—	—	—
814	ξ^2 *Sagittarii*	18 50.3	−21 16	3.67	2	3.8	3.5	—	—	—
815	ε *Coronae Aust.*	18 50.3	−37 16	5.69	1	5.9	5.5	5.3	—	—

TABLE III. *Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
816	*η Scuti*	18 50.4	− 6 0	4.78	1	—	5.4	—	—	—
817	*ζ Coronae Aust.*	18 54.3	−42 16	4.55	1	5.9	5.2	5.1	5.4	—
818	*ζ Sagittarii*	18 54.6	−30 3	2.77	3	2.9	3.1	2.7	—	—
819	*ι Aquilae*	18 55.0	− 5 55	4.50	1	—	3.8	—	—	—
820	*ο Sagittarii*	18 57.2	−21 55	3.97	2	4.1	3.8	—	—	—
821	*γ Coronae Aust.* ..	18 58.0	−37 14	4.11	1	5.0	4.6	4.5	5.0	—
822	*τ Sagittarii*	18 59.1	−27 51	3.63	2	3.8	3.6	—	—	—
823	*ζ Aquilae*	18 59.6	+13 41	3.24	1	—	—	—	—	—
824	*λ Aquilae*	18 59.6	− 5 4	3.40	1	—	3.3	—	—	—
825	*δ Coronae Aust.* ..	18 59.6	−40 41	4.45	1	5.4	5.0	4.7	5.1	—
826	*α Coronae Aust.* ..	19 1.0	−38 6	4.11	1	4.5	4.2	4.4	4.9	—
827	*β Coronae Aust.* ..	19 1.4	−39 32	4.11	1	4.5	4.1	4.2	4.8	—
828	*π Sagittarii*	19 2.3	21 13	3.09	3	3.1	3.1	—	—	—
829	*d Sagittarii*	19 10.3	−19 10	5.08	1	—	5.6	—	—	—
830	*β¹ Sagittarii*	19 13.7	−44 42	3.93	1	4.5	3.8	4.3	4.1	—
831	*β² Sagittarii*	19 14.2	−45 2	4.67	1	5.0	4.4	4.6	4.3	—
832	*ρ¹ Sagittarii*	19 14.4	−18 5	4.43	2	—	4.2	—	—	—
833	*υ Sagittarii*	19 14.6	−16 11	4.94	1	—	4.9	—	—	—
834	*α Sagittarii*	19 15.2	−40 51	4.02	1	4.5	4.0	4.3	4.4	—
835	*δ Aquilae*	19 19.2	+ 2 52	3.48	1	—	3.4	—	—	—
836	*γ Aquilae*	19 40.3	+10 18	2.86	3	—	—	—	—	—
837	*Lac. 8239*	19 43.3	−40 11	5.26	1	5.9	5.6	5.8	6.0	—
838	*α Aquilae*	19 44.7	+ 8 32	1.33	4	—	1.1	—	—	—
839	*ε Pavonis*	19 46.1	−73 14	3.82	1	3.5	4.0	4.2	4.7	—
840	*η Aquilae*	19 46.1	+ 0 41	4.01	1	—	var.	—	—	var.
841	*ι Sagittarii*	19 46.6	42 12	4.33	1	4.5	4.3	4.3	4.6	4.41
842	*ω Sagittarii*	19 48.2	−26 38	4.94	1	5.4	5.1	—	—	—
843	*β Aquilae*	19 49.2	+ 6 6	3.96	2	—	3.9	—	—	—
844	*b Sagittarii*	19 49.3	−27 30	4.94	1	5.4	4.6	—	—	—
845	*θ¹ Sagittarii*	19 51.6	−35 37	4.80	1	5.4	4.5	5.0	5.0	4.94
846	*c Sagittarii*	19 55.0	−28 0	4.94	1	5.4	4.7	—	—	—
847	*Lac. 8310*	19 55.2	−38 17	5.46	1	5.9	5.0	5.0	5.7	—
848	*δ Pavonis*	19 56.4	−66 30	3.62	1	3.5	3.5	3.6	3.4	3.52
849	*θ Aquilae*	20 4.8	− 1 11	3.40	1	—	3.0	—	—	—
850	*α¹ Capricorni*	20 10.7	−12 54	4.21	1	—	4.5	—	—	—
851	*α² Capricorni*	20 11.1	−12 56	3.53	1	—	3.6	—	—	—
852	*β Capricorni*	20 14.0	−15 10	3.10	1	—	3.1	—	—	—
853	*α Pavonis*	20 15.7	−57 8	2.13	7	2.0	2.1	2.1	2.3	2.13
854	*α Indi*	20 28.8	−47 43	3.22	4	2.9	3.1	3.2	3.3	3.14
855	*β Pavonis*	20 33.7	−66 39	3.58	2	2.9	3.3	3.6	3.3	—
856	*α Cygni*	20 37.2	+44 50	1.84	1	—	—	—	—	—
857	*β Indi*	20 45.0	−58 55	3.78	1	4.1	3.7	3.8	3.7	3.82
858	*γ Pavonis*	21 16.1	−65 56	3.63	2	4.1	4.5	4.4	4.8	var.?
859	*β Aquarii*	21 25.0	− 6 7	2.72	1	—	2.6	—	—	—
860	*ν Octantis*	21 27.5	−77 55	3.72	1	4.1	3.8	3.8	4.0	—
861	*γ Capricorni*	21 33.2	−17 13	3.58	1	—	3.7	—	—	—
862	*ι Piscis Aust.*	21 37.5	−33 36	4.43	1	5.0	4.4	4.7	4.6	—
863	*ε Pegasi*	21 38.0	+ 9 18	2.47	2	—	2.3	—	—	—
864	*δ Capricorni*	21 40.1	−16 41	2.96	1	—	2.8	—	—	—
865	*γ Gruis*	21 46.3	−37 57	3.06	7	3.1	3.0	3.2	3.3	3.13
866	*ο Aquarii*	21 56.8	− 2 46	5.00	1	—	4.9	—	—	—
867	*λ Gruis*	21 58.6	−40 9	4.56	1	5.0	4.7	4.8	4.6	—
868	*α Aquarii*	21 59.4	− 0 56	2.80	1	—	2.7	—	—	—

TABLE III—*Continued.*

No.	Name	R.A.	Dec.	h.	n.	B.	U.A.	S.M.P.	W.	Mean
		h m								
869	*ι Aquarii*	21 59.7	−14 29	4.29	1	—	4.4	—	—	—
870	*α Gruis*	22 0.3	−47 34	1.70	7	1.7	1.9	2.0	2.0	—
871	*μ Piscis Aust.*	22 1.1	−33 36	4.79	1	4.5	4.7	4.9	4.6	4.70
872	*υ Piscis Aust.*	22 1.1	−34 39	5.41	1	5.0	5.4	5.3	—	—
873	*μ¹ Gruis*	22 8.1	−41 58	5.02	1	5.4	5.0	5.2	5.0	5.12
874	*α Toucani*	22 9.9	−60 53	2.98	6	2.9	2.8	2.9	2.5	2.82
875	*θ Aquarii*	22 10.2	− 8 24	4.10	1	—	4.3	—	—	—
876	*ρ Aquarii*	22 13.6	− 8 27	5.34	1	—	5.6	—	—	—
877	*γ Aquarii*	22 15.2	− 2 1	3.78	1	—	3.9	—	—	—
878	*δ Toucani*	22 18.4	−65 36	4.10	2	4.5	4.8	5.0	—	—
879	*π Aquarii*	22 18.9	+ 0 45	4.48	1	—	4.9	—	—	—
880	*δ¹ Gruis*	22 21.8	−44 8	4.16	1	4.1	4.2	4.2	4.1	4.15
881	*δ² Gruis*	22 22.3	−44 23	4.21	1	4.1	4.4	4.5	4.1	4.26
882	*ζ Aquarii*	22 22.4	− 0 40	3.69	1	—	3.5	—	—	—
883	*σ Aquarii*	22 24.0	−11 19	5.28	1	—	5.1	—	—	—
884	*β Piscis Aust.*	22 24.4	−32 59	4.38	1	4.1	4.4	4.7	4.4	4.40
885	*ν Toucani*	22 24.5	−62 37	5.65	1	5.9	5.5	5.1	—	—
886	*η Aquarii*	22 28.9	− 0 46	3.93	1	—	4.1	—	—	—
887	*61 Aquarii*	22 29.1	−18 6	5.21	1	—	6.7	—	—	—
888	*β Octantis*	22 33.1	−82 2	3.93	1	5.0	4.4	4.6	4.5	—
889	*ε Piscis Aust.*	22 33.7	−27 42	4.33	1	4.1	4.3	—	—	—
890	*β Gruis*	22 35.2	−47 32	2.11	8	2.3	2.2	2.2	2.4	2.24
891	*19 Piscis Aust.* . . .	22 35.4	−30 1	5.13	1	—	6.5	6.7	6.6	—
892	*ρ Gruis*	22 36.2	−42 4	5.12	1	5.4	5.2	5.2	5.3	5.24
893	*η Gruis*.	22 37.9	−54 9	4.61	2	4.5	5.1	5.2	5.1	—
894	*ε Gruis*	22 41.0	−51 58	3.45	5	4.1	3.5	3.8	3.4	—
895	*τ Aquarii*	22 43.0	−14 15	4.06	1	—	4.2	—	—	—
896	*γ Piscis Aust.*	22 45.6	−33 32	4.66	1	4.5	4.6	4.9	4.6	4.65
897	*λ Aquarii*.	22 46.1	− 8 15	3.88	1	—	3.6	—	—	—
898	*δ Aquarii*	22 48.0	−16 29	3.13	2	—	3.2	—	—	—
899	*δ Piscis Aust.*	22 49.0	−33 12	4.73	1	4.5	4.4	4.3	4.4	—
900	*α Piscis Aust.*	22 50.7	−30 17	1.49	7	1.3	1.4	1.5	1.8	—
901	*ζ Gruis*	22 53.5	−53 25	4.07	1	4.1	4.0	4.3	4.3	4.15
902	*π Piscis Aust.*	22 56.6	−35 26	5.14	1	5.4	5.3	5.4	5 5	—
903	*β Pegasi*	22 57.7	+27 24	2.62	1	—	—	—	—	—
904	*α Pegasi*	22 58.5	+14 32	2.54	1	—	—	—	—	—
905	*θ Gruis*	22 59.8	−44 12	4.12	1	4.5	4.2	4.6	4.4	—
906	*c¹ Aquarii*	23 0.0	−24 25	4.93	1	4.5	4.4	—	—	—
907	*c² Aquarii*	23 2.8	−21 51	3.71	2	3.8	3.7	—	—	—
908	*c³ Aquarii*	23 3.2	−23 8	4.86	1	5.4	4.9	—	—	—
909	*ι Gruis*	23 3.3	−45 55	3.61	2	4.1	3.9	4.2	4.2	—
910	*γ Toucani*	23 10.1	−58 55	3.89	3	4.1	4.0	4.1	—	—
911	*b¹ Aquarii*	23 16.4	−20 47	4.18	1	4.1	3.9	—	—	—
912	*b² Aquarii*	23 19.5	−21 20	4.42	1	4.5	4.4	—	—	—
913	*b³ Aquarii*	23 26.7	−21 36	4.60	1	4.5	4.5	—	—	—
914	*ι Phoenicis*	23 28.3	−43 18	4.47	1	4.5	4.4	[5.2]	4.7	—
915	*Lac. 9543*.	23 32.7	−47 20	4.52	1	6.4	6.3	6.7	—	var ?
916	*δ Sculptoris*	23 42.4	−28 49	4.34	1	4.1	4.6	—	—	—
917	*η Toucani*	23 51.0	−64 59	4.82	2	5.4	5.3	5.5	4.7	var ?
918	*ε Toucani*.	23 53.4	−66 16	4.48	3	5.0	4.3	4.9	4.0	—
919	*ζ Sculptoris*	23 55.9	−30 25	4.62	1	4.5	5.2	5.3	5.0	—

HONGKONG OBSERVATORY,
January 15, 1900.

ON THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF THE BLACK BODY AT HIGH TEMPERATURES.[1]

By F. Paschen.

I was able to show in a previous communication[2] that the law of radiation of the black body proposed by W. Wien is the more completely confirmed by observations, within the range of temperature from 100° to 450° C., according as the arrangement of the experiment is more closely adapted to the theoretical premises. This law has also received an important support from the theoretical researches of M. Planck.[3] He derives the law from his electro-magnetic theory of radiation, and thereby introduces only one additional hypothesis, that for the definition of Entropy, which, although not regarded by him as fully established, is, however, probably more reliable than the assumptions which led Wien to the same results based on the kinetic theory of gases. According to Planck's presentation also, the law appears to be a rigorously valid law of nature, and its contents possess a general significance.

The problem remaining for experimental investigation is, first, to investigate the limits within which the law, and hence Planck's assumptions, are found to hold good; and second, to determine the constants of the law as accurately as possible.

From the results of the investigation of the region of lower temperatures I expected to find the law to be valid within very small limits of error at higher temperatures. My experiments showed, however, that many kinds of experimental difficulties arise here.

[1] *Sitzungsberichte der Berliner Akademie.* Session of the Physical-Mathematical Section on December 7, 1899.

[2] This Journal, **10**, 40, 1899.

[3] "Ueber irreversible Strahlungsvorgänge." *Sitzungsberichte der Berliner Akademie*, Session of May 18, 1899, p. 440.

While at low temperatures the production of radiation by the black body appears quite simple, and only a few special adaptations of the bolometer are necessary in order that the radiation of the longer wave-length shall be sufficiently absorbed, it was comparatively difficult for me to realize the radiation of the black body so perfectly at higher temperatures that its energy curve followed the law within the limits of the errors of observation.

I attempted to produce the radiation first by cavities with heated walls, and secondly, by causing the total radiation from a small incandescent surface to be repeatedly reflected by mirrors upon the surface itself. In the first arrangement, which seems to be the better, it is extremely difficult to heat with sufficient uniformity the walls of the cavity provided with an aperture. The greater the cavity and its aperture, and the higher the temperature, so much greater will be the difference in temperature at different points of the walls. Not until these differences were reduced to a small amount did the emergent radiation exhibit the energy curve of Wien's law with a sufficient degree of approximation.

With the second arrangement,—that of producing the radiation of the black body by reflection,—it is, indeed, possible to heat a small opaque surface uniformly, but the difficulty is to make the reflection somewhere nearly complete. Even if we can only correct by this the deviation[1] of the radiation of the surface from that of the black body at the same temperature,—a difference which can be reduced to a small amount by the use of a suitable surface for the incandescent layer,—I should nevertheless have been obliged to use an expensive reflecting hemisphere. The form of the image outside of the center of a reflecting sphere introduces further an irregularity, varying with the temperature, in the incandescence of a surface which is not very small. Finally the accurate measurement of a high temperature turns out be very difficult with this arrangement.

On gradually improving the arrangement of the experiment, according to both methods, I have found that each improvement

[1] *Wied. Ann.*, 60, 719, 1897.

reduces the amount of the departure of the results of the observations from the demands of theory, until finally results were obtained by both methods in complete agreement among themselves and also apparently in sufficient agreement with the results obtained at lower temperatures. The slight isolated deviations from the law found in the results now to be communicated are for the most part the relics of larger departures, and therefore only prove that certain defects in the arrangement were not sufficiently overcome. Should we desire to carry the accuracy of the measurements still further, however, it would be necessary to make a decided improvement in the arrangement, now only just sufficient, in order to establish the validity of the law within still smaller limits of error.

As it would lead us too far aside, if I were to detail all the researches which I have made in order to surmount the difficulties of a purely practical nature, I shall limit myself to the communication of the results obtained with the most suitable arrangements employed for the purpose.

The general arrangement of the spectroscopic apparatus remained the same as for the investigation at lower temperatures. In all the measures at high temperatures the exposed bolometer filament consisted of a strip of platinum 4 mm long, 0.3 mm wide, and $\frac{1}{2000}$ mm thick, which was covered with a thick layer of platinum black, and was placed at the center of the reflecting hemisphere previously described. The current in the bolometer was always maintained at the same strength, and the sensitiveness of the apparatus to radiation was varied by the introduction of resistances in the circuit of the galvonometer.

The temperature was always measured with a thermo-element of wires of platinum and platinum-rhodium of 0.15 mm diameter, which Mr. Holborn had kindly calibrated for me in 1898. All temperatures above 450° C. correspond to this calibration, but lower temperatures correspond to the scale of two mercury thermometers which had been certified by the *Reichsanstalt*. The thermo-electromotive forces were measured by compensation.

In measuring up an energy spectrum at those places where the energy rapidly changed (at small wave-lengths) I proceeded by steps of the apparent breadth of the bolometer strip ($3'$), in order to be able to accurately apply the correction for the impurity of the spectrum caused by the width of the bolometer strip and the equally wide slit. In the remaining part of the spectrum I selected a few places as free as possible from absorption and uniformly distributed. Along with the energy curve thus more completely observed I made check measurements at the same temperatures, determining the throw of the galvanometer at only eight or ten places, uniformly distributed in the spectrum. These served as a check on the more complete energy curves and for the most part they are not given in the tables.

The observations were at first compared with the theory in the same way as in my paper on lower temperatures. For this purpose the observed values of the wave-length λ_m and of the intensity J_m of the maximum of the energy curve were given in tables, and the relations obtained for these constants were tested, *viz.*:

$$\text{I.} \qquad \lambda_m \,.\, T = \text{constant.}$$

$$\text{II.} \qquad J_m / T^5 = \text{constant.}$$ [1]

In case the observed energy-curve differs appreciably from the theoretical requirement, *viz*:

$$\text{III.} \qquad J/J_m = \left\{ \frac{\lambda_m}{\lambda} e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^5,$$

this is indicated in a remark. In order to show how far the observed energy-curve follows this law, the observed points of

[1] In the determination of the values λ_m and J_m from an energy curve, all of the observed points of the curve are always employed, so that, $\lambda_m \,.\, T =$ constant at the same time expresses the fact that each wave-length is so displaced that the relationship $\lambda \,.\, T =$ constant is also fulfilled. In case relation II holds, the intensities of the wave-lengths therefore corresponding with each other are further proportional to T^5 (Wien's general law). The energy curves at different temperatures, expressed in logarithmic measure (with $\log J$ for ordinates and $\log \lambda$ for abscissas) are accordingly congruent, or J/J_m is the same function of λ/λ_m at all temperatures. This relation, together with I and II above, is identical with the relations as formulated by Wien.

different energy-curves are represented in a figure, along with the theoretical curve III. If the three conditions I, II, and III are fulfilled, Wien's general formula

$$\text{IV.} \qquad J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

is established. A more rigorous mode of investigating the law will be given in my conclusion.

I. MEASUREMENTS ON CAVITIES WITH HEATED WALLS.

A cavity of unglazed porcelain was made in the manner described by Lummer and Kurlbaum.[1] A porcelain cylinder of 5 cm diameter and 10 cm length having six cross walls was made in one piece. The length of the central radiating cavity was 5 cm, and it had two chambers both before and behind it. The apertures of the three walls in front had an area 0.6 sq. cm. A cylinder of platinum foil was wrapped around the porcelain cylinder, and was heated by an electric current. The radiating body was surrounded by an air space and by an asbestos covering. The losses of heat by conduction, chiefly due to the apertures, cause a lack of uniformity in the temperature of the walls of the cavity. The wall of the cylinder is hotter than the rear wall, and this again is hotter than the front wall, which is especially cooled off in the immediate neighborhood of the aperture. Without a protecting cover, the upper half of the radiating body is hotter than the lower half, in consequence of the rising warm air, and those parts of the platinum foil which are in contact with the porcelain differ from those which are not in contact. These irregularities increase with rising temperature, and further depend upon the external protection of the radiating body and upon the diaphragms placed in front of the aperture.

The thermo-element was introduced through the rear cross walls, and measured the temperature of the radiating part of the rear wall. As soon as the incandescent body has reached a stationary condition, the thermo-element almost completely vanishes in the background, even if very considerable difference in

[1] *Ber. der Phys. Gesellschaft zu Berlin*, 17, 106, 1898.

the temperature occur in the walls,—which may probably be explained by the near equality in the reflecting power for visible rays of the metal wires and of the porcelain.

In the three following series of measures the cavity was differently protected, and the light coming from the aperture was differently diaphragmed. For a given series the arrangement was kept as nearly as possible the same.

LARGE PORCELAIN CAVITY.

TABLE I

Temperature		$\lambda_m\ (\mu)$	$\lambda_m T$	J_m	$J_m T^5 \times 10^{15}$	
C.	Abs.					
413°.7	686°.5	4.222	2899	0.5420	3.553	
443.2	716.2	4.057	2905	0.6740	3.578	
643.8	916.8	3.199	2933	2.320	3.582	
693.0	966.0	3.044	2941	3.041	5.612	
907.3	1180.3	2.492	2941	8.464	3.695	Ch.[1]
933.0	1206.0	2.437	2940	8.560	3.764	
1048.1	1321.1	2.233	2950	15.03	3.737	Ch.
1053.4	1326.4	2.219	2945	15.65	3.807	
			Mean 2932	m. e. = 5		

TABLE II

411°.6	684°.6	4.235	2899	0.5114	3.398	
692.8	965.8	3.033	2928	2.855	3.397	
865.5	1138.5	2.556	2911	6.483	3.383	At the longest wave-lengths higher than the theoretical curve.
871.2	1144.2	2.575	2946	6.635	3.389	
958.2	1231.2	2.381	2931	9.528	3.370	Ch. Fits the theoretical curve poorly.
			Mean 2923	m. e. = 8		

TABLE III

374°.1	647°.2	4.508	2917	0.5419	4.774	Ch.
377.3	650.3	4.446	2889	0.5543	4.767	
654.0	927.0	3.155	2925	3.265	4.768	Ch.
664.6	937.6	3.113	2919	3.513	4.850	
845.2	1118.2	2.610	2918	8.534	4.888	
1043.4	1316.4	2.218	2921	18.79	4.741	Inaccurate on account of unsteady temperature.
			Mean 2915	m. e. = 5		

[1] Ch. denotes a check measure as distinguished from the more complete measures.

The third of this series may probably be regarded as the best in respect to arrangement. The differences in the values of $\lambda_m \cdot T$ and J_m / T^5 are larger in series I than would be expected from the residuals.

In order to obtain a more uniform temperature of the cavity, I selected the arrangement sketched in Fig. 1 *a* and *b*. A metallic crucible *T* (of copper or platinum), with thick walls,

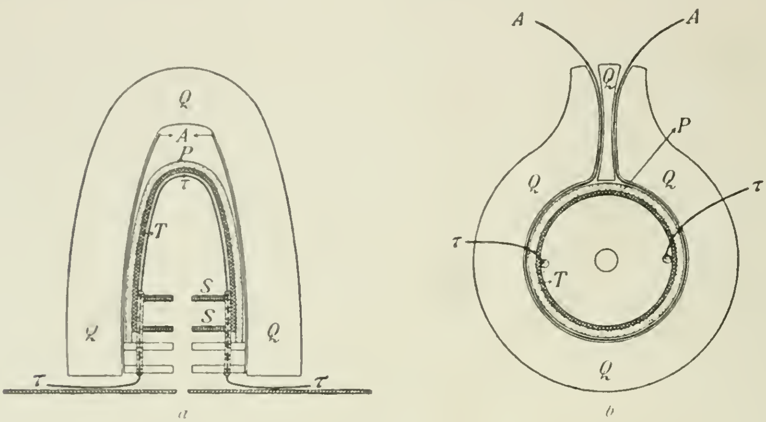


FIG. 1.

was separated into two chambers by two round separating walls *S*, of the same metal, with small round apertures. The rear chamber was the radiating cavity. A porcelain crucible *P*, with very thin walls, surrounded the metallic crucible. For the electric heating of the space I wrapped about this, as may be seen from the drawing, the platinum foil *A* which projected out from in front and from behind. *Q* was an asbestos cover. Two or three cross walls of asbestos were also attached. The length of the cavity amounted to about 3.5 cm, with an extreme breadth of 3 cm. The round apertures in *S* had a diameter of about 5 mm. As a result of the good conductivity in the metallic wall, and of the small dimensions of the cavity, a pretty uniform temperature was obtained in the interior of the cavity. The thermo-element *T* was introduced from in front through a thin small tube of porcelain. When the inner surface of the wall

was composed of metallic oxides (of copper or iron) the thermo-element did not disappear in the background unless the temperature of the cavity was practically uniform throughout, since the reflection of the visible rays was very different from the platinum wires and from the metallic oxides. It appeared dark on a bright ground when the metallic walls S were cooler than the back part of the crucible, and bright on a dark ground when they were warmer. The temperature distribution in the cavity could, however, be altered according to the mode of the external wrapping with asbestos, and it was possible to regulate this in such a way that the thermo-element almost entirely disappeared.[1]

If the interior metal wall was surrounded with porcelain, however, this appearance ceased to give a sensitive mode of judging of the inequalities of temperature in the cavity.

First arrangement. The metal crucible was of copper, which oxidized on incandescence in the air, and was almost entirely converted into oxide at the close of the experiment.

TABLE IV

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m\, T^5 \times 10^{15}$	
560°.0	833°.0	3.482	2900	1.583	3.946	
885.3	1158.3	2.525	2925	8.410	4.035	
501°.2	774°.2	3.748	2901	0.8532	3.069	Apertures smaller.
685.2	958.2	3.067	2939	2.464	3.050	Apertures smaller.
878.7	1171.7	2.502	2931	6.669	3.021	Apertures smaller.
			Mean 2919	m. e. = 8		

Second arrangement. The metal crucible was of platinum. The inner surface of the cavity was covered with a thin wall of porcelain. (A second porcelain crucible was fitted into the platinum crucible. The front cross wall of the cavity consisted of porcelain on the inside and platinum on the outside.)

[1] The wires could no longer be seen by an observer who did not know their location.

TABLE V

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m\ T^5 \times 10^{15}$	
411.1	684.1	4.212	2882	0.2015	1.344	Ch.
731.6	1004.6	2.896	2910	1.380	1.349	
732.0	1005.0	2.891	2904	1.394	1.360	Ch.
733.7	1006.7	2.884	2903	1.401	1.355	Ch.
1029.1	1302.1	2.233	2908	5.286	1.413	Ch.
1035.7	1308.7	2.223	2910	5.237	1.363	
1044.3	1317.3	2.217	2920	5.534	1.396	Ch.
1283.9	1556.9	1.887	2939	12.80	1.399	
			Mean 2910	m. e. = 6		

Third arrangement. The platinum crucible of the previous arrangement was covered with iron oxide after the removal of the porcelain wall.

TABLE VI a

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m\ T^5 \times 10^{15}$	
743.0	1016.0	2.858	2899	1.514	1.398	
1060.5	1333.5	2.188	2917	5.984	1.419	
467.6	740.6	3.864	2862	0.3221	1.446	
500.6	773.6	3.713	2873	0.4042	1.458	Ch.
687.4	960.4	3.010	2891	1.208	1.478	
731.4	1004.4	2.865	2878	1.507	1.474	Ch.
951.4	1224.4	2.367	2898	4.134	1.503	
961.2	1234.2	2.357	2908	4.331	1.508	Ch.
979.5	1252.5	2.310	2893	4.578	1.484	
982.0	1255.0	2.312	2902	4.639	1.489	Ch.
1294.3	1567.3	1.849	2898	14.39	1.523	At long wave-lengths deviations from the theoretical curve occur.
423.2	696.2	4.153	2891	0.4088	2.501	
737.5	1010.5	2.861	2891	2.795	2.655	
762.5	1035.5	2.791	2890	3.235	2.719	
791.6	1064.6	2.721	2897	3.632	2.655	The observed points lie above the theoretical curve at both ends of the curve.
1008.4	1281.4	2.271	2909	9.438	2.734	
1221.	1494.	1.967	2941	20.08	2.690	The curve is too high at the longer wave-lengths.
			Mean 2896	m. e. = 4		

The platinum crucible was freshly covered with iron oxide; aperture very small and incandescence very uniform.

TABLE VI b

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m T^{-5} \times 10^{15}$	
C.	Abs.					
701°.4	974°.4	2.960	2886	1.726	1.963	Ch.
706.3	979.3	2.951	2890	1.767	1.961	
992.7	1265.7	2.286	2893	6.557	2.011	
995.6	1268.6	2.282	2895	6.647	2.023	Ch.
1019.7	1292.7	2.246	2905	6.920	1.915	Ch.
1259.4	1532.4	1.907	2921	16.62	1.966	Ch.
1260.3	1533.3	1.906	2921	16.79	1.981	
			Mean 2901	m. e. = 5		

II. MEASUREMENTS OF THE RADIATION OF THE BLACK BODY BY THE METHOD OF REFLECTION.

The following arrangement of the incandescent body was chosen for the experiments now to be described (Fig. 2, *a*, *b*, *c*). The radiating sheet of platinum *P*, of 0.05 mm thickness, and of about 30 mm length and 16 mm width, and two shorter sheets of platinum *S*, insulated by mica *G*, are wrapped about a strip of platinum foil or platinum-iridium foil *A*, which heats *P* and *S*, being itself heated by an electric current. The thermo-element *T* is insulated by mica from the sheet *S* and from the parts of *P* furthest from the middle, and touches *P* with the junction from the inside, that is, is attached with the junction at the middle of *P*.[1]

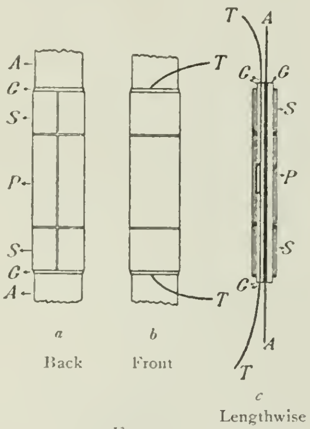


FIG. 2.

The piece *P* becomes uniformly incandescent as soon as a high temperature has been once suddenly produced, at which the water present in the mica quickly vaporizes and thereby

[1] In some of the experiments the junction was situated in the exterior surface, so that the conducting wires, insulated by a coating of iron oxide, passed through *P*; or the ends of the thermo-element which were not soldered together were allowed to touch *P* from within. The indications of temperature up to 800° C. were almost the same in all these arrangements.

uniformly inflates the strip of platinum P. The front surface of P was so adjusted in the central plane of a reflecting hemisphere that it was covered as closely as possible by its reflected image. The reflecting surface had a diameter of 15 cm and consisted of German silver; it had a good spherical figure and was admirably polished. It had a small aperture toward the incandescent surface P through which the radiation fell on the slit. The dimensions of the incandescent surface P are larger than was necessary to fill the diaphragm in front of the prism of the spectroscopic apparatus, on account of the imperfections of the image. The radiation from a portion of P only about 10 mm $\times$ 20 mm reaches the bolometer. In so far as the radiation of this smaller surface is reflected on neighboring parts of P and not on the surface itself, in consequence of the imperfections of the image, the radiation of these neighboring portions does, nevertheless, fall back upon the central portion. With a uniform incandescence of P this arrangement is accordingly equivalent to that in which the small surface only is present, and receives its own radiation completely returned upon itself.

First arrangements (for lower temperatures). The radiating sheet of metal is covered with platinum black. A bolometer strip 6′ wide is employed in the reflecting hemisphere. (See my paper on lower temperatures).

TABLE VII

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{14}$	
C.	Abs.					
99°.9	372°.9	7.730	2883	0.2737	3.797	
186.9	459.9	6.251	2875	0.7805	3.793	
190.7	463.7	6.211	2880	0.8136	3.797	
192.1	465.1	6.194	2883	0.8248	3.788	Larger deviations [1]
313.2	586.2	4.926	2887	2.620	3.787	
316.7	589.7	4.899	2890	2.683	3.761	
318.8	591.8	4.881	2889	2.730	3.760	
453.6	726.6	3.985	2896	7.642	3.773	
599.4	872.4	3.327	2902	18.93	3.747	
603.2	876.2	3.312	2901	19.56	3.787	Higher at the ends than the theoretical curve [1]
618.3	891.3	3.252	2899	21.30	3.760	
624.1	897.1	3.232	2900	21.93	3.773	Somewhat too high at the ends [1]
			Mean 2890	m. e. — 3		

[1] At the higher temperatures the sheet is somewhat irregular in its incandescence.

Second arrangement. The bolometer strip of 6′ breadth is replaced by one of 3′. The radiating body is the same as in the first arrangement.

TABLE VIIIa

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{15}$
C.	Abs.				
387°.0	660°.0	4.380	2891	0.5923	4.742
400.8	673.8	4.290	2891	0.6591	4.742
418.7	691.7	4.178	2890	0.7498	4.756
419.1	692.1	4.176	2891	0.7647	4.780
549.1	822.1	3.531	2904	1.783	4.743
557.3	830.3	3.495	2912	1.875	4.753
574.9	847.9	3.424	2903	2.095	4.779
633.5	906.5	3.214	2913	2.926	4.781
			Mean 2899	m. e. = 3.5	

Third arrangement. The platinum strip is covered with oxide of copper.

TABLE VIIIb

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{15}$
C.	Abs.				
560°.5	833°.5	3.488	2907	1.907	4.737
643.1	916.1	3.174	2908	3.095	4.797
656.9	929.9	3.126	2907	3.331	4.792
737.8	1010.8	2.877	2908	5.089	4.827
848.	1121.	2.596	2908	8.494	4.799
861.	1134.	2.559	2901	8.874	4.732
			Mean 2906	m. e. = 1	

The following measurements were obtained with a somewhat different adjustment of the radiating body and of the bolometer, and with a sheet of platinum P which was not uniformly incandescent in the reflecting sphere:

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{15}$	
C.	Abs.					
387°.2	660°.2	4.367	2883	0.6084	4.856	
646.6	919.6	3.159	2905	3.196	4.859	Observed curve is too high at the ends.
868.2	1141.2	2.565	2926	9.454	4.882	

A strip covered with iron oxide and uniformly incandescent gave:

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m T^5 \times 10^{15}$
750.5	1020.5	2.817	2900	5.655	4.891

It is very difficult to obtain a uniform incandescence of a larger surface at the higher temperatures with the arrangement described, if an image of the surface itself is reflected upon itself by the hemisphere. The central parts of the surface receive a greater amount of reflected radiation, and are better protected from loss of heat than the edges. I have obtained a fairly uniform condition of incandescence in the reflecting envelope with a surface 10 mm wide of platinum 0.1 mm thick, when the back side of the strip was protected from loss of heat by a cover of mica or asbestos. Outside of the center of the reflecting sphere the strip is then less incandescent than at the edge, and if its image is projected upon itself it will be uniformly incandescent at one definite temperature only. The strip to which the following results refer was most uniformly incandescent at about 1000° C. With this arrangement I could no longer obtain a reliable determination of the temperature with the thermo-element, since the mica insulation of the wires is insufficient at 1000°. For one set of energy curves I have determined the temperatures which seem to satisfy most closely laws I and II. The curves had very nearly the shape demanded by theory.

Fourth arrangement. A surface of 10×25 mm, blackened with iron oxide; the diaphragm in front of the prism smaller.

TABLE IX

Abs. Temp. assumed	λ_m (obs.)	$\lambda_m \cdot T$	J_m (obs.)	$J_m T^5 \times 10^{15}$	
834.9	3.468	2895	1.134	2.796	
1014.	2.862	2900	3.015	2.821	
1021.5	2.835	2896	3.129	2.815	
1273.	2.280	2903	9.478	2.833	The last end of the curve at long wave-lengths is a little too high.
1496.	1.945	2910	21.47	2.866	

It should be further mentioned that measurements, showing tolerable agreement with the experiments described and with the law, were made with a reflecting hemisphere of bronze, and with a different arrangement of the incandescent strip of platinum. As the arrangement was, however, defective in many respects,

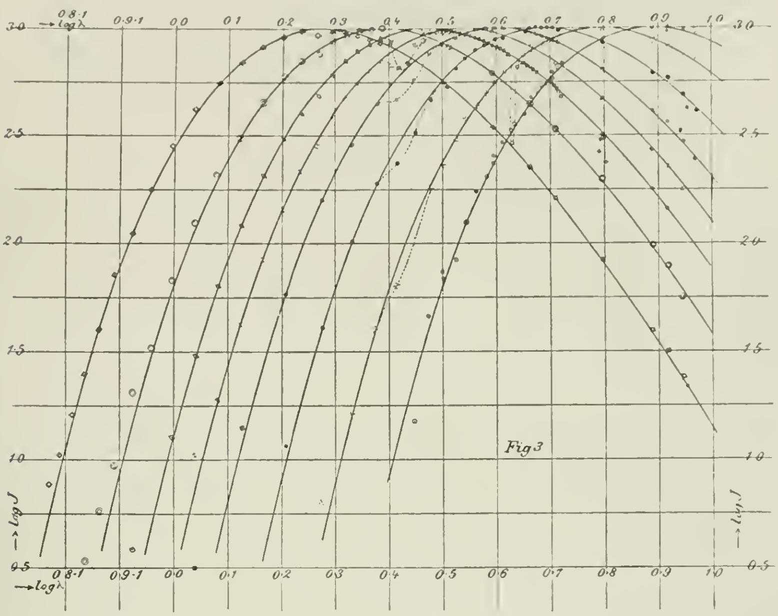


FIG. 3.

these measures need not be taken into consideration in comparison with those described.

It follows from the preceding results that the relations I and II have been found to hold good with a pretty close degree of approximation with most of the arrangements described. Somewhat larger deviations do indeed occur in a few of the series.

The observed points of a series of energy curves are represented with log J as ordinate and log λ as abscissa, in Fig. 3, and their symbols are given in the following list. For convenience of inspection log J_m is made the same in all the curves.

LIST OF THE ENERGY CURVES REPRESENTED IN FIG. 3.

Temp. C	λ_m	log λ_m	Table	Symbol
99.9	7.730	0.8882	VII	⊙ ⊙ ⊙
190.7	6.211	0.7932	VII	× × ×
318.8	4.881	0.6885	VII	• • •
453.6	3.985	0.6004	VII	◈ ⊡ ⊡
574.9	3.424	0.5345	VIIIa	⊟ |•| ⊟
700.3	2.951	0.4700	VI	⦶ ⦶ ⦶
933.0	2.437	0.3869	I	◎ ◎ ◎
1283.9	1.887	0.2758	V	⌖ ⌖ ⌖

These curves teach first the validity of the proposition that all energy curves represented logarithmically are congruent, and second, that law III holds good, as the observed points very closely follow its curve indicated by the dotted line. Consequently law IV appears to be confirmed in all its parts by the observations. A more rigorous test of the validity of the law can be obtained as follows:

If we write law IV in the form

$$\text{IVa.} \qquad \log (J \cdot \lambda^5) = \log c_1 - c_2 \cdot \log e \frac{1}{\lambda \cdot T},$$

the test of law IV may be reduced to that of a straight line, if we lay off $\log (J \cdot \lambda^5)$ and consider it as a function of $\frac{1}{\lambda T}$. The form of an energy curve $\left(\frac{1}{T} = \text{constant}\right)$ is hereby reduced to that of a straight line in the same way as in the case of an isochromatic line $\left(\frac{1}{\lambda} = \text{constant}\right)$ for which I had previously adopted this mode of representation. I gave in Table X the observed values of $\log (J \lambda^5)$ for a series of check measurements[1] which were made (1) with the reflecting hemisphere (copper oxide, Table VIIIb; or platinum black, Table VIIIa; designated in Table X by *CuO* or *Pt*); (2) with the platinum crucible cavity, which was covered with iron oxide[2] Table VI;

[1] These check measures were mostly made in connection with the measurements of the complete energy curves, the results of which are given in the proper tables. All the measures were made at exactly the same wave-lengths.

[2] All of the intensities J observed with this arrangement have been multiplied by the factor 2.421, in order that they may be comparable with the intensities with the other arrangements. The factor is the ratio of the values of J_m/T^5 in Tables VI and VIII.

TABLE X.

	Abs. T	$\lambda (\mu)$ / $\frac{1}{T}$ $\frac{1}{\lambda}$ 0.00	0.9104 1.0983	1.208 0.8278	1.589 0.6294	1.971 0.5073	2.316 0.4318	3.247[1] 0.3080	3.910 0.25574	4.588 0.21797	5.124[2] 0.19517	6.263 0.15967	7.736 0.12927	8.244 0.12131	Energy curves C_1	Energy curves C_2
H	1532.4	06526	0.618 −25	1.742 −14	2.562 −11	3.060 −15	3.373 −12	3.891 −3	4.115 +4	4.280 +15	4.374 +14	4.524 +17	4.663 +31	. . .	150300	14600
H	1292.7	07735	0.814−1 +11	1.098 −26	2.078 −14	2.669 −18	3.035 −21	3.641 −20	3.898 −18	4.089 −12	4.203 −8	4.379 −6	4.542 +8	. . .	142200	14520
H	1268.6	07883	0.718−1 +17	1.064 +18	2.050 +17	2.649 +7	3.025 +8	3.642 +10	3.904 +11	4.098 +17	4.214 +20	4.377 +6	4.542 +20	. . .	147900	14480
CuO	1134	08818	0.062−1 +10	0.561 +4	1.646 −16	2.330 −11	2.756 −5	3.449 −3	3.742 +1	3.960 +8	4.088 +15	4.267 −9	4.473 +28	4.456 +27	140900	14420
CuO	1121	08921	0.042−1 +60	0.499 −15	1.614 −8	2.304 −4	2.729 −4	3.431 0	3.727 +2	4.944 +7	4.072 +6	4.276 +10	4.470 +33	4.511 +29	144400	14510
CuO	1010.8	09893	. . .	0.986−1 −11	1.232 −3	1.992 −6	2.472 +4	3.250 +9	3.580 +12	3.820 +16	3.960 +13	4.186 +18	4.373 +15	4.417 +10	148100	14540
H	974.4	10263	. . .	0.835−1 +34	1.101 +16	1.876 −2	2.362 −6	3.158 −11	3.493 −15	3.740 −14	3.884 −16	4.119 −11	4.326 −2	. . .	148900	14540
CuO	929.9	10754	. . .	0.546−1 −1	0.894 0	1.718 −4	2.236 +2	3.078 +4	3.434 +5	3.695 +9	3.847 +7	4.093 +12	4.302 +14	4.352 +11	147600	14530
CuO	916.1	10916	. . .	0.469−1 +7	0.842 +13	1.664 −5	2.190 0	3.043 0	3.409 +6	3.671 +9	3.825 +5	4.074 +10	4.278 +4	4.323 −5	148000	14540
Pt	906.5	11031	. . .	0.407−1 +4	0.783 −1	1.626 −6	2.156 −2	3.023 +3	3.390 +6	3.655 +8	3.810 +4	4.063 +10	4.280 +15	4.347 +27	148900	14570
Pt	847.9	11794	. . .	0.011−1 +8	0.482 +2	1.386 −2	1.947 −4	2.874 +2	3.264 +3	3.545 +3	3.712 0	3.977 0	4.199 −3	4.255 −6	146100	14510
CuO	833.5	11997	. . .	0.872−2 −24	0.395 −3	1.307 −16	1.890 −5	2.830 −2	3.227 −1	3.514 0	3.683 −4	3.955 −1	4.179 −6	4.239 −7	145900	14530
Pt	830.3	12044	. . .	0.892−2 +19	0.373 −8	1.294 −14	1.873 −9	2.824 0	3.223 +2	3.510 +2	3.681 0	3.950 −1	4.186 +4	4.234 −9	147700	14560
Pt	822.1	12164	. . .	0.849−2 +38	0.336 +3	1.265 −5	1.845 −5	2.799 0	3.204 +3	3.497 +6	3.664 −2	3.936 −2	4.174 +2	4.220 −12	145400	14520
Pt	692.1	14449	. . .	. . .	0.453−1 +26	0.558 +19	1.231 +4	2.363 +6	2.836 +3	3.176 −1	3.383 −2	3.708 0	3.971 −14	4.050 −8	143200	14450
Pt	673.8	14841	. . .	. . .	0.283−1 +14	0.414 +1	1.118 −1	2.282 +2	2.771 +2	3.116 −7	3.332 −5	3.647 −22	3.943 −10	4.009 +15	142200	14460
Pt	660.0	15152	. . .	. . .	. . .	0.334 +21	1.046 +11	2.223 +3	2.717 −1	3.073 −7	3.287 −11	3.629 −8	3.908 −20	3.9[illegible] −14	142200	14450
Isochromatic curves C_1			103200	138100	141100	141200	141500	144700	148500	147700	148900	145600	154800	146300		
Isochromatic curves C_2			14100	14460	14470	14480	14500	14500	14580	14550	14580	14510	14860	14660		

Mean values

	Formula IVa	Energy curves	Isochromatic curves
C_1	146030	145870	145300
C_2	14531	14514	14520

[1] At this wave-length an absorption averaging 2 per cent. is corrected by a constant factor 1.020.

[2] All the intensities J observed with this arrangement have been multiplied by 2.421 to render them comparable with the intensities with the other arrangement. This factor is the ratio of the values $J_m/T5$ in Table VI and Table VIII.

designated in Table X by large H). The numbers below give the difference, in units of the third decimal, of the observed values, and computed according to formula IVa (O.–C.) The following values of the constants were employed in the computation :

$$c_1 = 146030$$
$$c_2 = 14531$$

The numbers of the horizontal row represent an energy curve, which calculated as such, yield the constants printed at the right. The numbers in the vertical column represent an isochromatic curve, which calculated as such yields the constants printed below. We can, of course, locate each individual series within small limits of error on the corresponding straight line in the same manner as we may all of the numbers of all of the series of the comprehensive formula. But even the differences outstanding in this case correspond for the most part to only a few per cent. in the value of $J\lambda^5$. A deviation of 4, 9, 13, 17, 21, 25, etc., units of the third decimal correspond to 1, 2, 3, 4, 5, 6, etc., per cent. in the value of $J\lambda^5$. Within these limits of error the law is in any case demonstrated by this calculation.

TABLE XI

Abs. T.	$\lambda (\mu)$ / $\frac{1}{\lambda}$; $\frac{1}{T}$ 0.00	7.738 0.12924	6.263 0.15967	4.663 0.21444	3.354 0.2982	2.279 0.4389	1.887 0.5300	Energy curves C_1	Energy curves C_2
723.0[1]	1383	4.684 +4	4.422 +6	3.939 −1	3.207 −7	1.993 −1	1.206 0	634600	14450
577.1	1733	4.394 −2	4.066 +1	3.468 −2	2.559 0	1.021 −7	0.047 +9	636700	14465
462.4	2162	4.042 −5	3.631 −3	2.892 +1	1.758 +4	0.847−1 +4		634100	14445
373.0	2681	3.627 0	3.116 +1	2.190 −3	0.783 −1			633100	14445
Isochromatic curves[2]	C_1	629600	642900	630300	623000	633700	600000[3]		
	C_2	14440	14490	14450	14430	14460	14370		

[1] The wave-lengths belonging to this temperature were: 7.740μ, 6.264, 4.664, 3 356, 2.2181, 1889μ, and are calculated with these values.

[2] The isochromatic curves are here calculated on the basis of equal weight for all observations, (as in the computation of Formula IVa). Different weights were assigned in the earlier calculations.

[3] By an oversight the value 738800 was previously given at this point.

	Formula IVa	Mean values: Energy curves	Mean values: Isochromatic curves
C_1 - -	633000	634600	631900
C_2 - - -	14450	14450	14450

To facilitate comparison with the results at the lower temperatures I have calculated the set of observed numbers[1] previously communicated in the same manner, and give them in Table XI. We see that these observations yield a still closer validity of the law, as was to be expected from the theoretically better arrangement of the experiments.

Wien's law may accordingly be regarded as demonstrated as well as the difficulties of the experiments admit, within the range of wave-length from 9.2μ to 0.7μ (and in the research carried on in conjunction with Wanner,[2] to 0.5μ), and within the range of temperature from 1300° C. to 100° C.[3]

I consider the determination of the constant c_2 of the law as the final object of my researches. For lower temperatures I determined its value from experiments on cavities as $14455\mu \times$ degree of the absolute or of the Celsius scale.

From the radiation at lower temperatures produced by reflection Table VII gives

$$c_2 = 5 \times \lambda_m \,.\, T = 5 \times 2890 = 14450 \text{ with a mean error} = 15.$$

The following value is yielded by the observations now communicated at higher temperatures. We have found as mean values of $\lambda_m \,.\, T$:

	Cavities: Table I	II	III	IV	V	VIa	VIb
$\lambda_m \,.\, T$ =	2932	2923	2915	2919	2910	2896	2901
Mean error =	4	8	5	8	6	4	5

	Reflection: Table VIIIa	VIIIb
$\lambda_m \,.\, T$ =	2899	2907
Mean error =	3.5	1

[1] Paper on lower temperatures. [2] This JOURNAL, **9**, 300, 1899.

[3] Since in the observations of lower temperatures the radiation of the space in front of the slit contributes a considerable amount with the slide let down, which amount is added to the observed intensity, the lower temperature limit is the more correct.

This mean is

	$\lambda_m . T$	$c_2 = 5 \times \lambda_m . T$
with equal weights of all values -	2911	14555
with the omission of the values which seem least reliable in Tables I and II	2907	14535

The last value is in accord with that on which the calculation in Table X is based, *viz.*, $c_2 = 14531$.

The results of individual series deviating most from the means are, omitting Tables I and II:

Table	Temp. C.	$\lambda_m . T$	$c_2 = 5 \times \lambda_m . T$
VIa -	467.6	2862	14310
VIa -	1221	2941	14705

I estimate the highest possible error of the mean value of $\lambda_m . T$ to be 16; and hence that of c_2 to be 80.

A somewhat larger value is therefore obtained from the experiments at higher temperatures than from those at lower temperatures. We shall not be disposed to attach an excessive weight to the values of the constant c_2 found at higher temperatures when we consider the great experimental difficulties at high temperatures, and in particular the rather imperfect arrangement for producing the radiation, and further the still considerable uncertainties which seem to be possible in the determination of higher temperatures, according to the experiments of Holborn, Wien, and others. In my opinion the value $c_2 = 14455$ found with lower temperatures and relatively perfect arrangements is to be regarded as the more reliable,[1] similarly to the case of the determination of the constants of the law of total radiation for which a precise arrangement at low temperatures is preferred.

[1] I do not give a comparison of my results with those published by O. Lummer and E. Pringsheim (*Verhandlungen der Deutschen Physikalischen Gesellschaft*, 1, No. 1), as it was indicated in an address by Herr Pringsheim at the last meeting of the German Scientists' Association that the measurements are still being continued.

THE VARIABLE VELOCITY OF ε *LEONIS* IN THE LINE OF SIGHT.

By W. S. Adams.

I wish to call attention to the evident variation in the radial velocity of ε *Leonis*, a third magnitude star of Type II a. Three spectrograms taken with the Yerkes refractor in February and the first part of March gave velocities of +10.4, +1.2, and −1.5 kilometers, respectively. As a range of four kilometers is considered extreme in measures of a star of this type with the dispersion and the camera employed, it was apparent that the velocity varied, and subsequent plates confirmed this variation. Fifteen plates have been secured, the results of which are as follows:

1900,	Feb.	11	+10.4 km	1900,	March	30	−1.0 km
	"	26	+1.2		"	31	−5.9
	March	6	−1.5		April	2	−1.8
	"	9	−3.0		"	3	−5.7
	"	22	−10.6		"	6	+3.0
	"	23–15 h	−3.4		"	7	−12.9
	"	23–19 h	−0.2		"	9	−0.1
	"	26	+3.6				

The total range given by these observations is 23.3 km, and the period is apparently very short, probably in the neighborhood of two and a quarter days.

All of these plates were obtained with the Brashear three-prism spectrograph attached to the 40-inch refractor and with a camera of 456 mm focal length. For a comparison spectrum the titanium spark was employed, which gives in the region measured (λ 4450–4650) a remarkably large number of fine sharp lines, well adapted to close measurement and conveniently spaced for the application of the Cornu-Hartmann interpolation formula. The measures upon the plates have been made in both directions, but no systematic variations have been detected between the values obtained with red to right and red to left.

The probable errors of the measures of course vary greatly with the quality of the two spectra and the number of lines suitable for measurement. The average probable error for the series is about ±0.30 km: the two most critical plates, those of February 11 and April 7, give ±0.18 km and ±0.23 km for 15 lines and 11 lines, respectively. It should, however, of course be noted that the probable error is of value mainly as evidence of the consistency of the measures, and not as the probable limit of error in the result obtained for a given plate. This is due to the fact that systematic errors even under the best conditions enter to such an extent as to considerably outweigh it.

In view of the apparent short period of the velocity variation it would seem probable that ϵ *Leonis* is a comparatively close system, and hence peculiarly subject both to tidal disturbances and to the perturbations caused by the flattening of the components, and already noted by Tisserand and Bélopolsky in the cases of *Algol* and α^1 *Geminorum*.

YERKES OBSERVATORY,
April 17, 1900.

THE CURVATURE OF THE SPECTRAL LINES IN THE SPECTROHELIOGRAPH.

By W. S. Adams.

When the new spectroheliograph of the Yerkes Observatory was designed the question of the curvature of the spectral lines became of great importance as determining the form of the slits to be used in correcting the distortion of the solar image. That this curvature must necessarily be very great is apparent from the fact that to include the entire image of the Sun in the focal plane of the forty-inch objective a slit nearly eight inches in length is required, together, of course, with prisms of corresponding height. In the course of the determination of the curvature some results were arrived at which may be of interest as bearing upon the character of the familiar Ditscheiner formula.[1]

In order to secure axes of reference for measurement a series of very fine wires were stretched across the first slit and fastened by means of "Rowland's universal wax." These were represented upon the photographic plate by sharp lines, the relative distances of which could be determined with great accuracy, and which consequently were well adapted to serve as axes of abscissae. For the corresponding axis of ordinates the edge of the exposure produced by the widened second slit was taken. To insure perpendicularity the slits were first set as nearly parallel as possible and a slight correction for slope was applied later to the measures. These last were made upon the aluminium line at λ 3944, K itself being too broad and diffuse, and are accurate to 0.01 mm, the nature of the line and the slightly fuzzy edge of the exposure preventing closer measurement. The distances between the axes of abscissae, however, are accurate to 0.001 mm.

[1] Scheiner's *Astronomical Spectroscopy* (Frost), p. 15.

It at once became desirable to compare these measures with the results given by the constants of the instrument, and here it was evident that the approximate character of the Ditscheiner formula would probably debar its use. In order to develop a rigorous expression I had recourse to the method used by Professor Lord for the derivation of the ordinary formula,[1] and, taking the equations obtained by him in the first part of his article, substituted in them rigorous values for the angles involved. The process then consisted merely of solving for the unknown quantity, the sign of the radical being determined by the conditions of the problem. Using Professor Lord's notation the result is

$$\frac{X}{f} = -\sin\left[-\sin^{-1}\left(n\sin\frac{A}{2}\right)+\sin^{-1}\left\{-n\cos A\sin\frac{A}{2} + \sqrt{\sin^2 A\,\frac{Z^2(n^2-1)}{f'^2-Z^2}+n^2\sin^2\frac{A}{2}(\cos A+1)^2}\right\}\right],$$

in which f and f' denote the focal lengths of the camera and collimator lenses respectively. The prisms are considered to be in the position of minimum deviation.

This is the result for a single prism. For a second prism it is clear that the conditions are different in that we are obliged to consider the curved form of the image emerging from the first prism. The procedure, however, is as before except that the angle of projection of the incident ray upon the principal section of the prism (x) can no longer be considered constant. The result, which is somewhat complicated, is as follows:

$$\frac{X}{f} = -\sin\left[-\sin^{-1}\left(n\sin\frac{A}{2}\right)+\sin^{-1}\left\{-\cos A\sin\left[\sin^{-1}\left(n\sin\frac{A}{2}\right)-(x'-x'_0)\right] + \sqrt{\sin^2 A\,\frac{Z^2(n^2-1)}{f'^2-Z^2}+2n^2\sin^2\frac{A}{2}(1+\cos A)-\sin^2 A\sin^2\left[\sin^{-1}\left(n\sin\frac{A}{2}\right)-(x'-x'_0)\right]}\right\}\right],$$

where

[1] This Journal, 5, 348, May 1897.

$$x' - x'_0 = -\sin^{-1}\left(n \sin\frac{A}{2}\right) + \sin^{-1}\left\{ -n\cos A \sin\frac{A}{2} + \sqrt{\sin^2 A \frac{Z^2(n^2-1)}{f'^2 - Z^2} + n^2 \sin^2\frac{A}{2}(\cos A + 1)^2} \right\}.$$

With the last formula and the instrumental constants

$$n = 1.64906 \text{ (for K)}\,;\quad A = 60^\circ\,;\quad f = 952^{\mathrm{mm}}\,;\quad f' = 956^{\mathrm{mm}},$$

the values of the curvature were computed for fourteen lines extending outward from the center of the photographic plate. These values, together with those obtained from the Ditscheiner formula, are gathered in the following table and compared with the measured results. As usual Z denotes distance from the middle of the spectrum, and X the departure of the spectral line from a right line. The unit is the millimeter:

Z	X			Residuals		
	Measured	Rigorous formula	Ditscheiner formula	M.–R.	M.–D.	R.–D.
4.761	0.05	0.050	0.044	±0.00	+0.01	+0.006
12.037	0.29	0.286	0.280	±0.00	+0.01	+0.006
16.825	0.55	0.551	0.548	±0.00	±0.00	+0.003
25.718	1.28	1.277	1.280	±0.00	±0.00	−0.003
30.375	1.78	1.783	1.786	±0.00	−0.01	−0.003
35.711	2.46	2.465	2.468	−0.01	−0.01	−0.003
40.341	3.14	3.143	3.150	±0.00	−0.01	−0.007
49.172	4.68	4.681	4.680	±0.00	±0.00	+0.001
56.724	6.24	6.243	6.228	±0.00	+0.01	+0.015
62.632	7.62	7.622	7.595	±0.00	+0.02	+0.027
71.155	9.86	9.866	9.800	−0.01	+0.06	+0.066
77.890	11.87	11.867	11.744	±0.00	+0.13	+0.123
83.775	13.75	13.755	13.585	±0.00	+0.17	+0.170
88.674	15.45	15.455	15.220	−0.01	+0.23	+0.235

An inspection of these results shows that the parabola of the Ditscheiner formula crosses the curve of the rigorous formula at two points, and that up to the second of these the correspondence of the two is close. The general conclusion may be drawn that for any work except that requiring an exceptionally long slit the ordinary formula is quite sufficient.

YERKES OBSERVATORY,
March 1900.

A SECOND SPECTRUM OF HYDROGEN BEYOND λ 185 $\mu\mu$.

By Victor Schumann.

I showed in 1893[1] that by no means the whole of the radiation of hydrogen was included in the region previously known, which embraced the visible spectrum and the adjacent ultra-violet, but that on the contrary hydrogen develops its principal energy beyond the wave-length 185 $\mu\mu$. Until recently I knew of only a single spectrum in this region of previously unrecognized refrangibility, and this was the spectrum produced when electric discharges are passed through a Plücker tube filled with hydrogen at moderate pressure. The mode of the discharge, which is well known to have a great effect on the visible spectrum and the adjoining ultra-violet, passed directly over to the new hydrogen region without any trace of difference, in spite of numerous experiments I made in the attempt to show the contrary. Thus its original condition was maintained even on the introduction into the circuit of a Leyden jar and an air gap, except that a moderate change took place in its energy in general. It was never possible to recognize a disappearance or a broadening of the lines, or their transformation into a continuous spectrum, which the long-known hydrogen spectrum readily displays at a pressure of from a few centimeters upward.

Now I have recently found that the metallic spectra beyond λ 185 $\mu\mu$, which are still provisionally connected with discharges in hydrogen at atmospheric pressure, exhibit a large number of lines in common. Continued experiments showed that these lines are due to hydrogen. While at the first comparison of these two spectra in this region, one is inclined to regard them as related, on account of their similarity, it is striking that a careful comparison leads to the fact that they are in complete

[1] *Sitzungsberichte* der K. Akad. der Wiss. in Wien. *Mathemat.—Naturwiss. Classe.* 102, Abth. IIa, 415–475, 625–694.

agreement in respect to neither their distribution of energy nor their refrangibility. Deviations can be demonstrated to exist throughout.

In this second spectrum of hydrogen also no tendency to a broadening of the lines is appreciable, in spite of the high pressure and the greater potential of the discharge necessary to its production. This fact deserves the more attention, as we should expect the opposite from the behavior of the rhythmical hydrogen lines, the broadening of which increases with the refrangibility under suitable experimental conditions.

I have not yet succeeded in photographing by itself this spectrum corresponding to atmospheric pressure. At present I recognize it only as an attendant phenomenon of the metallic spectra of this region. The possibility is not excluded that several of its lines are suppressed by the presence of metallic vapors.

Its activity is a function of the wave-length. Its lines are entirely absent, those of the metal alone appearing, when the spark is unusually short. This occurred in my experiments with a separation of the electrodes of from 0.1 to 0.2 mm. But exceptions are found in this respect, as was shown in the spectrum of copper.

In general the hydrogen radiations at atmospheric pressure are decidedly feebler in their action than those of the Plücker tubes. It is therefore questionable whether they can be followed so far into the ultra-violet as those of the tube, to which I was able in 1895 to assign an extensive field of radiation beyond the estimated wave-length 100 $\mu\mu$.

Minor Contributions and Notes

PLANS OF AMERICAN ECLIPSE PARTIES.

The following circular letter was issued by the Eclipse Committee of the Astronomical and Astrophysical Society of America on March 29, 1900:

Dear Sir: In response to a circular letter issued by this committee on February 10, 1899, preliminary statements have been received from many astronomers regarding the work which they expect to undertake in connection with the total solar eclipse of May 28, 1900. The committee's expression of a desire for some measure of coöperation met with favorable consideration. The more important suggestions for eclipse observations offered by these astronomers may be found in the preliminary report of the committee, which was published in the Astrophysical Journal for October 1899.

With these and other suggestions in mind the committee now begs leave to offer the following remarks:

1. Observations made without a definite purpose, and not based upon a careful consideration of the end to be attained, will rarely be of scientific value. The more important observations which may be undertaken are enumerated in the paragraphs below.

2. *Relative position of the Sun and Moon.*—(This will require the observer's position and the time to be known.) (*a*) From visual observations of contacts, especially contacts II and III, made at stations near the central line, and at stations near the boundaries of the shadow path. (*b*) From photographs of the solar crescent obtained with long-focus instruments at stations on the central line, within one minute of the contacts, the exact moments of exposure being noted. The program of exposures should be symmetrical with reference to the middle of the eclipse. Snap shots of the solar crescent taken a moment before the Sun is entirely covered, and the moment after it reappears, with a large camera, may also be useful if the photographer can give the moments to the nearest second of standard time. The slowest plates should be used, and the exposures given by automatic shutters. With a camera ratio of $\frac{a}{f} = \frac{3}{480}$, an exposure of 0.001 second is abundant. Excessive exposures cannot be expected to yield results comparable to visual observations of the contacts.

3. *Drawings of the corona* (made at the telescope).—Attention should be directed to the structure of the inner corona (especially the regions near large prominences) and to the general outline of the corona. Drawings of details should not attempt to cover large areas.

4. *Color of prominences.*—Compare prominences seen with a telescope and determine relative degree of redness of each. Slight variations in tint may not be important, as from the Purkinje phenomenon the brightest prominences should seem the reddest. White prominences may perhaps be seen. Provision must be made for determining the position angles of the prominences observed.

5. *Photographs of the corona.*—Attention may be called to three classes of camera lenses used for recording the corona. (*a*) Lenses for which $\frac{a}{f}$ is $>$ or $= \frac{1}{6}$, especially adapted to recording the details of the outer corona and the extensions. A wide range of exposures, from $\frac{1}{100}$ second up, should prove useful. The greatest extent of corona hitherto recorded was on triple-coated plates, exposure 20 seconds. Some observers consider that long exposures on very slow plates, fully developed, should be effective in distinguishing the long streamers on the bright-sky background. Some also believe that the faint extensions will be best photographed with lenses for which $\frac{a}{f}$ is considerably less than $\frac{1}{6}$. (*b*) Lenses in which $\frac{a}{f}$ is about $\frac{1}{15}$, especially adapted to recording the details of the mid-corona. The rear lens of a photographic doublet may be used alone to give a long focus. With rapid plates, exposures of $\frac{1}{60}$ second should easily record the prominences, $\frac{1}{10}$ to $\frac{1}{5}$ second the inner corona, and a range of longer exposures the mid-corona. (If equatorial mounting and clock-work are wanting, exposures should not exceed one or two seconds for $f = 60$ inches, and similarly in other cases.) (*c*) Objectives of relatively great focal length, giving details of structure in the inner corona. The objective may be pointed directly at the Sun and used with a moving photographic plate; or it may receive light from a suitable heliostat. With $\frac{a}{f} = 196$, an exposure of $\frac{1}{4}$ second, on slow plates, has recorded the prominences. Revolving diaphragms immediately in front of the photographic plate and concentric with the Moon's image, may be used to reduce the equivalent exposure time of the inner corona.

Before and after the total phase it will be interesting to repeat, and extend as far as possible, the successful experiments made at the Chile and Indian eclipses, of photographing the corona many seconds before and after totality.

In all photographic work care must be taken to provide for the accurate orientation of the coronal image. A satisfactory method consists in exposing a plate to the sky on a dark night, allowing the stars to trail across the plate.

For cameras in which $\frac{a}{f}$ is large, β *Herculis* will give a suitable trail. *Mercury* and ϵ *Tauri* will appear on most photographs of the corona, and will also serve as an excellent means of orientation.

6. *Photographic search for an intra-mercurial planet.* — The lenses recommended for this purpose in Harvard College Observatory *Circular* No. 48 have an aperture of 3 inches and a focal length of 11 feet 4 inches. It is stated in the *Circular* that with such lenses stars as faint as the eighth magnitude can be photographed during the eclipse in an exposure of one minute. Portrait lenses of large aperture and rapid rectilinear lenses may also be used. The sky surrounding the Sun may well be divided among several observers, as few individuals will be provided with enough lenses to cover the whole region. At least two comparable photographs of a given part of the sky should be made by each observer. The length of exposure which can advantageously be employed with each lens must take into account the strength of the bright-sky background; and while the brightness of the sky varies at different eclipses, some idea of the requirements of the problem may be gained by photographing a portion of the evening sky in which a second magnitude star, whose position is known, is just visible.

7. *Distribution of "coronium."* — Observations of the corona with a direct-vision prism in the eyepiece of a field glass, or with an objective prism or grating attached to a telescope used either visually or photographically, should show whether the image corresponding to the green line of this gas has a well-defined structure, or a nearly uniform intensity. The *dispersive power should be strong* in order to reduce the strength of the continuous spectrum background.

8. *Position of the green coronal line.* — The wave-length of the line can best be determined from photographs taken with a powerful prismatic spectrograph. Direct visual observations with a suitable spectroscope and telescope will, however, suffice to show whether the line coincides with the chromospheric line at 1474 *K*. Accurate determinations of the wave-lengths of the bright lines near λ 399 and λ 423 are also desirable.

9. *The spectrum of the "flash"* may be photographed with an objective prism or a spectograph used in conjunction with a telescope. A concave grating spectroscope or an objective grating might also be employed. For this work stations should be selected not far from the northern and southern boundaries of the shadow path. If possible the observations should include a study of the dark line spectrum, before the flash at second contact and after the flash at third contact, and a study of the coronal spectrum.

10. *The heat radiation of the corona* should be measured in the streamers and rifts, and at various distances from the Sun's limb.

11. *Photometric observations of the corona* at various distances from the limb should be made photographically, in view of the brief duration of totality.

12. *Photometric observations of Mercury* during the total phase are recommended by Professor Müller in an article published in the ASTROPHYSICAL JOURNAL for March 1900.

Eclipse photographers cannot be too strongly cautioned against a dense development of the plate. This has been the great fault with most of the negatives at previous eclipses. The image should be slowly developed and kept as thin and transparent as possible.

All photographic plates used in the eclipse problems should be "backed" to prevent halation. Water-color lampblack, reduced to the consistency of paste and applied to the back of the plate, will answer well for this purpose. Or the back of the plate may be flowed with a mixture of normal collodion · 100 parts, chrysoïdine 3 parts.

For valuable information regarding probable weather conditions, local time and duration of totality, etc., reference is made to *Weather Bureau Bulletin* No. 209 and the Supplement to the *American Ephemeris* for 1900.

In some of the work outlined above, notably that referred to in paragraphs (3) and (6), it is desirable that there should be coöperation among observers. It is also of great importance that the eclipse stations should be distributed along a considerable length of the path of totality, so that local cloudiness may not affect all parties. For the purpose of securing for general publication accurate information regarding proposed observations, and in the hope of bringing about some degree of coöperation, the Committee requests that replies be sent to the following questions:

1. Do you intend to observe the eclipse?

2. If so, what site do you expect to occupy?

3. If any other location would be equally convenient, kindly state in what region.

4. If possible, kindly name the members of your party.

5. What observations will be made by the members of your party? What instruments will be employed?

6. If your present plans do not prevent, are you willing to coöperate with other parties, especially in undertaking observations such as those named in paragraphs (3) and (6) above?

Please be kind enough to send replies at your earliest convenience to the Secretary of the Committee.

Very respectfully,

SIMON NEWCOMB, *Chairman*
GEORGE E. HALE, *Secretary*
E. E. BARNARD,
W. W. CAMPBELL.
} Eclipse Committee.

From the replies received by the Committee, the following information regarding the plans of eclipse parties has been derived:

NORFOLK, VA. NATIONAL GEOGRAPHIC SOCIETY.

NEAR NORFOLK, VA.

PARTY FROM BROWN UNIVERSITY.

Members. Winslow Upton, in charge; Frederic Slocum, John Edwards, Howard D. Kenyon, and others.

Principal instruments. Two 4-inch telescopes, one 2½-inch telescope, several cameras mounted equatorially, combined transit and zenith telescope.

Proposed observations.[1] —(2) (a), (b); (3); (5) (a), (b).

SOUTH OF NORFOLK, VA.

PARTY FROM GEORGETOWN COLLEGE OBSERVATORY.

Members.—J. G. Hagen, in charge; Messrs. Zwack, Brosnan, Hedrick, and others.

Proposed observations.—(3); (5).

NEAR RALEIGH, N. C.

PARTY FROM TRINITY COLLEGE, DURHAM, N. C.

Members.—C. W. Edwards, in charge; W. H. Pegram, two students.

Principal instruments.—Double mirror heliostat, large spectrometer, 3½-inch telescope, cameras, concave grating spectroscope of 10 feet radius.

Proposed observations.—(2); (3); (5); possibly (6).

PINEHURST, N. C.; BARNESVILLE, GA.; GRIFFIN, GA. (near the northern limit of totality.)

PARTIES FROM THE U. S. NAVAL OBSERVATORY.

Members.[2]— S. J. Brown, in charge; (1) Observatory staff: Messrs. A. N. Skinner, T. J. J. See, M. Updegraff, and W. S. Eichelberger; G. A. Hill, T. I. King, and F. B. Littell; W. M. Lawton, Hammond, and G. H. Peters. (2) Associate members: Messrs. J. S. Ames, L. E. Jewell, N. E. Dorsey, Hoff, Reese, and Krouse, of Johns Hopkins University; Messrs. Henry Crew and R. R. Tatnall, of the Northwestern University; H. C. Lord, of Ohio State University; F. L. Chase, of Yale University; J. K. Rees and S. A. Mitchell, of Columbia University; Mr. A. L. Colton, formerly of the Lick Observatory; Mr. Charles A. Post, of Bayport, L. I. Dr. W. J. Humphreys, of the University of Virginia, will also probably occupy a station near the limit of totality in South Carolina, near Winnsboro.

Principal instruments.—Telescopes and cameras ranging from a 40-foot photoheliograph down to one of 4 inches aperture and 8 inches focal length;

[1] The numerals refer to the numbered paragraphs of the above circular.

[2] Definite information as to the assignment of the different observers to the various stations has not been received.

concave gratings of 21 and 10 feet radius; objective gratings, slitless spectroscopes, and large prismatic camera; polariscopes.

Proposed observations.—(2) (a), (b); (3); (5) (a), (b), (c); (7); (8); (9); polarization of the bright line spectrum of the corona; general polarization of the continuous spectrum; velocity, size, and direction of shadow bands.

WADESBORO, N. C.

1. PARTY FROM BRITISH ASTRONOMICAL ASSOCIATION.

Members.—Rev. J. M. Bacon, in charge; seven other members.

Principal instruments.—Photographic telescope of 4.1 inches aperture and 60 inches focal length; telephoto lens; four equatorially mounted cameras; telescopic kinematograph; slitless spectroscope; instrument for recording photographically the brightness of the sky at the zenith.

Proposed observations.—(5) (a), (b); (7); (11); brightness of sky; shadow bands.

2. PARTY FROM PRINCETON UNIVERSITY.

Members.—C. A. Young, in charge; Messrs. Brackett, Magie, Libbey, Reed, McClenahan, Russell, Fisher, and others.

Proposed observations.—(2) (a); (3); (4); (5); (7); (8).

3. PARTY FROM SMITHSONIAN INSTITUTION.

Members.—S. P. Langley, in charge, with about twelve assistants.

Principal instruments.—Two coelostats; large spectro-bolometer with photographic registration; photographic telescopes of 12 inches aperture and 135 feet focal length, 5 inches aperture and 40 feet focal length, 6 inches aperture and 7½ feet focal length, etc.

Proposed observations.—(2) (b); (3); (5) (a), (b), (c); (6); (9); (10); bolographs of the spectrum of the corona.

4. PARTY FROM VASSAR COLLEGE.

Members.—Mary W. Whitney, in charge; Caroline E. Furness, several students.

Principal instruments.—Three 3-inch Clark telescopes.

Proposed observations.—(3).

5. PARTY FROM YERKES OBSERVATORY.

Members.—George E. Hale, in charge; E. E. Barnard, E. B. Frost, F. Ellerman, G. W. Ritchey. Volunteers: A. S. Flint, Washburn Observatory; H. M. Goodwin and A. A. Noyes, Massachusetts Institute of Technology; G. I. Isham, Chicago; and others.

Principal instruments.—Coelostat with two mirrors; photographic telescope of 6 inches aperture and 62 feet focal length; bolometer and 20-inch

concave mirror for measuring the heat radiation of the corona;[1] concave gratings of 10 feet and 5 feet radius; plane gratings; prismatic cameras; several cameras mounted equatorially; Heyde universal instrument; 3-inch equatorial.

Proposed observations.—(2) (a); (5) (a), (b), (c); (6); (7); (9); (10).

NEWBERRY, S. C.

PARTY FROM UNITED STATES WEATHER BUREAU.

Members.—Cleveland Abbe, in charge; Frank H. Bigelow.

Principal instruments.—Four-inch telescope, polariscopes.

Proposed observations.—(3); (5); polarization of the sky.

The Weather Bureau will secure standard meteorological observations from a large number of its regular stations in the southern and southwestern states, special orders being issued for this purpose.

WASHINGTON, GA.

PARTY FROM BLUE HILL METEOROLOGICAL OBSERVATORY AND MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Members.—A. L. Rotch, in charge; A. E. Burton, of the Massachusetts Institute of Technology, with several students; A. E. Douglass, of Lowell Observatory; assistants from Blue Hill Observatory.

Principal instruments.—Five-inch equatorial; magnetic and meteorological apparatus.

Proposed observations.—(1); (3); shadow bands; eclipse wind; zodiacal light; magnetic observations.

UNION POINT, GA.

PARTY FROM CHABOT OBSERVATORY.

Members.—Charles Burckhalter, in charge; seven assistants.

Principal instruments.—Two 4-inch telescopes of 15 feet focal length, with device for giving suitable exposure for all parts of the corona.

Proposed observations.—(5) (b), (c).

FORSYTHE, GA.

PARTY FROM RENSSELAER POLYTECHNIC INSTITUTE.

Members.—C. W. Crockett, in charge.

Principal instrument.—Three and one half-inch equatorial.

Proposed observations.—(3).

[1] Through the courtesy of Professor Langley this apparatus will be used in conjunction with one of the coelostats of the Smithsonian party.

THOMASTON, GA.

PARTY FROM LICK OBSERVATORY.

Members.—W. W. Campbell, in charge; C. D. Perrine, and others.

Principal instruments.— Photographic telescope of 5 inches aperture and 40 feet focal length, for recording prominences and inner corona; photographic telescope of 5 inches aperture and 6½ feet focal length; Dallmeyer 6-inch camera; Willard 5½-inch portrait lens, and other cameras; slit spectrograph with three prisms; spectrograph with objective grating: spectrograph with two objective prisms; photometric apparatus, visual telescope, etc.

Proposed observations.—(2)(a); (5)(a) and (5)(b); (6); (7); (8); (9); (11).

GREENVILLE, ALA.

PARTY FROM HARVARD COLLEGE OBSERVATORY.

Members.—W. H. Pickering, in charge; W. H. Attwill, J. R. Edmands.

Principal instruments.—Cameras from 0.04 inches to 6 inches aperture, and from 1 inch to 11 feet 4 inches focal length; 4-inch and 5-inch telescopes.

Proposed observations.—(3); (5); (6); (11).

FORT DEPOSIT, ALA.

PARTY FROM ALLEGHENY OBSERVATORY.

Members.— F. L. O. Wadsworth, in charge; J. A. Brashear, and others.

Principal instruments.—Two 4-inch cameras; one 4½-inch camera, and another 3-inch, both for curved plates; a 9½-inch reflector of 38 inches focus; a 20-inch reflector of 100 feet focus, besides smaller camera lenses. Spectroscopic apparatus will consist of three large glass prisms of 3-inch by 4½-inch face, and numerous gratings.

Proposed observations.—(5) (a), (b), (c); (6); (7); (8); (9).

INDIVIDUAL OBSERVERS.

MEXICO (near M. C. R. R.).

Miss Rose O'Halloran.

Principal instruments.— Four-inch refractor of 60 inches focus; 2½-inch portrait lens.

Proposed observations.—(5) (b); (3).

NEAR NORFOLK, VA.

John N. Stockwell.

Principal instruments.— Field glass and appliance for measuring angles.

Proposed observations— General visual observations.

A PHOTOGRAPHIC SEARCH FOR AN INTERMERCURIAL PLANET.[1]

It has not been, in general, the policy of the Harvard College Observatory to send expeditions to observe total eclipses of the Sun. First, since in case of cloudy weather, no return is obtained for the expenditure of a sum of money which is often large. Secondly, if clear, the results in many cases are only a series of pictures of the corona and protuberances which add but little to our knowledge of them. Therefore, when officers of this Observatory have observed eclipses, it has generally been largely at their own expense. When, however, a new problem presents itself, some aid is rendered from the funds of the Observatory, in the construction of instruments and for similar expenses. The following plan for observing the Eclipse of May 28, 1900, has been prepared by Professor W. H. Pickering:

It is a fact capable of demonstration that the faintness of a star that may be photographed with a given instrument, against a bright background of sky, depends, within certain limits, directly on the length of the focus of the lens, and is independent of its aperture.

In the *Harvard Observatory Annals*, 18, 104, it was shown, if the place in which to look for the pole star is known, that three minutes after it first becomes visible to the naked eye in the evening, the light of the sky in its immediate vicinity is of about the same photographic intensity as that of the sky surrounding the Sun at the time of a total solar eclipse.

Starting with these two fundamental facts, a series of experiments has been undertaken with a photographic lens having an aperture of 3 inches, and a focal length of 11 feet 4 inches. The curves adopted were those employed in an ordinary landscape lens, and it was found that the field was large enough to cover nine 8×10 photographic plates arranged in three rows of three each. This result was only obtained, however, by attaching the plates to the interior of a concave surface of double curvature, and thus obtaining a curved field.

By giving an exposure of one minute in the region of the pole, with this instrument, three minutes after the pole star first became visible, it was found that the light of the sky was sufficient to darken the plate appreciably, but not so much as to prevent stars of the eighth magnitude appearing with sufficient intensity to be found by a careful search, in the larger part of the field of view.

Three similar lenses have now been ordered, and the four will be placed upon one mounting, in such a manner as to photograph a region extending

[1] *Harvard College Observatory Circular* No. 48.

for sixteen degrees on either side of the Sun, and having a breadth of ten degrees throughout its length. Throughout nineteen degrees of its length every portion of the region will appear upon two separate plates.

The satellites of *Mars*, *Jupiter*, and *Saturn* all revolve very nearly in the equatorial planes of their primaries, and in the same manner *Mercury* revolves very nearly in the equatorial plane of the Sun, which is inclined about seven degrees to the plane of the ecliptic. It is therefore reasonable to suppose that bodies still nearer to the Sun would revolve in the same plane. It so happens that the Earth passes through this plane about one week after the date of the solar eclipse of next May, so that there is a strong probability that if an intermercurial planet exists, it will appear somewhere upon the narrow line forming the projection of this plane upon the celestial sphere. It will be seen, therefore, that the date of this eclipse is especially favorable for the proposed search.

We have very good evidence, from the visual observations hitherto made, that no intermercurial planet brighter than the third or fourth magnitude exists. We possess no evidence whatever for or against the existence of fainter bodies in this region having sufficient size to be properly called planets. We are reasonably certain that the immediate vicinity of the Sun is filled with countless bodies of such size as to be properly described as meteors.

If we assume that at its average brightness, *Mercury* is of the first magnitude, and that the albedo of an intermercurial planet is the same as that of *Mercury*, we shall find that at the distance of *Mercury* from the Sun, a body of the eighth magnitude would be 120 miles in diameter. If its distance from the Sun was but one half as great, its diameter would be 60 miles, and if but one quarter as great, or nine million miles, it would be 30 miles in diameter. Judging by the analogous case of *Jupiter*, the existence of such a small planet is quite possible.

Should such a body exist, and should it appear upon the plates, which it is proposed to expose somewhere in the State of Alabama, we should still be entirely at a loss to compute the orbit, or to determine the distance of the body from the Sun. If, however, other photographs of it should be obtained with a similar apparatus, in Spain or Algeria, we should then be enabled to compute an approximate orbit, based on the assumption that it moved in a circular path. It might then be found again at the following eclipse, which occurs a year later, and a more accurate elliptical orbit could be computed for it. While it is desirable that the duplicate apparatus should also be furnished with four lenses, this is not necessary, and in case the planet should be found upon our plates, two lenses, one photographing the region on each side of the Sun, would be all that would be necessary to independently make the discovery, and furnish the elements necessary to compute the circular

orbit. It is in the hope of inducing some European observer to supply himself with this apparatus, that the present article has been written.

The foregoing plan appears to be of sufficient importance to justify aid from the Observatory. Preparations have, therefore, been made to give it a careful trial. It is hoped that this early publication may permit similar observations to be made at a second station sufficiently distant to reduce the danger of failure from clouds, and if an intermercurial planet should be found, to furnish an approximate determination of the form of its orbit.

EDWARD C. PICKERING.

February 13, 1900.

PHOTOGRAVURE & COLOR CO. N.Y.

THE TRIFID NEBULA IN SAGITTARIUS

Photographed with the Crossley Reflector of the Lick Observatory

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XI JUNE, 1900 NUMBER 5

THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

By James E. Keeler.

The Crossley reflector, at present the largest instrument of its class in America, was made in 1879 by Dr. A. A. Common, of London, in order to carry out, and test by practical observation, certain ideas of his respecting the design of large reflecting telescopes. For the construction of the instrument embodying these ideas, and for some fine astronomical photographs obtained with it, Dr. Common was awarded the gold medal of the Royal Astronomical Society in 1884.

In 1885, Dr. Common, wishing to make a larger telescope on a somewhat similar plan, sold the instrument to Edward Crossley, Esq., F. R. A. S., of Halifax, England. Mr. Crossley provided the telescope with a dome of the usual form, in place of the sliding roof used by its former owner, and made observations with it for some years; but the climate of Halifax not being suitable for the best use of such a telescope, he consented, at the request of Dr. Holden, then Director of the Lick Observatory, to present it to this institution. The funds for transporting the telescope and dome to California, and setting them

up on Mt. Hamilton, were subscribed by friends of the Lick Observatory, for the most part citizens of California. The work was completed, and the telescope housed in a suitable observatory building, in 1895.[1]

On taking charge of the Lick Observatory in 1898, I decided to devote my own observing time to the Crossley reflector, although the whole of my previous experience had been with refracting telescopes. I was more particularly desirous of testing the reflector with my own hands, because such preliminary trials of it as had been made had given rise to somewhat conflicting opinions as to its merits.[2] The result of my experience is given in the following article, which is written chiefly with reference to American readers. If I have taken occasion to point out what I regard as defects in the design or construction of the instrument, I have done so, not from any desire to look a gift horse in the mouth, but in the interest of future improvement, and to make intelligible the circumstances under which the work of the reflector is now being done and will be done hereafter. The most important improvements which have suggested themselves have indeed already been made by Dr. Common himself, in constructing his five-foot telescope. The three-foot reflector is, in spite of numerous idiosyncracies which make its management very different from the comparatively simple manipulation of a refractor, by far the most effective instrument in the Observatory for certain classes of astronomical work. Certainly no one has more reason than I to appreciate the great value of Mr. Crossley's generous gift.

The Crossley dome (Plate VI) is about 350 yards from the main Observatory, at the end of a long rocky spur which extends from the Observatory summit toward the south, and on which are two of the houses occupied by members of the Observatory

[1] For a more complete history of this part of the subject, see Dr. Holden's articles in *Pub. Ast. Soc. Pacific*, 7, 197 *et seq.*, 1895.

[2] The difficulties here referred to, about which a good deal has been written, seem to have had their origin in the fact that it was impossible, at the time of the preliminary trials, to provide the observer with an assistant, while the Crossley reflector is practically unmanageable by a single person.

PLATE VI

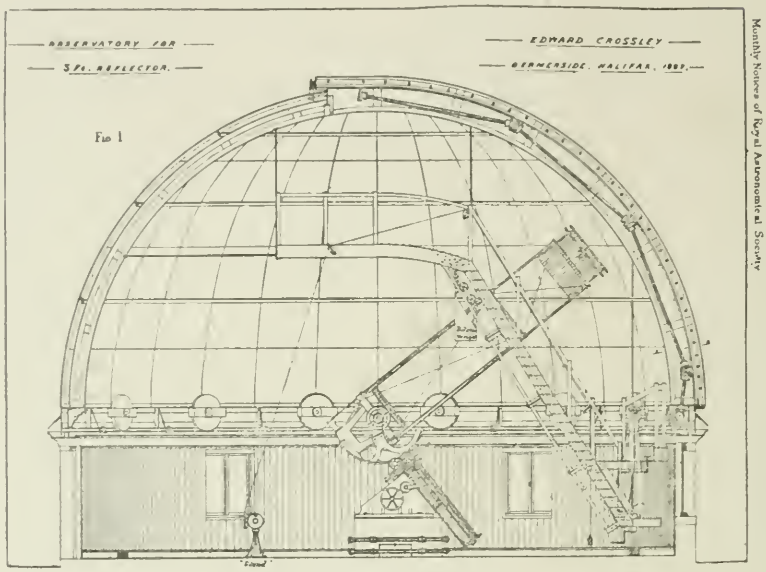
DOME OF THE CROSSLEY REFLECTOR

staff. It is below the level of the lowest reservoir, "Huyghens," which receives the discharge from the hydraulic machinery of the 36-inch refractor, and therefore the water engine furnished by Mr. Crossley for turning the dome cannot be used, unless a new water system — overflow reservoir, pump and windmill — is provided. In this respect a better site would have been a point on the south slope of "Kepler,"—the middle peak of Mt. Hamilton — just above the Huyghens reservoir. No addition to the present water system would then have been needed. The slope of the mountain at this place might cut off the view of the north horizon, but since the telescope cannot be turned below the pole, this would be a matter of no consequence. Water-power for the dome is not, however, really necessary.

The cylindrical walls of the dome, 36¼ feet inside diameter, are double, and provided with ventilators. Opening into the dome, on the left of the entrance, are three small rooms, one of which has been fitted up as a photographic dark room, and another, containing a sidereal clock and a telephone which communicates with the main Observatory, as a study, while the third is used for tools and storage. There is also a small room for the water engine, in case it should be used. The dome is at present supplied with water from only the middle reservoir, Kepler, which is reserved for domestic purposes and is not allowed to pass through the machinery.

The dome itself, 38 feet 9 inches in diameter, is made of sheet-iron plates riveted to iron girders. It also carries the wooden gallery, ladders, and observing platform, which are suspended from it by iron rods. The apparatus for turning the dome consists of a cast-iron circular rack bolted to the lower side of the sole-plate, and a set of gears terminating in a sprocket-wheel, from which hangs an endless rope. As the dome does not turn easily, it has been necessary to multiply the gearing of the mechanism so that one arm's-length pull on the rope moves the dome only about one inch. In some positions of the telescope the dome cannot be moved more than six or eight inches at a time without danger of striking the tube, and

this slowness of motion is then not disadvantageous. It is only when the dome has to be moved through a considerable angle, as in turning to a fresh object, or in photographing some object which passes nearly through the zenith, that the need for a mechanical means of rotation is felt.



THE CROSSLEY REFLECTOR.

The observing slit, 6 feet wide, extends considerably beyond the zenith. It is closed by a double shutter, which is operated by an endless rope. The upper part, within the dome, is also closed by a hood, or shield, which serves to protect the telescope from any water that may find its way through the shutter, and which is rolled back to the north when observations are made near the zenith. I have recently fitted the lower half of the slit with a wind-screen, which has proved to be a most useful addition. It is made of tarpaulin, attached to slats which slide between the two main girders, and is raised or lowered by halliards, which belay to cleats on the north rail of the gallery. A more detailed description of the dome has been given in an

article by Mr. Crossley,[1] from which the reduced figure in Fig. 1[2] has been taken.

The mounting of the three-foot reflector has been very completely described and illustrated by Dr. Common,[3] so that only a very general description need be given here. The most important feature of the mounting is that the telescope tube, instead of being on one side of the polar axis, as in the usual construction, is central, so that the axis of the mirror and the polar axis are in the same line when the telescope is directed to the pole The declination axis is short, and is supported by a massive goose-neck bolted to the upper end of the polar axis. The mirror is placed just *above* the declination axis. Its weight, and the weight of the whole tube and eye-end, are counterpoised by slabs of lead, placed in two iron boxes between which the goose-neck of the polar axis passes. The great advantage of this arrangement, and the controlling principle of the design, is that the telescope is perfectly free to pass the meridian at all zenith distances. No reversal of the instrument is needed, or is indeed possible.

For long-exposure photography, the advantage above referred to is obvious, but it is attended by certain disadvantages. One of these is that a very much larger dome is required than for the usual form of mounting. Another is the great amount of dead weight which the axes must carry; for the mirror, instead of helping to counterpoise the upper end of the tube, must itself be counterpoised. When anything is attached to the eye-end (and in astrophysical work one is always attaching things to the eye-end of a telescope), from ten to twenty times as much weight must be placed in the counterpoise boxes below the declination axis. Where room is to be found for the weights required to counterpoise the Bruce spectrograph, is a problem which I have not yet succeeded in solving.

In his five-foot reflector, Dr. Common has caused the telescope tube to swing between two large ears, which project

[1] *Mon. Not. R. A. S.*, 48, 386.

[2] Kindly lent by the Astronomical Society of the Pacific.

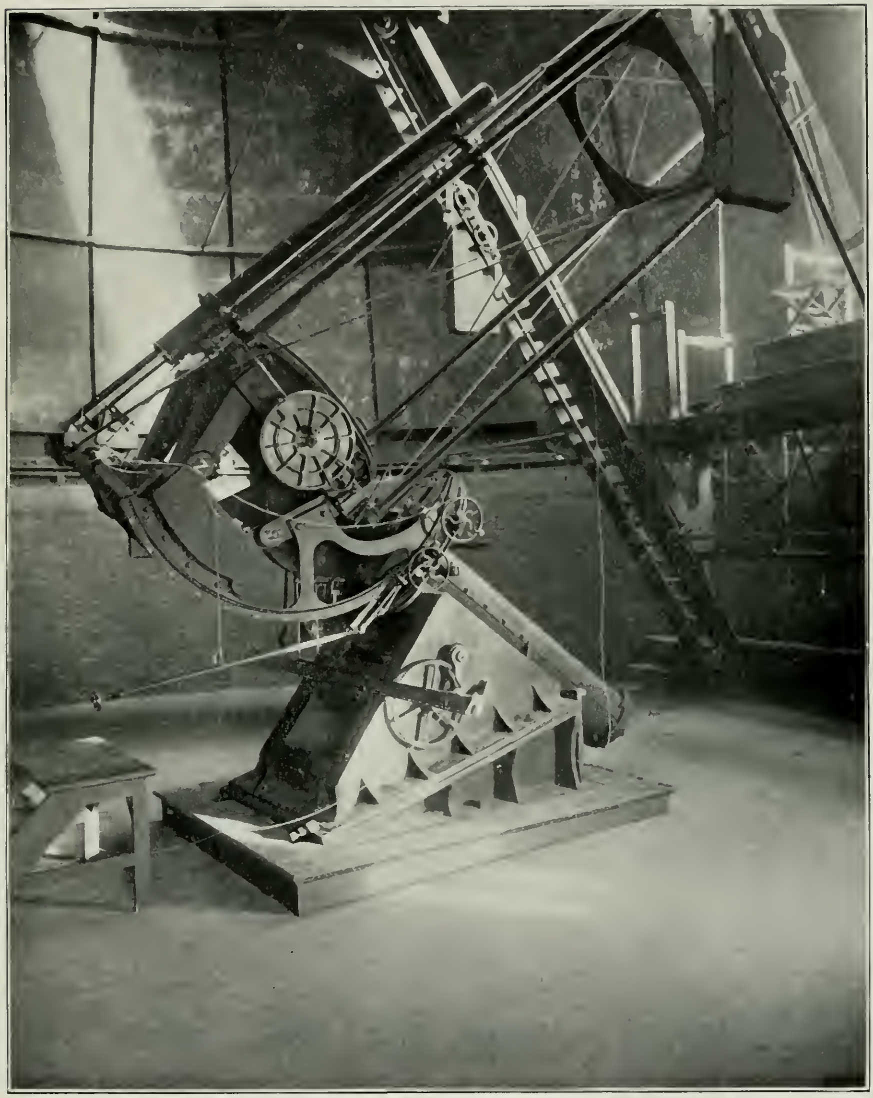
[3] *Mem. R. A. S.*, 46, 173.

from the upper end of the boiler-like polar axis, the pivots constituting the declination axis being near, but above, the lower end of the tube. The mirror therefore helps to counterpoise the upper end of the tube. This I regard as a distinct improvement. The danger of large masses of metal near the mirror injuring the definition is, in my opinion, imaginary; at least there is no such danger on Mt. Hamilton, where the temperature variations are unusually small. Experience with the Crossley reflector, as well as with the other instruments of the Lick Observatory, shows that the definition depends almost entirely on external conditions.

My first trials of the reflector, as first mounted at the Lick Observatory, showed that the center of motion was inconveniently high. Among other difficulties arising from this circumstance, the spectroscope projected beyond the top of the dome, so that it had to be removed before the shutter could be closed. In July 1898 the pier was therefore cut down two feet. This brought the eye-end down nearly to the level of the gallery rail, where it was at a convenient height for the observer when sitting on a camp-stool, and it made all parts of the mounting more accessible. Toward the north and south, the range of the telescope, being limited in these directions by the construction of the mounting, was not affected by the change, but the telescope cannot now be used at such low altitudes as formerly, near the east and west points of the horizon. The only occasion likely to call for the use of the reflector in these positions is the appearance of a large comet near the Sun, and after some consideration, I decided to sacrifice these chances for the sake of increasing the general usefulness of the instrument. Except in rare cases, all observations are made within three hours of the meridian.

To adapt the mounting to the latitude of Mt. Hamilton, a wedge-shaped casting, shown in Plate VII, had been provided, but through some error, arising probably from the fact that the telescope had been used in two different latitudes in England, the angle of the casting was too great. When the pier was cut down its upper surface was therefore sloped toward the south, in order

PLATE VII



THE CROSSLEY REFLECTOR

to compensate the error in the casting. Plate VII shows the instrument very nearly as it is at the present time.

The polar axis of the Crossley reflector is a long, hollow cylinder, separated by a space of about one eighth of an inch from its concentric casing. The idea was to fill this space with mercury, and float the greater part of the thrust of the axis, the function of a small steel pin at the lower end being merely to steady the axis. But this mercury flotation, as applied to the Crossley telescope, is a delusion, as I think Mr. Crossley had already found. The mercury, it is true, relieves the thrust to some extent, but it greatly increases the already enormous side pressure on the steel pin at the bottom, thus creating a much greater evil than the one it is intended to remedy. The workmen who set up the mounting inform me that the small bearing at the lower end of the polar axis is badly worn, as I should expect it to be. Instead of putting mercury into the space intended for it, I have therefore poured in a pint or so of oil, to keep the lower bearing lubricated. For the reasons indicated above, the force required to move the telescope in right ascension is perhaps five times greater than it should be. The lower end of the polar axis ought to be fitted with ball bearings to take the thrust, and with a pair of friction wheels on top; but it would be difficult to make these changes now. It should be observed that the disadvantages of the mercury flotation are considerably greater at Mt. Hamilton than at the latitude for which the telescope was designed.

As already stated above, the range of the telescope is limited on the south by the construction of the mounting. The greatest southern declination which can be observed is 25°. In England this would doubtless mark the limit set by atmospheric conditions, but at Mt. Hamilton it would be easy to photograph objects 15° farther south, if the telescope could be pointed to them.

The original driving-clock having proved to be inefficient, at least without an electric control, a new and powerful driving-clock was made by the Observatory instrument maker, from

designs by Professor Hussey. In its general plan it is like that of the 36-inch refractor. The winding apparatus, contained in the large casting of the original mounting, has no maintaining power, and cannot easily be fitted with one. The clock could in no case be wound during a photographic exposure, on account of the tremors attending the operation, but it would be somewhat more convenient to have the stars remain on the plate during the winding. With a little practice, however, one can wind the clock without actually stopping it, though the object must afterwards be brought back to its place by means of the slow motion in right ascension.

Two finders have recently been fitted to the Crossley reflector. One has an object-glass of four inches aperture and eight feet six inches focal length, with a field of about 1° 2′, which is very nearly the photographic field of the main telescope. Its standards are bolted to one of the corner tubes of the reflector, as shown in Plate VII. The other finder has a three-inch objective and a large field. It had not been mounted when the photograph for the plate was made.

When a telescope is used for photographing objects near the pole, with long exposures, the polar axis must be quite accurately adjusted, for otherwise the centers of motion of the stars and of the telescope will not agree, and the star images will be distorted. It is true that with a double-slide plate-holder, like the one used with the Crossley reflector, one star — namely the guiding star — is forced to remain in a fixed position with respect to the plate; but the differential motion of the other stars causes them to describe short arcs, or trails, around this star as a center. A considerable part of the spring of 1899 was spent in efforts to perfect the adjustment of the polar axis, an operation which, on account of the peculiar form of the mounting, offers unusual difficulties.

In the first plan which was tried, the reflector was used as a transit instrument. The inclination of the declination axis was determined with a hanging level which had been provided by Mr. Crossley, the hour circle and polar axis being very firmly

clamped. The clock correction being known from the records kept at the Observatory, the collimation and azimuth constants were found by the usual formulæ. This method failed to give satisfactory results, and it was found later that the declination and polar axis were not exactly at right angles.

There is only one part of the sky on which the telescope can be reversed; namely, the pole. A method which promised well, and on which some time was spent, consists in photographing the pole (the declination axis being horizontal) by allowing the stars near it to trail for ten or fifteen minutes, then turning the polar axis 180° and photographing the pole again on the same plate. Half the distance between the images gives the error of the polar axis, which if the plate is properly oriented, is easily resolved into horizontal and vertical components; while the distance of each image from the center of the plate is this error increased or diminished by twice the deviation of the telescope axis. In this case the vertical component depends upon the reading of the declination circle, and the horizontal component gives the error of collimation. This method failed, however, to give consistent results, mainly on account of instability of the mirror, and was abandoned.

The use of the large mirror for purposes of adjustment was finally given up, and the axis was adjusted by observations of *Polaris* with the long finder, in the usual manner. In order to reach the star at lower culmination the finder tube had to be thrown out of parallelism with the main telescope.

The base-plate having no definite center of rotation in azimuth, and the wedges and crowbars used for moving it being uncertain in their action, a watch telescope, provided with a micrometer eyepiece, was firmly secured to the mounting throughout these operations, in such manner that a mark on the southern horizon could be observed through one of the windows of the dome. The errors of the polar axis were finally reduced to within the limits of error of observation.

The movable hour circle and driving wheel of the Crossley reflector has two sets of graduations. The driving screw having

been thrown out of gear, the circle is turned until the outer vernier indicates the sidereal time, whereupon the driving screw is thrown into gear again. The inner vernier is then set to the right ascension of the object which it is desired to observe. As an inconsistency, of minor importance, in the design of the mounting, I may note that the slow motion in right ascension changes the reading of the outer vernier instead of that of the inner one. In practice, however, no inconvenience is caused by this construction.

In the early experiments and photographic work with the Crossley telescope, irregularities in driving were a source of great annoyance. Dr. Roberts, in laying down the conditions which should be fulfilled by a good photographic telescope, says that a star should remain bisected by a thread in the eyepiece for two minutes at a time. The Crossley telescope was so far from fulfilling this condition that a star would not keep its place for two consecutive seconds; and the greatest alertness on the part of the observer did not suffice to ensure round star images on a photographic plate. It was obvious that the fault did not lie with the driving clock; in fact, many of the sudden jumps in right ascension, if explained in this way, would have required the clock to run backward; nevertheless the clock was tested by causing its revolutions to be recorded on a chronograph at the main Observatory, together with the beats of one of the standard clocks. For this purpose a break-circuit attachment was made by Mr. Palmer. The errors of the clock were in this way found to be quite small.

The principal source of the irregularities was found in the concealed upper differential wheel of the Grubb slow motion. This wheel turned with uncertain friction, sometimes rotating on its axis, and sometimes remaining at rest. After it was checked the driving was much better, and was still farther improved by repairing some defective parts of the train. Small irregularities still remain. They seem to be partly due to inaccuracies in the cutting of the gears, or of the teeth of the large driving wheel, and partly to the springing of the various parts, due to the very

considerable friction of the polar axis in its bearings. The remaining irregularities are so small, however, that they are easily corrected by the screws of the sliding plate-holder, and with reasonable attention on the part of the observer, round star images are obtained with exposures of four hours' duration.

The large mirror, the most important part of the telescope, has an aperture of three feet, and a focal length of 17 feet 6.1 inches. It was made by Mr. Calver. Its figure is excellent. On cutting off the cone of rays from a star, by a knife-edge at the focus, according to the method of Foucault, the illumination of the mirror is very uniform, while the star disks as seen in an ordinary eyepiece are small and almost perfectly round. They are not, I think, quite so good as the images seen with a large refractor; still, they are very good indeed, as the following observations of double stars, made recently for this purpose, will show.

Several close double stars were examined on the night of April 17, 1900, with a power of 620. The seeing was four on a scale of five. The magnitudes and distances of the components, as given in the table, are from recent observations by Professor Hussey with the 36-inch refractor.

Star	Mag.	*d.*	Result of Obs.
OΣ 208 (ϕ *Urs. Maj.*)	5.0, 5.5	0″.35	Not resolved; too bright.
OΣ 249, AB	7.2, 8.0	0.54	Easily resolved.
OΣ 250	7.7, 8.0	0.44	Resolved.
OΣ 267	8.0, 8.2	0.30	Just resolved at best moments.

Although the theoretical limit of resolution for a three-foot aperture is not reached in these observations, I do not think the mirror can do any better.

The small mirror, or flat, at the upper end of the tube, is circular, the diameter being nine inches. Its projection on the plane of the photographic plate is therefore elliptical; but the projection of the mirror and its cell on the plane of the great mirror is very nearly circular.

The small mirror, acting as a central stop, has the effect of diminishing the size of the central disk of the diffraction pattern,

at the expense of an increase in the brightness of the system of rings. To this effect may be due, in part, the inferiority of the reflector for resolving bright doubles, as compared with a refractor of the same aperture. For photographic purposes, it is evident that the mirror is practically perfect.

The upper end of the tube can be rotated, carrying with it the flat and the eye-end. Whenever the position is changed, the mirrors have to be re-collimated. In practice it is seldom necessary to touch the adjusting screws of the mirrors themselves. The adjustment is effected by means of clamping and butting screws on the eye-end, and a change of the line of collimation, with respect to the finders and the circles, is avoided. The operation is generally referred to, however, as an adjustment of the mirrors.

For adjusting the mirrors there are two collimators. One of these is of the form devised by Mr. Crossley.[1] It is very convenient in use, and is sufficiently accurate for the adjustment of the eye-end when the telescope is used for photographic purposes, inasmuch as the exact place where the axis of the large mirror cuts the photographic plate is not then a matter of great importance, so long as it is near the center. Moreover, as stated farther below, the direction of the axis changes during a long exposure. The other collimator is of a form originally due, I think, to Dr. Johnstone Stoney. It consists of a small telescope, which fits the draw-tube at the eye-end. In the focus of the eyepiece are, instead of cross-wires, two adjustable terminals, between which an electric spark can be passed, generated by a small induction machine, like a replenisher, held in the observer's hand. The terminals are at such a distance inside the principal focus of the objective, that the light from the spark, after reflection from the flat, appears to proceed from the center of curvature of the large mirror. The rays are therefore reflected back normally, and form an image of the spark which, when the mirrors are in perfect adjustment, coincides with the spark itself. The precision of this method is very great. It is

[1] *Mon. Not. R. A. S.*, 48, 280, 1888.

in fact out of proportion to the degree of refinement attained in other adjustments of the reflector, for a slight pressure of the hand on the draw-tube, or movement of the telescope to a different altitude, instantly destroys the perfection of the adjustment. I have provided these collimators with an adapter which fits the photographic apparatus, so that one can adjust the mirrors without having to remove this apparatus and substitute for it the ordinary eye-end carrying the eyepieces.

For visual observation the Crossley telescope is provided with seven eyepieces, with powers ranging from 620 downward. The lowest power is only 60, and consequently utilizes only 12 inches of the mirror, 9 of which are covered by the central flat. It is therefore of little value, except for finding purposes. The next lowest power utilizes 28 inches of the mirror. The other eyepieces call for no remark.

But, while the Crossley reflector would doubtless be serviceable for various kinds of visual observations, its photographic applications are regarded as having the most importance, and have been chiefly considered in deciding upon the different changes and improvements which have been made.

The interior of the dome is lighted at night by a large lamp, which is enclosed in a suitable box or lantern, fitted with panes of red glass, and mounted on a portable stand. In order to diffuse the light in the lower part of the dome, where most of the assistant's work is done, the walls are painted bright red; while to prevent reflected light from reaching the photographic plate, the inner surface of the dome itself, the mounting, and the ladders and gallery are painted dead black. The observer is therefore in comparative darkness, and not the slightest fogging of the plate, from the red light below, is produced during a four-hours' exposure. On the few occasions when orthochromatic plates are used, the lamp need not be lighted.

Experiments have shown that the fogging of the photographic plate, during a long exposure, is entirely due to diffuse light from the sky, and is therefore unavoidable. For this reason the cloth curtains which lace to the corners of the telescope

tube, enclosing it and shutting out light from the lower part of the dome, have not been used, since their only effect would be to catch the wind and cause vibrations of the telescope. They would probably have little effect on the definition, and at any rate could not be expected to improve it.

For photographing stars and nebulæ the Crossley reflector is provided with a double-slide plate-holder, of the form invented by Dr. Common.[1] This apparatus, which had suffered considerably in transportation, and from general wear and tear, was thoroughly overhauled by the Observatory instrument-maker. The plates were straightened and the slides refitted. A spring was introduced to oppose the right ascension screw and take up the lost motion — the most annoying defect that such a piece of apparatus can have — and various other improvements were made, as the necessity for them became apparent. They are described in detail farther below.

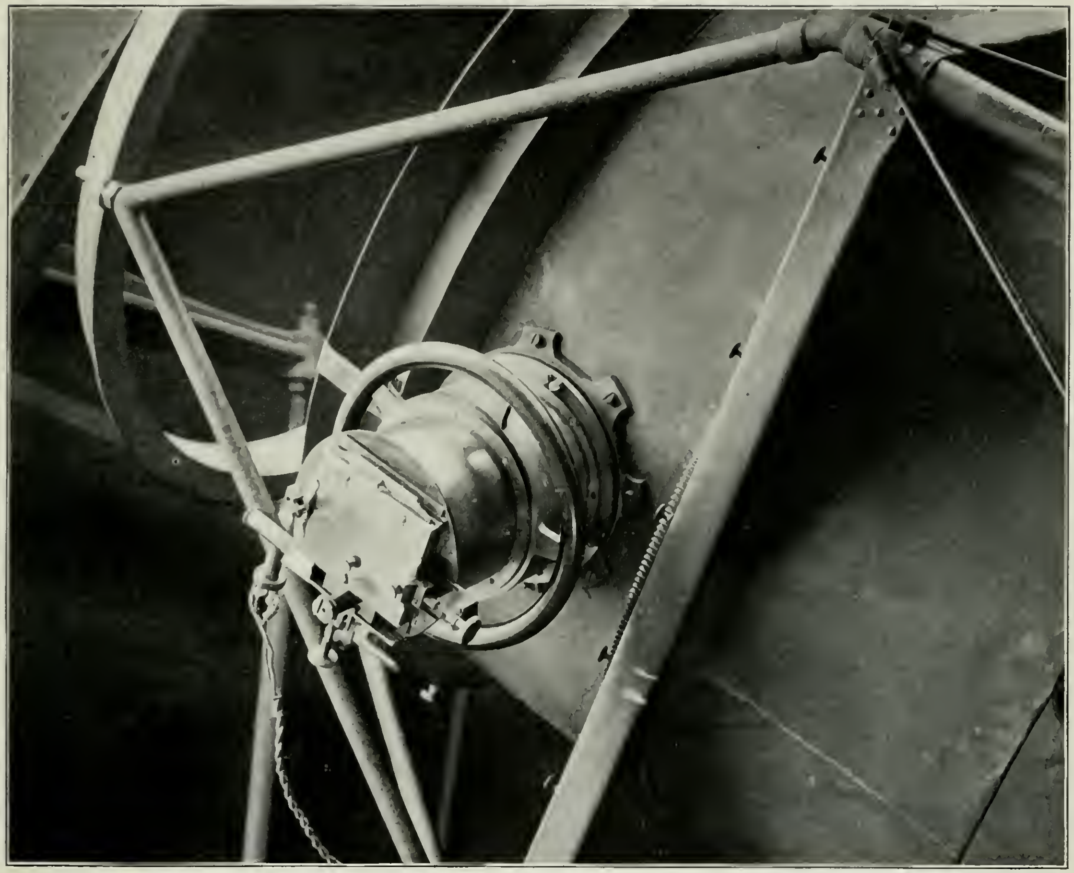
The present appearance of the eye-end is shown in Plate VIII. The plate-holder is there shown, however, on one side of the tube, and its longer side is parallel to the axis of the telescope. This is not a good position for the eye-end, except for short exposures. In practice, the eye-end is always placed on the north or south side of the tube, according as the object photographed is north or south of the zenith. The right ascension slide is then always at right angles to the telescope axis, and the eye-end cannot get into an inaccessible position during a long exposure.

As the original wooden plate-holders were warped, and could not be depended upon to remain in the same position for several hours at a time, they were replaced by new ones of metal, and clamping screws were added, to hold them firmly in place. The heads of these screws are shown in the plate, between the springs which press the plate-holder against its bed.

To illuminate the cross-wires of the guiding eyepiece, a small electric lamp is used, the current for which is brought

[1] *Mon. Not. R. A. S.*, 49, 297. The construction here described is not followed exactly in the Crossley apparatus. The guiding eyepiece slides freely when not held by a clamp. Pin-holes for preventing fogging are unnecessary when red light is used.

PLATE VIII



DOUBLE-SLIDE PLATE HOLDER OF THE CROSSLEY REFLECTOR

down from the storage battery at the main Observatory. The coarse wires have been replaced by spider's webs,[1] and reflectors have been introduced, to illuminate the declination thread. A collimating lens, placed at its principal focal distance from the incandescent filament of the lamp, makes the illumination of the wires nearly independent of their position on the slide, and a piece of red glass, close to the lens, effectually removes all danger of fogging the plate. The light is varied to suit the requirements of observation by rotating the reflector which throws the light in the direction of the eyepiece.

In long exposures it is important for the observer to know at any moment the position of the plate with reference to its central or zero position. For this purpose scales with indexes are attached to both slides; but as they cannot be seen in the dark, and, even if illuminated with red light, could not be read without removing the eye from the guiding eyepiece, I have added two short pins, one of which is attached to the lower side of the right ascension slide, and the other to its guide, so that the points coincide when the scale reads zero. These pins can be felt by the fingers, and with a little practice the observer can tell very closely how far the plate is from its central position. It would not be a very difficult matter to improve on this contrivance, say by placing an illuminated scale, capable of independent adjustment, in the field of the eyepiece, but the pins answer every purpose. The declination slide is changed so little that no means for indicating its position are necessary.

In this apparatus, as originally constructed, the cross-wires of the guiding eyepiece were exactly in the plane of the photographic plate. The earlier observations made with the Crossley reflector on Mt. Hamilton showed that this is not the best position of the cross-wires. The image of a star in the guiding eyepiece, which, when in the middle of its slide, is nearly three inches from the axis of the mirror, is not round, and its shape

[1] It so happens that the tension of the vertical thread is such that it begins to slacken when the temperature falls to within about 2° of the dew point. The thread thus forms an excellent hygrometer, which is constantly under the eye of the observer. When the thread becomes slack, it is time to cover the mirrors.

varies as the eyepiece is pushed in or drawn out. In the plane of the photographic plate (assumed to be accurately in focus), it is a crescent, with the convex side directed toward the center of the plate. This form of image is not suitable for accurate guiding. Outside this position the image changes to an arrow-head, the point of which is directed toward the axis, and this image can be very accurately bisected by the right ascension thread. As the construction of the apparatus did not allow the plane of the cross-wires to be changed, the wooden bed of the plate-holder was cut down, so as to bring the wires and the plate into the proper relative positions.

After some further experience with the instrument, still another change was made in this adjustment. It was found that the focus often changed very perceptibly during a long exposure, and while the arrow-head image above described was suitable for guiding purposes, its form was not greatly affected by changes of focus. Between the crescent and the arrow-head images there is a transition form, in which two well-defined caustic curves in the aberration pattern intersect at an acute angle. The intersection of these caustics offers an excellent mark for the cross-wires, and is at the same time very sensitive to changes of focus, which cause it to travel up or down in the general pattern. The bed of the plate-holder was therefore raised, by facing it with a brass plate of the proper thickness.

Why the focus of the telescope should change during a long exposure is not quite clear. The change is much too great to be accounted for by expansion and contraction of the rods forming the tube, following changes of temperature, while a simple geometrical construction shows that a drooping of the upper end of the tube, increasing the distance of the plate from the (unreflected) axis of the mirror, cannot displace the focus in a direction normal to the plate, if it is assumed that the field is flat. The observed effect is probably due to the fact that the focal surface is not flat, but curved. During a long exposure, the observer keeps the guiding star, and therefore, very approximately, all other stars, in the same positions relatively to the

plate; but he has no control over the position of the axis of the mirror, which, by changes of flexure, wanders irregularly over the field. The position of maximum curvature, therefore, also varies, and with it the focus of the guiding star relatively to the cross-wires, where the focal surface is considerably inclined to the field of view. It is certain that the focus does change considerably, whatever the cause may be, and that the best photographic star images are obtained by keeping the focus of the guiding star unchanged during the exposures. This is done by turning the focusing screw of the eye-end.

In making the photographs of nebulæ for which the Crossley telescope is at present regularly employed, it was at first our practice to adjust the driving-clock as accurately as possible to a sidereal rate, and then, when the star had drifted too far from its original position, on account of changes of rate or of flexure, to bring it back by the right ascension slow motion, the observer either closing the slide of the plate-holder or following the motion of the star as best he could with the right-ascension screw. Lately a more satisfactory method, suggested by Mr. Palmer, has been employed. The slow motion in right ascension is of Grubb's form,[1] and the telescope has two slightly different rates, according to whether the loose wheel is stopped or allowed to turn freely. The driving-clock is adjusted so that one of these rates is too fast, the other too slow. At the beginning of an exposure the wheel is, say, unclamped, and the guiding star begins to drift very slowly toward the left, the observer following it with the screw of the plate-holder. When it has drifted far enough, as indicated by the pins mentioned further above, the wheel is clamped. The star then reverses its motion and begins to drift toward the right; and so on throughout the exposure. The advantages of this method over the one previously employed are, that the star never has to be moved by the slow motion of the telescope, and that its general drift is in a known direction, so that its movements can be anticipated by the observer. In this way photographs are obtained, with four

[1] *Mon. Not. R. A. S.*, 48, 352.

hours' exposure, on which the smallest star disks are almost perfectly round near the center of the plate, and from 2″ to 3″ in diameter.

The star images are practically round over a field at least 1 inch or 16′ in diameter. Farther from the center they become parabolic, but they are quite good over the entire plate, 3¼ by 4¼ inches.

From these statements it will be seen that small irregularities in driving no longer present any difficulties. But certain irregular motions of the image still take place occasionally, and so far it has not been possible entirely to prevent their occurrence.

It was found that the declination clamp (the long slow-motion handle attached to which is shown in Plate VII) was not sufficiently powerful to hold the telescope firmly during a long exposure. A screw clamp was therefore added, which forces the toothed-declination sector strongly against an iron block just behind it, thus restoring, I think, the original arrangement of the declination clamp as designed by Dr. Common. This clamp holds the tube very firmly.

The irregularities to which I have referred consist in sudden and unexpected jumps of the image, which always occur some time after the telescope has passed the meridian. These jumps are sometimes quite large—as much as one sixteenth of an inch or 1′. They are due to two causes; flexure of the tube, and sliding of the mirror on its bed. When the jump is due to sudden changes of flexure, the image moves very quickly, and vibrates before it comes to rest in its new position, and at the same time there is often heard a slight ringing sound from the tension rods of the tube. There seems to be no remedy for the sudden motions of this class. The tension rods are set up as tightly as possible without endangering the threads at their ends or buckling the large corner tubes. A round telescope tube, made of spirally-wound steel ribbon riveted at the crossings, would probably be better than the square tube now in use.

Jumps due to shifting of the mirror are characterized by a gentle, gliding motion. They can be remedied, in part, at least,

by tightening the copper bands which pass around the circumference of the mirror within its cell. This will be done the next time the mirror is resilvered.

All that the observer can do when a jump occurs is to bring back the image as quickly as possible to the intersection of the cross-wires. If all the stars on the plate are faint, no effect will be produced on the photograph; but stars of the eighth magnitude or brighter will leave short trails. The nebula, if there is one on the plate, will, of course, be unaffected.

Before beginning an exposure the focus is adjusted by means of a high-power positive eyepiece. An old negative, from which the film has been partially scraped, is placed in one of the plate-holders, and the film is brought into the common focus of the eyepiece and the great mirror. The appearance of the guiding star, which varies somewhat with the position of the guiding eyepiece on its slide, is then carefully noted, and is kept constant during the exposure by turning, when necessary, the focusing screw of the eye-end. For preliminary adjustments a ground-glass screen is often convenient. On it all the *DM.* stars, and even considerably fainter ones, as well as the nebulæ of Herschel's Class I, are easily visible without a lens.

Plates are backed, not more than a day or two before use, with Carbutt's "Columbian backing," which is an excellent preparation for this purpose. During the exposure the observer and assistant exchange places every half hour, thereby greatly relieving the tediousness of the work, though two exposures of four hours each, in one night, have proved to be too fatiguing for general practice. At the end of the first two hours it is necessary to close the slide and wind the clock.

The brightness of the guiding star is a matter of some importance. If the star is too bright, its glare is annoying; if it is too faint, the effort to see it strains the eye, and changes of focus are not easily recognized. A star of the ninth magnitude is about right. In most cases a suitable star can be found without difficulty.

In such an apparatus as that described above, the amount by which the plate may be allowed to depart from its zero position is subject to a limitation which has not, I think, been pointed out, although it is sufficiently obvious when one's attention has been called to it. It depends upon the fact that the plate necessarily moves as a whole, in a straight line which is tangent to a great circle of the sphere, while the stars move on small circles around the pole. The compensation for drift, when the plate is moved, is therefore exact at the equator only.

Let the guiding star have the declination δ_1, and let a star on the upper edge of the plate (which, when the telescope is north of the zenith, and the eye-end is on the north side of the telescope, will be the southern edge), have the declination δ_2. Then if the guiding star is allowed to drift from its zero position through the distance d, the other star will drift through the distance $d\,\frac{\cos\,\delta_2}{\cos\,\delta_1}$. If the guiding star is followed by turning the right ascension screw, the upper edge of the plate, as well as the guiding eyepiece, will be moved through the distance d. Hence there will be produced an elongation of the upper star, represented by

$$e = d\left(\frac{\cos\,\delta_2}{\cos\,\delta_1} - 1\right)$$

from which $$d = \frac{e\,\cos\,\delta_1}{\cos\,\delta_2 - \cos\,\delta_1}\,.$$

Now, in the Crossley reflector, the upper edge of the plate and the guiding eyepiece are just about $3\frac{2}{3}$ inches, or $1°$, apart. If e is given, the above formula serves to determine the maximum range of the slide for different positions of the telescope.

It has been stated farther above that the smallest star disks, on a good photograph, are sometimes not more than $2''$ in diameter, or in a linear measure, about $\frac{1}{20}$ mm. An elongation of this amount is therefore perceptible. There are many nebulæ in high northern declinations, and there are several particularly fine ones in about $+70°$. If, therefore, we take $\delta_2 = 70°$, $\delta_1 = 71°$, e=0.05, and substitute these values, we find d=1.0 mm, which

is the greatest permissible range of the plate in photographing these nebulæ. Before I realized the stringency of this requirement, by making the above simple computation, I spoiled several otherwise fine negatives by allowing the plate to get too far from the center, thus producing elongated star images.

There is a corresponding elongation in declination, the amount of which can be determined by an adaptation of the formula for reduction to the meridian, but it is practically insensible.

On account of the short focal length of the three-foot mirror, the photographic resolving power of the telescope is much below its optical resolving power. For this reason the photographic images are less sensitive to conditions affecting the seeing than the visual images. On the finest nights the delicate tracery of bright lines or caustic curves in the guiding star is as clear and distinct as in a printed pattern. When the seeing is only fair these delicate details are lost, and only the general form of the image, with its two principal caustics, is seen. A photograph taken on such a night is not, however, perceptibly inferior to one taken when the seeing is perfect. When, however, the image is so blurred that its general form is barely distinguishable, the photographic star disks are likewise blurred and enlarged, and on such nights photographic work is not attempted.

The foregoing account of the small changes which have been made in the Crossley telescope and its accessories may appear to be unnecessarily detailed, yet these small changes have greatly increased the practical efficiency of the instrument, and therefore, small as they are, they are important. Particularly with an instrument of this character, the difference between poor and good results lies in the observance of just such small details as I have described.

At present the Crossley reflector is being used for photographing nebulæ, for which purpose it is very effective. Some nebulæ and clusters, like the great nebula in *Andromeda* and the *Pleiades*, are too large for its plate ($3\frac{1}{4} \times 4\frac{1}{4}$ in.), but the great majority of nebulæ are very much smaller, having a length

of only a few minutes of arc, and a large-scale photograph is required to show them satisfactorily. It is particularly important to have the images of the involved stars as small as they can be made.

Many nebulæ of Herschel's I and II classes are so bright that fairly good photographs can be obtained with exposures of from one to two hours; but the results obtained with full light-action are so superior to these, that longer exposures of $3\frac{1}{2}$ or 4 hours are always preferred. In some exceptional cases, exposures of only a few minutes are sufficient. The amount of detail shown, even in the case of very small nebulæ, is surprising. It is an interesting fact that these photographs confirm (in some cases for the first time) many of the visual observations made with the six-foot reflector of the Earl of Rosse.

Incidentally, in making these photographs, great numbers of new nebulæ have been discovered. The largest number that I have found on any one plate is thirty-one. Eight or ten is not an uncommon number, and few photographs have been obtained which do not reveal the existence of three or four A catalogue of these new objects will be published in due time.

Some of the results obtained with the Crossley reflector, relating chiefly to particular objects of some special interest, have already been published.[1] The photographs have also per-

[1] The following list includes all papers of interest.

"Photographic Observations of Comet I, 1898 (Brooks), made with the Crossley Reflector of the Lick Observatory," *A. J.* No. 451, **19**, 151; see also *Ap. J.*, **8**, 287.

"The Small Bright Nebula near *Merope*," *Pub. A. S. P.*, **10**, 245.

"On Some Photographs of the Great Nebula in *Orion*, taken by means of the Less Refrangible Rays in its Spectrum," *Ap. J.*, **9**, 133. See also *Pub. A. S. P.*, **11**, 70; *Ap. J.*, **10**, 167; *A. N.*, 3601.

"Small Nebulæ discovered with the Crossley Reflector of the Lick Observatory, *Mon. Not. R. A. S.*, **59**, 537.

"The Ring Nebula in *Lyra*," *Ap. J.*, **10**, 193.

"The Annular Nebula H. IV. 13 in *Cygnus*," *Ap. J.*, **10**, 266; see also *Pub. A. S. P.*, **11**, 177.

"On the Predominance of Spiral Forms among the Nebulæ," *A. N.*, 3601.

"The Distribution of Stars in the Cluster *Messier 13* in *Hercules*," (by H. K. Palmer), *Ap. J.*, **10**, 246.

"The Photographic Efficiency of the Crossley Reflector," *Pub. A. S. P.*, **11**, 199; *Observatory*, **22**, 437.

mitted some wider conclusions to be drawn, which are constantly receiving further confirmation as the work progresses. They may be briefly summarized as follows:

1. Many thousands of unrecorded nebulæ exist in the sky. A conservative estimate places the number within reach of the Crossley reflector at about 120,000. The number of nebulæ in our catalogues is but a small fraction of this.

2. These nebulæ exhibit all gradations of apparent size, from the great nebula in *Andromeda* down to an object which is hardly distinguishable from a faint star disk.

3. Most of these nebulæ have a spiral structure.

To these conclusions I may add another, of more restricted significance, though the evidence in favor of it is not yet complete. Among the objects which have been photographed with the Crossley telescope are most of the "double" nebulæ figured in Sir John Herschel's catalogue (*Phil. Trans.*, 1833, Plate XV). The actual nebulæ, as photographed, have almost no resemblance to the figures. They are, in fact, spirals, sometimes of very beautiful and complex structure; and, in any one of the nebulæ, the secondary nucleus of Herschel's figure is either a part of the spiral approaching the main nucleus in brightness, or it cannot be identified with any real part of the object. The significance of this somewhat destructive conclusion lies in the fact that these figures of Herschel have sometimes been regarded as furnishing analogies for the figures which Poincaré has deduced, from theoretical considerations, as being among the possible forms assumed by a rotating fluid mass; in other words, they have been regarded as illustrating an early stage in the development

"New Nebulæ discovered photographically with the Crossley Reflector of the Lick Observatory," *Mon. Not. R. A. S.*, **60**, 128.

"The Spiral Nebula *H. I. 55 Pegasi*," *Ap. J.*, **11**, 1.

"Photographic Observations of Hind's Variable Nebula in *Taurus*, made with the Crossley Reflector of the Lick Observatory," *Mon. Not. R. A. S.*, **60**, 424.

"Use of the Crossley Reflector for Photographic Measurements of Position," *Pub. A. S. P.*, **12**, 73.

"Discovery and Photographic Observations of a New Asteroid 1899 FD.," *A. N.* 3635.

"Elements of Asteroid 1889 FD.," (by H. K. Palmer), *A. N.*, 3635.

of double star systems. The actual conditions of motion in these particular nebulæ, as indicated by the photographs, are obviously very much more complicated than those considered in the theoretical discussion.

While I must leave to others an estimate of the importance of these conclusions, it seems to me that they have a very direct bearing on many, if not all, questions concerning the cosmogony. If, for example, the spiral is the form normally assumed by a contracting nebulous mass, the idea at once suggests itself that the solar system has been evolved from a spiral nebula, while the photographs show that the spiral nebula is not, as a rule, characterized by the simplicity attributed to the contracting mass in the nebular hypothesis. This is a question which has already been taken up by Professor Chamberlin and Mr. Moulton, of the University of Chicago.

The Crossley reflector promises to be useful in a number of fields which are fairly well defined. It is clearly unsuitable for photographing the Moon and planets, and for star charting. On the other hand, it has proved to be of value for finding and photographically observing asteroids whose positions are already approximately known.

One of the most fruitful fields for this instrument is undoubtedly stellar spectroscopy. Little has been done in this field as yet, with the Crossley reflector, but two spectrographs, with which systematic investigations will be made, have nearly been completed by the Observatory instrument maker. One of these, constructed with the aid of a fund given by the late Miss C. W. Bruce, has a train of three 60° prisms and one 30° prism, and an aperture of two inches; the other, which has a single quartz prism, will, I have reason to expect, give measurable, though small, spectra of stars nearly at the limit of vision of the telescope.

The photogravure of the Trifid nebula which accompanies this article (see frontispiece) was made from a photograph taken with the Crossley reflector on July 6, 1899, with an exposure of three hours. It was not selected as a specimen of the work of

the instrument, for the negative was made in the early stages of the experiments that I have described and the star images are not good, but rather on account of the interest of the subject. At the time the photogravures were ordered no large scale photograph of the Trifid nebula had, so far as I am aware, ever been published.[1] The remarkable branching structure of the nebula is fairly well shown in the photogravure, though less distinctly than in the transparency from which it was made. The enlargement, as compared with the original negative, is 2.9 diameters, (1 mm = 13″). The fainter parts of the nebula would be shown more satisfactorily by a longer exposure.

[1] Since then a photograph by Dr. Roberts has appeared in *Knowledge*, **23**, 35, Feb. 1900.

THE PHYSICAL MEANING OF THE STAR-MAGNITUDE.

By R. de Kövesligethy.

By assuming Pogson's numerical value for the constant of the Fechner psycho-physical law, we get the definition of the star-magnitude in the form

$$m - m_0 = -2.5 \log \frac{J}{J_0} \tag{1}$$

which, for stars equal to or fainter than the sixth magnitude, represents tolerably well the connection between the measured intensity J and J_0 and the estimated magnitude m, m_0 of any two stars. This definition, however, involves merely subjective physiological elements, inasmuch as the visual intensity depends not only upon the objective continuous spectrum of the star, but varies also with the limiting wave-lengths of perception and with the sensibility of the eye, which is itself a function of the wave-length. Thus even for another suitable choice of the Pogson constant the differences of magnitude seem altered when the perceiving medium of the spectrum is changed; *e. g.*, when the photographic film is used instead of the normal eye.

The question of a physical definition of the star magnitude is therefore still open, though in many cases it might be of considerable value, especially for the estimation of processes operating in new stars. Of course I do not here allude to the fact sometimes invoked in investigations of the dimensions of the stellar system, that stars of equal intensity seem fainter by *one* magnitude if removed to a 1.585-fold distance, but I seek the changes in the physical condition of a star whose brightness has varied by *one* magnitude.

Let

$$i = \Lambda f(\lambda, \mu) \tag{2}$$

be the intensity of a continuous spectrum between the wave-lengths λ and $\lambda + d\lambda$. Then the visual intensity becomes

$$I = \Lambda \int_{\lambda_1}^{\lambda_2} s f(\lambda, \mu) \, d\lambda, \tag{3}$$

where the sensibility of the eye, s, is a function of the wave-length, while λ_1 and λ_2 denote the limits of the visual spectrum. The function $\Lambda f(\lambda, \mu)$ contains at least two independent parameters, as I have shown elsewhere[1] (since it would lead to an equation of dispersion without any constant). Let Λ be the total intensity of the spectrum and μ the wave-length of the maximum of intensity. Thus we have

$$\int_0^\infty f(\lambda, \mu)\, d\lambda = i .$$

The analytical form of the sensibility function s is quite unknown (for geometrical representation, see this JOURNAL, **11**, 18–22). Numerically it might be determined by comparisons of the visual intensity with energy observations made with an absolutely black bolometer. Its differential equations could be measured in a simpler way by the probable error with which a homogeneous light-field of given wave-length can be inserted on the appropriate place of a continuous spectrum. Here we can get on without the explicit expression of the sensibility, inasmuch as a mean value of it between the limits of the spectrum may be placed as a factor before the integral. Thus the visual intensity becomes

$$I = \sigma \Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda\ , \tag{4}$$

and σ remains constant for all stars of the same spectral composition, that is of the same type, while it varies from type to type.

This variation is very little, however, and, beyond the limits of exactness of photometric measures, may be neglected in practice. Indeed all photometric measures, even of differently colored stars, are usually made by comparison with a few standard stars, and likewise the same color of comparison star was always used in the *Potsdamer Durchmusterung*, being about equal to the mean color of the second stellar type.

[1] *Monthly Notices*, 58, No. 3, 115; *Grundzüge einer theoretischen Spektralanalyse* Halle a/S. 1890, p. 151.

Thus we become—at least in practice—independent of the unknown coefficient σ, and the Pogson formula gives for the same star whose magnitude changed by *one* order ($m = m_o + 1$):

$$1 = -2.5 \log \frac{\Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda}{\Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda}, \tag{5}$$

as conditional equation for the changes necessary in Λ_o and μ_o to produce such effect. Taking into consideration what we have said of the sensibility factor, this equation holds good, even if the change of light was accompanied by a variation of the color amounting to an interval of two types.

The two parameters of the spectrum, Λ and μ, are functions of the temperature and the density,[1] and, though known functions, do not allow a uniform solution of the above equation. The manner in which the temperature and density changed remains in every case quite arbitrary, and thus a uniform physical definition of the star magnitude cannot be given.

The question becomes resolvable in a uniform manner, however, if we assume that the radiating body is an absolutely black one. Inasmuch as the superficial layer, which produces the greatest part of the continuous spectrum, may be regarded thick enough, our hypothesis holds also good for the fixed stars.

Now we know two analytical expressions for the continuous spectrum; the one generally applicable for any body is by the author:[2]

$$i = \frac{4}{\pi} \mu \Lambda \frac{\lambda^2}{(\lambda^2 + \mu^2)^2}; \tag{6}$$

the other, which refers only to absolutely black bodies, is that of Wien-Paschen,[3] and may be written in the form

$$i = \frac{5^4 \mu^4}{6} \Lambda \lambda^{-5} e^{-\frac{5\mu}{\lambda}}. \tag{6'}$$

[1] "Ueber die beiden Parametergleichungen der Spektralanalyse," *Math. u. Naturw. Ber. aus Ungarn*, 16, 1–49.

[2] *Grundzüge e. theor. Spektralanalyse*, p. 157.

[3] This JOURNAL, 10, 40.

For an absolutely black body, Λ obeys the Stefan law of radiation, while μ is given by the Wien formula. If θ denotes the absolute temperature, then

$$\frac{\Lambda}{\Lambda_0} = \frac{\theta^4}{\theta_0^4}\,, \text{ and } \mu\theta = 2880\,, \tag{7}$$

provided that μ is given in thousandth parts of the millimeter. It may be said, in brief, that I was familiar with this ten years before the publication of Wien's *Verschiebungsgesetz*, and that I determined its constant provisionally by the observations of G. Müller[1] on the absorption of the air, in a most troublesome way, as 2088. Similarly I used this equation and the value of the constant for the determination of the temperature of the Sun's chromosphere. It gave 1800°, or, with the present value of the constant, 2475° of the absolute scale, being 1.1631 the wave-length of the maximum intensity of an absolutely black body having the same temperature as the chromosphere.[2]

Putting now

$$F_1(\mu) = \frac{4}{\pi}\mu\int_{\lambda_1}^{\lambda_2}\frac{\lambda^2}{(\lambda^2+\mu^2)^2}d\lambda$$

$$= \frac{2}{\pi}\left[\text{arc tang}\,\frac{\mu(\lambda_2-\lambda_1)}{\mu^2+\lambda_1\lambda_2} - \mu\,\frac{(\lambda_2-\lambda_1)(\mu^2-\lambda_1\lambda_2)}{(\lambda_1^2+\mu^2)(\lambda_2^2+\mu^2)}\right]\,, \tag{8}$$

and

$$\frac{\lambda_1}{\mu} = \text{tang}\,\phi_1\,;\ \frac{\lambda_2}{\mu} = \text{tang}\,\phi_2\,,$$

we get in a more simple form

$$F_1(\mu) = \frac{2}{\pi}\Big[(\phi_2-\phi_1) - \sin(\phi_2-\phi_1)\cos(\phi_2+\phi_1)\Big]\,,$$

and similarly in the other equation

$$F_2(\mu) = \frac{5^4\mu^4}{6}\int_{\lambda_1}^{\lambda_2}\lambda^{-5}e^{-\frac{5\mu}{\lambda}}d\lambda$$

$$= \frac{125}{6}\left\{e^{-\frac{5\mu}{\lambda_2}}\left(\frac{\mu^3}{\lambda_2^3}+\frac{3\mu^2}{5\lambda_2^2}+\frac{6\mu}{25\lambda_2}+\frac{6}{125}\right)\right.$$

$$\left.-\,e^{-\frac{5\mu}{\lambda_1}}\left(\frac{\mu^3}{\lambda_1^3}+\frac{3\mu^2}{5\lambda_1^2}+\frac{6\mu}{25\lambda_1}+\frac{6}{125}\right)\right\}\,, \tag{8'}$$

[1] *Astr. Nachr.*, **103**, 241, 1882. [2] *Grundzüge*, p. 185.

we have generally

$$I = \Lambda F(\mu) \,, \tag{9}$$

and thus the Pogson equation (5) becomes

$$\log \mu - \frac{1}{4} \log F(\mu) = 0.1 + \log \mu_o - \frac{1}{4} \log F(\mu_o) \,. \tag{10}$$

Thus if we know the average values of μ_o for the individual stellar types we may calculate the variations in μ necessary to produce a change from *one* magnitude in the luminosity of the star. Early spectral-photometric observations of thirty-four fixed stars[1] led me to the following values, corrected for the effect of the atmosphere:

Type of the star:		I.	II.	III.	
Average μ_o	=	0.45	0.53	0.60,	(11)

which give, according to the two hypotheses made for $F_1(\mu)$ and $F_2(\mu)$, the limits of the visual spectrum being assumed at $\lambda_1 = 0.39$ and $\lambda_2 = 0.76$, the equations

I. type ;	$\log \mu - \frac{1}{4} \log F_1(\mu)$		$+0.0923 = 0$	and	$\log \mu - \frac{1}{4} \log F_2(\mu)$	$+0.1679 = 0$	
II.	"	"	$+0.0082 = 0$	"	"	$+0.0881 = 0$	(12)
III.	"	"	$-0.0592 = 0$	"	"	$+0.0200 = 0$;	

The solutions for the three types are, accordingly,

I. type	$\mu = 0.546$	and	$\mu = 0.551$	
II.	0.636		0.633	(13)
III.	0.715		0.706,	

where the first values refer to the first hypothesis for $F_1(\mu)$, the other column to the second hypothesis $F_2(\mu)$.

For facilitating the solution of (10) we give here a short numerical table of the function $F_1(\mu)$ and $F_2(\mu)$.

μ	Log $F_1(\mu)$	Log $F_2(\mu)$	μ	Log $F_1(\mu)$	Log $F_2(\mu)$
0.40	9.4073	9.6827	0.65	9.2354	9.5412
45	3821	6842	70	1931	4818
50	3507	6671	75	1499	4162
55	3150	6357	80	9.1066	9.3454
60	9.2762	9.5930			

The part of the energy falling in the visual spectrum is considerably greater if the spectrum is assumed in the exponential form instead of the algebraic form. But in both cases the

[1] *Beobacht. angest. am astrophys. Observ. O-Gyalla*, 9, 21–41, 1888.

intensity vanishes for $\mu=0$ and $\mu=\infty$, and reaches a maximum value for $\mu=0.309$ and $\mu=0.429$ respectively. These values, therefore, correspond to the impression of the purest white in the mixed light of the source. I remark that the gradual increasing and the subsequent decreasing of the intensity with increasing temperature is proved, at least in single cases, by observations of Lucas.

In virtue of the second part of equation (7), the solutions (13) correspond to the following temperatures:

Type.	θ_0	θ_1	θ_2	$\frac{\theta_0}{\theta_1}$	$\frac{\theta_0}{\theta_2}$	
I.	6400°	5274°	5230°	1.213	1.224	
II.	5434	4527	4547	1.200	1.195	(14)
III.	4799	4025	4080	1.191	1.176	
		Mean of temperature ratio:		1.201	1.198.	

Therefore, if a white star changes in magnitude by *one* order, it is the same as if its superficial temperature fell from 6400° to 5274° or to 5230° respectively. The temperatures in the two hypotheses are almost identical, and the ratio of them is also sensibly the same, but shows a slight variation from type to type. If we therefore assume on the two hypotheses that a variation of *one* magnitude corresponds to an increase of temperature in the ratio of 1 : 1.201 or of 1 : 198, the equation (1) may be written in this form:

$$m-m_0=-12.58\log\frac{\theta}{\theta_0},\text{and } m-m_0=-12.74\log\frac{\theta}{\theta_0}, \quad (15)$$

and the assumption of an average ratio gives for the stars of the first and third type errors of 0.056 and 0.044, of 0.118 and 0.103 mag., respectively, which on my hypothesis do not reach, on the other scarcely exceed, the limits of exactness of photometric measures.

We have, then, the following theorem:

When the light of a star in form of a point, of nearly absolute black character, increases by one magnitude, then, independently of the color of its light, its superficial temperature has lessened by 20 per cent.

We may further remark, that the "superficial" temperatures (*i. e.*, temperature of the layer, which produces the principal part of the continuous spectrum) given in (14) under θ_0 represent lower limits. For if any body and an absolutely black body have the same wave-length of intensity maximum, the latter is always of lower temperature.

The above investigation may be of some use in its application to new stars. The star *T Coronae* increased by three magnitudes in about two and one half hours; that is to say, during this time its temperature was elevated about $0^\circ.5$ per second, a most probable quantity, whether we assign the apparition to a collision with a cosmic cloud or to an eruption of the still fluid inner magma.

O-Gyalla Observatory,
February 1900.

ON THE ESCAPE OF GASES FROM PLANETARY ATMOSPHERES ACCORDING TO THE KINETIC THEORY.

By G. Johnstone Stoney.

PART II. MORE DETAILED EXAMINATION.

Mr. Cook in America[1] and Professor Bryan in England[2] have endeavored to determine the rate of outflow of gaseous molecules from atmospheres, by regarding as applicable to that part of the atmosphere from which the outflow takes place, probability laws which refer to the distribution of the speeds of the molecules in a portion of uniform gas surrounded by similar gas. In Part I of the present paper some general considerations have been adduced tending to show that this use of the aforesaid laws is not legitimate; and in the following pages it is proposed to institute a closer scrutiny, of such a kind as will bring the assumptions involved fully into view, and exhibit in a clear light the contrast between them and what is really going on in nature. In order to do this it will be convenient to inquire successively into the molecular velocities of the lower and of the upper regions of the atmosphere.

MOLECULAR MOVEMENTS IN THE LOWER STRATA OF THE ATMOSPHERE.

1. Maxwell's law and its successors the Boltzmann-Maxwell law, and the law with the additional term introduced by Professor Bryan, are probability laws, and as such depend essentially on the solution of a functional equation. This method of proof does not ascertain that a law of the kind exists, but only that if such a law had existed under the conditions assumed in the proof, it would have been that law which emerges when we solve the functional equation. The question as to whether a law of

[1] See the January number of this Journal.

[2] See the *British Association Reports* for 1893 and 1899; and the *Proceedings of the Royal Society* for April 1900.

the kind exists is one which the proof of the probability law gives no answer. It is there assumed.

2. This is consistent with what we know of such laws, that *the law is not really complied with* for any finite value whatever of N, using N for the number of events whose distribution is under consideration. But the probability law is nevertheless of value whenever it can be shown that for values of N above a certain limit N_0, the actual distribution is unlikely to differ much from that laid down by the law. We can assure ourselves that this is the case in many instances. We find, however, that the number N_0—the smallest number of events whose distribution will exhibit a sufficient approach to the law to be of use—has very different values in different problems. In some it is a moderate number; while in others (including, as we shall see, the case of molecular speeds) it is an enormously large number.

3. In some few instances the value of N_0 may be foreseen; as when the law concerns the throwing of dice. Here the probability of each individual event is known, the causes of the events are alike with respect to the chance as to which of certain definite possible events shall follow, and we have all the data required to compute the value of N_0 which suffices to render probable any desired degree of approximation between the actual distribution and that furnished by the probability law.

But in most cases we have to find N_0 by experiment. Thus, if the matter to be investigated is the distribution of shots on a target, we must have recourse to experiment to determine what number N_0 of shots will have to be fired under given conditions in order that it may be probable that the actual distribution shall sufficiently accord with that indicated by the law; and if we vary the conditions the number N_0 is found to vary.

4. In both these cases the individual events are independent of one another, and their causes are alike. Where this is so, the number N_0 is usually not a very large number. But when the problem has reference to the distribution of the speeds with which gaseous molecules pursue their free paths, the individual events neither are independent nor are their causes alike with

regard to the probability of the consequences that will ensue. Each speed is the outcome of an intricate web of preceding speeds and of associated events which are all the more complex because of exchanges which take place during the encounters between the kinetic energy of the molecules regarded as missiles and the energy of those wholly different internal events which influence and are influenced by heat radiations and other kinds of electrical activity. The many causes which have thus to combine to develop one of the molecular speeds may, and frequently do, coöperate to produce in it an exaggerated effect. *There is, accordingly, a continual recrudescence of abnormal speeds always springing up within a gas.*

5. The frequency of combinations that produce exaggerated effects is consequent upon the marvelous rapidity and complex variety with which encounters succeed one another in the career of every molecule. Thus the number of encounters met with by each molecule of the air about us at the bottom of the atmosphere is some seven or eight millions in every thousandth of a second of time, according to Maxwell's determinations. It follows from this that a combination of events as unusual as the throwing of doublets thirteen times in succession upon unloaded dice, the probability of which is less than 1 in 13,000 millions, does on the average determine the speed of *every* molecule of the air about us rather more frequently than once every two seconds.

A most instructive experiment which places in a clear light (*a*) the frequent occurrence of exceptional events in gases, (*b*) the fact that they may reach every molecule, and (*c*) the conspicuousness of their effects, may be made by exposing one mixture of equal volumes of hydrogen and chlorine to diffused light and another to direct sunshine. In diffused light the gases slowly combine into hydrochloric acid gas, in other words, the exchange of chemical atoms between two encountering molecules is, in diffused light, consequent upon some very unusual kind of encounter which occurs excessively rarely from the molecular point of view. In intense light this kind of

encounter is still infrequent from the molecular standpoint, but so much less so than before that the combination of the gases now seems to our coarse human senses to take place with explosive rapidity. In either case, after the lapse of a sufficient time the reaction is complete; every molecule is now hydrochloric acid, *i. e.*, the very unusual kind of encounter has reached *every* molecule of the hydrogen and chlorine which was originally present.

6. It thus appears that the speed of a molecule in one of its free paths is the outcome of an intricate body of causes, some of which are preceding molecular speeds in the gas, and others are agencies of a different character; and further that these various factors sometimes find themselves so interwoven that they produce exaggerated effects. When in this way an abnormal speed has been developed in a molecule, the subsequent encounters of that molecule — which at the bottom of the atmosphere no molecule is able to avoid — tend in almost every case to tone down the irregularity. Now this toning down must have been effectual upon a sufficient majority of the irregularities that present themselves, in order that the distribution of the speeds may approximately conform to such a law as Maxwell's. This requires a very large number of molecular speeds to be embraced in our survey; so that in gases, N_0 is likely to be a vastly higher number than in the simple cases with which we are more familiar, such as those referred to in paragraph (3). This inference is borne out by experiment.

7. Let ΔV be a small volume of air under standard temperature and pressure, containing n molecules and surrounded by air subjected to the same conditions; and let Δt be a small duration such that in it each molecule on the average meets with n' encounters: then will

$$N = n \, . \, n'$$

be the number of free-paths described in the time Δt, by the molecules which were, at the beginning of that time, within the volume ΔV.

We have to find by experiment the value of N_0, *i. e.*, the smallest value which N can have consistently with a recognizable

approximation between the actual distribution of speeds, and that indicated by the probability law. Now several experiments, both in ordinary air and in attenuated air, seem to show that the limiting number N_0 is approached in ordinary dry air (which consists mainly of nitrogen, oxygen and argon)

when ΔV is the cube of 0.1 of a millimeter,
and Δt is the $\frac{1}{25}$ of a second of time.

We have next to turn these into the corresponding number of free paths. The estimates and determinations of the number of molecules present in a gas under standard temperature and pressure, differ a good deal from one another, but may all be summed up in the statement that

> In a cubic centimeter of gas under standard temperature and pressure there are —
>
> *several* trillions of molecules;

where we use the word "several" in a special and very convenient sense to mean a number which differs in the various determinations and the precise value of which is, therefore, not known, but which certainly stands somewhere between the limits 10 and 1000.

For our present purpose we shall find it convenient to extend this statement to the air around us at the bottom of the atmosphere, whatever the indications of the thermometer and barometer for the time being may be. This can be effected by slightly enlarging the range within which the number represented by "several" is to be sought. When thus modified the statement becomes

> In air or gas, subjected to any of the temperatures and pressures we have experience of at the bottom of the atmosphere, there are in each cubic centimeter —
>
> *several* trillions of molecules;

where "several" stands for some number between 8 and 1100.

Hence in ΔV (ΔV being the cube of the tenth of a millimeter) the number of molecules of ordinary air is

$$n = \textit{several} \text{ billions.}$$

Again, as each molecule in atmospheric air under, or nearly under, standard conditions, meets with about 7000 or 8000 millions of encounters per second (see Maxwell's paper), it follows that in the time Δt ($\frac{1}{25}$ of a second) each molecule of air at the bottom of the atmosphere meets with about

$$n' = 300 \text{ millions of encounters.}$$

Hence the number of free paths in the time Δt, of the molecules that at the commencement of that time were within the volume ΔV, is about—

$$N_0 = n \cdot n' = \textit{several} \text{ times } 300 \text{ trillions,}$$

where "several" means a number between 8 and 1100, the value of which is not more exactly known.

8. This then seems to bring us into the neighborhood of the number of individual molecular speeds which must be included in our survey, in order that the distribution of these speeds at the bottom of our atmosphere, and in the $\frac{1}{25}$ of a second, may approximate to obeying a probability law such as the Maxwell law, the Boltzmann-Maxwell law, or the Bryan-Boltzmann-Maxwell law. No doubt, if we assign a different value to Δt, we may get a somewhat different value for N_0; but it will always be an immense number. This is because the assumption that is made in the proof of Maxwell's law — that the proportion of molecules whose speed lies between v and $v + dv$, is dv multiplied by a function of v only — is too great a simplification of the far more complex function which it really is.[1]

9. The state of affairs at the bottom of the atmosphere will perhaps be more fully apprehended, if we represent it to ourselves under a symbolical form.

Out of N free paths, the actual number whose speed lies between v and $v + dv$

$$= N(\pi + \delta)\, dv$$

where π is the probability function — Maxwell's law or one of

[1] See the first few pages of a paper on "The Kinetic Theory of Gas," by the present writer, in the *Proceedings of the Royal Dublin Society* for June 26, 1895, or in the *Philosophical Magazine* for October 1895.

the others — and δ is the function which represents the deviation of the actual from the computed distribution.

Now π is assumed by Maxwell to be a function of v only, and similar assumptions are made with reference to the other laws ; from which it follows that δ must be a very complex function of many variables — v, n, n', θ, τ, etc., where v is the speed, n the number of molecules in ΔV, n' the number of encounters which each molecule meets with in the time Δt, θ the average duration of a free path, τ the average duration of an encounter, and where etc. stands for other variables depending on whether the encounters are between molecules of the same or of different gases, and on the events which happen during encounters — which in turn depend on the kind of gas or gases, and on the electro-magnetic waves or other electrical agencies operating through the ether. It is essential that those variables shall be included which when suitably combined will furnish its function δ whatever is the proper mathematical expression of the *cumulative* effect which arises when opportunity offers. Cumulative effects are partly a consequence of the influence which the velocities exercise over all their successors, and partly due to the varying foreign agencies which also intervene. Whenever the many factors more or less coalesce, then there emerges a more or less cumulative effect. Now we can conceive this to be expressed mathematically in function δ, although nothing that any mathematician could write down would at all adequately represent it. In such cases we must have recourse to experiment or observation to discover what, at any given time, is the actual outcome of function δ. Moreover, this function δ is not fixed : it fluctuates, and has had a very different value at some times from what it has had at others. In the present state of the Earth it is larger in sunshine than in shade ; perhaps markedly different during a thunderstorm or when there is an auroral display from what it is at other times ; probably sensitive to all the influences which cause upheavals of molecules into stratum V[1] ; and so on.

[1] See paragraph 12, p. 364

When the events whose distribution is being studied are so much alike as they are in the problems spoken of in paragraph (3), function δ is little more than a mere function of V and N, and one which dwindles away in value according as N increases; but where the events whose distribution is sought are due to complex, differing, and fluctuating causes, function δ may hold a much more conspicuous position in its relation to function π, and one which is likely to become predominant for certain values of v, especially at certain times.

10. The general lesson which we learn from this way of presenting the subject is that probability laws, such as Maxwell's, when applied to the distribution of molecular speeds, may be useful guides in reference to those values of v, which correspond to large values of π; but should not be trusted, even in the lower strata of the atmosphere, in reference to the values of v which assign small values to π. Now it is only with these very values of v that we are concerned when studying the possibility of the escape of gas from an atmosphere.

11. At the bottom of the atmosphere, where δV is a volume of air surrounded by air at almost exactly the same temperature and pressure, there is practically no chance that an irregularly moving molecule can avoid that torrent of subsequent encounters which is what in the air about us tones down and as it were swamps all irregularities, and brings about such an approach towards a *general* conformity with Maxwell's law as is possible; but the essentially different environment in which a molecule finds itself in the upper reaches of the atmosphere, provide it with new opportunities, to the study of which we have now to invite attention.

MOLECULAR MOVEMENTS IN THE UPPER STRATA OF THE ATMOSPHERE.

12. *Definitions.*—It is convenient to picture the atmosphere as consisting of 26 strata distinguished from one another by the letters of the alphabet, A being the stratum at the bottom in which we live and Z being the stratum at the top.

When we describe a molecule which has left the Earth as having escaped from a specified stratum, what is to be understood is that it was within that stratum that the molecule met with its last gaseous encounter, whereupon it ceased to be a molecule of gas, and started on its new career as a body traveling by itself through space.

13. Now it is manifest that the escape of gas from an atmosphere can only take effect from its upper regions, since a molecule situated low down has no appreciable chance of making its way outwards without meeting with encounters on the way. In fact it will presently appear that we can definitely locate the escape as coming exclusively from three strata which we shall call the ultimate stratum of the atmosphere Z; next to it the penultimate stratum Y; and farther down the ante-penultimate stratum X. Beneath these come the other strata of the atmosphere, from A to W, from none of which has a gaseous molecule any prospect of being able to make its way outwards without encountering some other molecule.

14. *Definitions of these three strata.* — Stratum Z, at the boundary of the atmosphere, *is characterized by an almost total absence of gaseous encounters* within it. The molecules that tenant it have been thrown up from below, and are so separate that they scarcely ever come across one another while within that stratum. Most of them describe elliptic trajectories relatively to the Earth, and fall back into the penultimate stratum: if any have risen into it in hyperbolic trajectories these leave the Earth altogether: so may some that rise into it in elliptic trajectories if they are sufficiently long to carry the molecule within the perturbating influences of the Sun or Moon.

Next under stratum Z comes the penultimate stratum Y throughout which encounters do take place, and which is characterized by the circumstance that *the molecules which occupy it are all within effectual striking distance of stratum Z* and of the space beyond. By the molecule's being within effectual striking distance of the ultimate stratum and of the void beyond, is meant that if a molecule after an encounter anywhere within stratum Y,

finds itself traveling with suitable velocity as regards speed and direction, it has, in the upper ranges of stratum Y a great, in its middle region a moderate, and in its lowest portion an appreciable chance of escaping outwards *without meeting with any further encounter* within stratum Y.

The rest of that part of the atmosphere from which molecules sometimes escape lies farther down than the penultimate stratum, and may be called the ante-penultimate stratum X. Molecules after an encounter within this stratum, whatever their speed and direction, have but a slender prospect of avoiding further encounters; but *as this does sometimes happen*, though not often, stratum X must be included among those from which the escape of gas is possible.

15. These three strata are all of great depth, by reason of the attenuation of the air at great altitudes: they also obviously pass insensibly into one another, but may nevertheless be as distinct in the atmosphere as are the chin, the cheek, the temple and the forehead of the human countenance, which like them have no definite lines of demarcation.

16. Now it is apparent from the definitions of these strata, that *nearly the whole of the actual escape* of gas from the Earth consists of molecules which met with their last encounter within the penultimate stratum. An altogether trifling proportion have emanated from the adjoining strata above and below — few from the ultimate stratum because there are few encounters there, and few from the ante-penultimate stratum because few molecules that have encountered within this stratum can escape a subsequent encounter; and practically none come directly from any of the strata that lie farther down. Accordingly what we have to do is to consider what velocities may arise within the penultimate stratum. In doing this we shall have to bear in mind that a molecule within that stratum moving swiftly outward is traveling toward regions where there are fewer and fewer encounters, so much so that it may get off without meeting with any encounter; while a molecule within the stratum that is traveling swiftly inward is plunging into regions where encounters are inevitable.

17. It would be rash to guess how many miles deep this penultimate stratum is, though we can see that it is of great depth. And if we make Δt 4 or 5 seconds, which is a reasonable value to assign to it in view of the time which a molecule with the requisite speed would take to escape, the corresponding ΔV, in order to provide a sufficient number of free paths during the time Δt, of the molecules that at the beginning of that time were within the space ΔV, may on a probable hypothesis be a cube of some 10 or 20 kilometers, *i. e.*, a cube of from 6¼ to 12½ miles. This then may be taken as about the smallest volume within which any law can be recognized governing the distribution of the speeds that occur within it, in periods of time as short as Δt. And that law, if we could discover it, would doubtless be *utterly unlike Maxwell's law* or either of its successors. It would be a resultant of distinct laws belonging to the different directions, in which the behavior of molecules traveling outward is discriminated from the behavior of molecules traveling inward, and in which a due allowance has been made for the circumstance that the molecules of the lighter gases whenever a sufficient speed is engendered in them, are likely, if the direction of their flight is outward, to place themselves beyond the reach of all subsequent encounters; thus destroying in their case the machinery which lower down in the atmosphere brings about a partial conformity to such a law as Maxwell's.

18. Another and very important factor which would need to be taken into account in determining the distribution of speeds at great elevations in the atmosphere, is that the molecules in those regions are exposed to the full glare of a sunshine which is not moderated by having passed through the Earth's atmosphere, and that whatever internal activity is thereby excited within the molecules during their comparatively long flights, will on their next encounter contribute energy towards the speed with which the two colliding molecules will fling asunder. How much energy may be introduced in this way is revealed to us by the intensely dark terrestrial lines of oxygen in the solar

spectrum, and by the very remarkable fact that all radiations from the Sun of wave-lengths less than 0.3 of a micron, are completely taken up and intercepted by the upper portions of our atmosphere, so that none of them reach its lower strata. It is also brought home to us by the hydrochloric experiment cited above in paragraph 5, where unusual events become much more numerous in direct sunshine.

19. In view of all these facts the present writer came to the conclusion more than thirty years ago (*a*) that *it is a mistake* to suppose that *Maxwell's* law governs the distribution of speeds in that upper portion of the atmosphere from which alone molecules escape; and (*b*) that the true law of distribution, whatever it is, has but a partial connection with the rate of outflow (1) on account of the size which has to be attributed to ΔV, (2) on account of the very different behavior of outflowing and inflowing molecules, (3) on account of the magnitude and fluctuating character of the deviation function δ, described in paragraph 9, and (4) on account of the opportunity which happens now and then to each molecule of the lighter gases, of withdrawing from those subsequent encounters which would keep it within the jurisdiction of the law.

20. My first attempt was to try whether the rate of escape can be determined by the help of a law of the same kind as Maxwell's, in which the vectors as well as the scalar values of the velocities were taken into account. But this, too, proving to be insufficient, I turned to the *a posteriori* method of investigation, and sought to gain some insight into what the outflow has actually been, by studying its effects. The observations principally made use of are

1. That the Moon is now without an atmosphere;
2. That the Earth has been able to retain the vapor of water in its atmosphere;

both of which are data that may be depended upon.

21. It would be very satisfactory if we could also investigate the problem in the more *a priori* way that has been attempted by Messrs. Cook and Bryan; but for the reasons

stated above this seems to the present writer to be quite beyond the reach of the Molecular Physics we as yet know.

APPENDIX.

ON THE BEHAVIOR OF HELIUM IN THE EARTH'S ATMOSPHERE.

The questions whether helium in former times escaped from the Earth, and whether it is still escaping, have a double interest—they are of interest in themselves, and of interest because they seem to answer the further question whether we are to attribute the polar snows of *Mars* to water or to carbon dioxide.

The attempts hitherto made to obtain *a priori* answers to questions of this kind from the kinetic theory of gas, have, as appears from the discussion of them in the preceding pages, afforded no real light. They have in fact had to be deductions from illegitimate premises, based upon the assumption that the events as they occur in nature are far simpler than they really are. Nor does a valid investigation upon these lines seem even possible; the knowledge we as yet possess of molecular physics is too fragmentary and defective to deal adequately with the problem, and our acquaintance with the penultimate stratum of the atmosphere, which is the situation from which almost the whole of the real escape takes place, is imperfect.

It becomes then of importance to proceed by the other possible method, the *a posteriori* method, which rests upon such data as are furnished by experiments, *i. e.*, by observation of the actual conditions under which helium exists upon the Earth. The facts brought to our knowledge in this way appear to indicate that helium is still slowly escaping from the Earth, and *a fortiori* was able to escape more freely in earlier cosmical ages. The data upon which this conclusion rests are now fuller and more definitely known than they were when the present writer ventured, some years ago, to put forward the view as probable. They seem to change what was then probability into what is now almost certainty. These more precise data have been generously furnished to me by my friend, Professor Ramsay, and are as follows:

1. The proportion by volume of argon in dry atmospheric air is about 1 per cent. of the whole; the volume of neon (to which the present notes will not further refer) may be taken as about a thousandth part of the volume of argon, and the volume of helium as about $\frac{1}{10}$ to $\frac{1}{20}$ of the volume of neon. Accordingly the volume of helium in dry air is something like from $\frac{1}{10000}$ to $\frac{1}{20000}$ of the volume of argon.

2. Both argon and helium are supplied to the atmosphere by hot springs; argon generally by all hot springs which contain atmospheric gases, and helium by some of them.

3. The argon in such springs, like the oxygen and nitrogen, may be simply gas which had previously been removed from the atmosphere by water. A liter of water under ordinary conditions will absorb:

about 45 cc of the oxygen of the air in contact with it,
about 15 cc of its nitrogen,
about 40 cc of its argon,[1]
about 14 cc of its helium.[1]

Hence in rain we should expect to find the following proportions preserved:

$$\frac{20.9}{100} \times 4.5 \text{ of } O_2$$

$$\frac{78.1}{100} \times 1.5 \text{ of } N_2$$

$$\frac{1}{100} \times 4 \text{ of } A$$

$$\left.\begin{array}{ll} \text{and from} & \dfrac{1}{1,000,000} \times 1.4 \\ \text{to} & \dfrac{1}{2,000,000} \times 1.4 \end{array}\right\} \text{ of } He$$

So far as oxygen, nitrogen, and argon are concerned, these proportions are sufficiently nearly those in which the gases are present in the springs referred to. But in those springs in which

[1] For the determinations made by Herr Estreicher in Professor Ramsay's laboratory, see *Zeitschrift für physikalische Chemie*, 31, 148, 1899.

helium also has been detected it seems to be present in quantities about $\frac{1}{10}$ of the argon — that is, in a quantity which is nearly from 3000 to 6000 times more than is consistent with its having been derived from the atmosphere.

4. This great excess of helium in some springs has doubtless a mineral origin, some minerals, chiefly uranium compounds, containing much helium, which they give up when heated. On the other hand there does not appear to be any comparable mineral source of argon.

5. Hence, on the whole, the argon which is being supplied to the atmosphere by hot springs seems to be argon which had previously been withdrawn from the atmosphere, and which is being restored to it. Whereas, in contrast to this, there seems to be a continuous transfer of additional helium from the solid earth to the atmosphere always going on.

Thus the facts seem to indicate with some emphasis:

1. That the excessively small quantity of helium in the atmosphere is helium on its way outward.

2. That it would have become a larger constituent of the atmosphere by reason of the influx from below if there had been no simultaneous outflow from above.

3. That the rate of this outflow is presumably equal to the rate of supply; and therefore such as would suffice, in a few thousand,[1] or at least in a few million years, to drain away the

[1] The total quantity of helium in the Earth's atmosphere, according to Professor Ramsay's determination of its percentage in air at the bottom of the atmosphere, would, if reduced to standard temperature and pressure, nearly occupy something between a cube of ten miles and a volume half that size. The actual volume is, in fact, a little less than this, inasmuch as the computation has been made without taking into account that part of the reduction of density of the helium as it ascends through the atmosphere, which is caused by its being able to escape from the top.

Now, so far as can be judged from the observations upon such hot springs as have been examined, it seems likely that the rate at which helium is at present being filtered from the Earth into the atmosphere is such as would fill a cube of ten miles with helium at standard temperature and pressure in something like one or two thousand years, or perhaps in a less time. Hence the time which the helium at present in the atmosphere would take to escape if the supply from below were cut off, may be taken to be a period of this order of magnitude.

stock of helium in the atmosphere if the source of supply from below could be cut off.

Hence the evidence goes to show (1) that argon is not able to escape from the Earth; (2) that helium is escaping from the Earth, and consequently that water can probably escape from *Mars*, and that the polar snows of that planet are presumably carbon dioxide.

London, April 1900.

ON THE PROGRESS MADE IN THE LAST DECADE IN THE DETERMINATION OF STELLAR MOTIONS IN THE LINE OF SIGHT.

By H. C. Vogel.

After the early attempts at the determination of the component in the line of sight of the motion of the stars by means of the spectroscope, which were made in 1868 by Huggins, in London, and in 1871 by myself at Bothkamp, on a few stars, extensive observations of this kind were conducted at the Observatory at Greenwich, extending over a period of thirteen years. The great persistence exhibited by Maunder in these observations, which were placed in his charge, is worthy of the more recognition in view of the slight interest which astronomers then generally had in the physical side of astronomy, and especially in view of their skeptical attitude toward the application of the spectroscope for determinations of motion. There was, indeed, some basis for this, inasmuch as a contention had arisen among the physicists as to whether the so-called Doppler's principle, which was recognized as experimentally correct for sound waves, and which also permitted an easy theoretical explanation, could be justifiably transferred directly to light waves. The striking proofs[1] of the admissibility of extending Doppler's principle to moving sources of light, furnished by the astrophysicists in the course of time, gradually silenced their opponents, foremost among whom were Van der Willigen and Spée. We must not omit to mention, however, that an exhaustive theoretical treatment and explanation of the problem has not been given even up to the present time.

The protracted Greenwich observations of the stellar motions, which included forty-eight of the brightest stars, have demonstrated that direct observations with medium sized instruments

[1] I would refer to the introduction to Part I of Vol. VII of the Publications of the Potsdam Observatory, as well as to Dunér's *Recherches sur la Rotation du Soleil.*

cannot furnish results whose uncertainty is of a less order than the average motion of the stars themselves. When the dispersion is just sufficient to permit the definite recognition of the displacement, which at most is slight, the intensity of the spectrum of even the brightest stars in a medium sized instrument is too low to permit even a tolerably accurate measurement. A further reason for the small success doubtless lay in the unsuitability of the apparatus, which was especially lacking in stability.

When I made the first attempt in 1887, with the assistance of Professor Scheiner, to record photographically the displacements of the lines in stellar spectra, and then to measure them as accurately as possible on the spectrograms, it very soon appeared that this constituted a very marked advance in the determination of these motions, which are so significant in stellar astronomy. The accuracy of the observations was increased more than eightfold with the apparatus constructed in 1888; the probable error, which in the Greenwich observations averaged ± 22 km for an evening, being brought down in the Potsdam observations to an average of ± 2.6 km. We may therefore fairly say that the determination of motions in the line of sight thus first received a substantial basis in the spectrographic method, and thereby the widest prospects were opened for a period of new investigations and discoveries.

This success is doubtless due in the first instance to the application of photography; but we must not leave out of consideration that it is also in part due to the fact that a complete departure from previous principles was made in the construction of the apparatus, and an instrument was completed exclusively for this definite purpose, possessing the greatest possible stability. While spectroscopes are even yet constructed so that they can serve for many purposes, — permitting variations in the dispersion, and allowing measurements to be made in the most widely separated parts of the spectrum, — the Potsdam instrument photographs only a small portion of the spectrum in the neighborhood of the hydrogen line $H\gamma$. Its dispersion was so chosen that with sufficient sharpness of the spectrum a difference

between the setting on a line of the star spectrum and one of the comparison spectrum could be determined under the measuring machine with an accuracy corresponding to a motion of but a few kilometers. Where the spectrum of the star permitted, it was further arranged so that not only the position of a single line in the stellar spectrum was referred to that of a corresponding line in the comparison spectrum, but several lines were employed for the determination of the displacement. The accurate identification of the lines in the star spectrum was also attempted during the measurement by direct comparison with a plate of the solar spectrum.

I can restrict myself to these general statements, since the apparatus, as well as the methods of measuring the plates and reducing the observations, are sufficiently well known from the full descriptions given in Part I of Volume VII of the *Publications of the Astrophysical Observatory.* But I desire to add something for the accurate description of the state of this branch of science at the conclusion of our series of observations at the beginning of the decade just past, and I also cannot withhold a critique of our observations from the standpoint gained by wider experience.

I would first point out that several of the precautions taken in the observations, which might appear as carried too far, were conditioned by an unsuitable telescope. The Potsdam 11-inch refractor is of very light construction, and has a wooden tube which occasions a strong effect of temperature in the alteration of the focal length. Although the wooden tube is conical, it has a very appreciable flexure.

In consequence of the slight stability of the telescope, an arrangement had to be thought out for bringing the star accurately upon the slit and holding it there in the proper position during the exposure of the photographic plate. This was accomplished by the simple means of simultaneously observing with a small telescope the image of the slit, illuminated by a Geissler tube, and the image of the star, reflected from the front face of the first prism. This method of guiding

appears to have been adopted later by all observers who have undertaken determinations of velocities by the spectrographic method.

It is no longer indispensable with some of the more recent instruments, as for instance the great Potsdam refractor, since a guiding telescope of nearly the same focal length is attached to the telescope itself.

Although it was my endeavor to develop something entirely new in constructing the stellar spectrograph in 1888, in respect to the prisms I was under the influence of the times, and chose the compound prisms known as Rutherfurd's, which then were considered the most excellent, and in fact possessed the advantage of giving less deviation for the same dispersion, and hence less curvature of the spectral line, and which caused less loss by reflection on account of the less oblique incidence of the rays. Extensive investigations made here at a later time have shown that simple prisms are preferable, and elsewhere the use of prism systems has continually decreased. In observations with the spectrograph we have had the experience that strains occur in the cemented prisms at temperatures below -2°, causing a diffuseness of the spectra. A consequence of the above mentioned large flexure of the wooden tube of the 11-inch refractor may be that in many positions of the telescope the collimator and the prism are not fully utilized; except for a slight loss of light, this would have no injurious effect on the observations under conditions of good adjustment, perfect surfaces of the prisms, and complete homogeneity of the masses of glass used for the prisms.

As above stated, however, tensions in the prisms and consequent departures from homogeneity in the glass have shown themselves distinctly at low temperatures, which have caused not only a diffuseness of the spectra and a consequent diminished accuracy, but also have not been without effect on the displacement of the lines. Although this effect may have been exceedingly slight in most cases, it may neverthless have become appreciable in unusual positions of the telescope, such

as occur for stars near the pole, and possibly the large discrepancies for a few stars between our observations and those of other observers may be thus explained. In our observations we have always used the telescope in that position only (east or west) in which the optical axis of the collimator and telescope were adjusted. It was not possible to determine subsequently the effect a of lack of coincidence of the two axes on the displacement of lines in the spectra; this, moreover, could hardly have been determined at the time of observation, since it depends on the changes of temperature in the prisms, and the tension thereby occasioned, as well as upon the declination and the hour angle. The determination of the true temperature of the prisms is in general beset with difficulties, and the determination of the temperature of the separate parts of the prisms is, therefore, hardly to be thought of. We might infer that the errors arising would balance each other in a great number of observations made under the greatest variety of conditions. It would, however, probably be possible to obtain later data as to the errors dependent upon the position of the spectrograph, from a comparison of the Potsdam observations with the results obtained by other observers and with other instruments.[1] Such errors are hardly to be expected in the newer instruments with simple prisms, and having greater stability of the telescope. We have most carefully studied the effect of the temperature on the focal length of the objective of the refractor and upon the focal

[1] At present I am acquainted with determinations of the motions of only something more than half of the objects observed in Potsdam, made by Bélopolsky, Campbell, Newall, and Lord, for the great part of which I am indebted to letters of Messrs. Bélopolsky and Campbell. The comparison of these with the Potsdam observations leads to the following provisional results. The departure of the Potsdam results from those of the above observers averages $\pm$ 2.8 km for 23 objects, and the comparison shows with some certainty that the Potsdam values on the average are 1.2 km too large for stars of negative motion, and 0.7 km too small for those of positive motion. This constant departure is more clearly pronounced if we consider Campbell's observations only. Taking the mean from 19 objects the departure V. S. — C. = $\pm$ 2.7 km; the negative velocities were found on the average to be 2.5 km larger, and the positive velocities 1.6 km smaller by Vogel and Scheiner than by Campbell. If we apply these average values to the observations, the mean value of the difference V. S. — C. comes out as $\pm$ 2.0 km.

length of the objectives of the collimator and camera, as well as the varying dispersion of the prism with the temperature, and we have taken them into account both in the observations and in the reductions of the measures.

The hydrogen spectrum was in nearly all cases used as a comparison spectrum, though attempts were made to use the magnesium line λ 448 $\mu\mu$, which possesses the advantage of being very sharp in some star spectra while the $H\gamma$ line is broad and diffuse; but without success, since this line is diffuse in the spark spectrum. The iron spectrum may also be used, and I have pointed out the advantages gained in the use of this spectrum, as well as that of other metallic substances, have described the method of observation, and have illustrated it by an example—the spectrum of *Sirius*.[1] It would have been better for this purpose to have used the spectrum of *a Cygni* or of a star of the second class, because the lines in the spectrum of *Sirius* are exceedingly fine. For *Sirius* the probable error of the motion in the line of sight obtained from the difference between a single line in the star spectrum and a single line in the comparison spectrum is $\pm$ 1.34 km, so by the use of nine such pairs the probable error of the measurements of the plate would not be more than $\pm$ 0.45 km.

The observations could be made on only forty-seven of the brighter stars, since the spectrum of a star fainter than 2.3 mag. with an exposure of about an hour did not have sufficient intensity to be measured with accuracy. With a longer exposure the practically unavoidable changes in temperature produced such an influence on the spectrograph that the accuracy of the observation was impaired. As a guard against accidental errors it was intended that each star should be observed on at least two nights. At the time when these observations were begun, the possibility that a star might show changes in its motion in the line of sight, in a short time, had not been thought of; yet out of these forty-seven stars four had already been found to have a periodic variation during the progress of the work.

[1] *Sitz. d. k. Acad. d. W. Berlin* **28**, 533, 1891.

The object of this work on the motion of stars in the line of sight, completed in 1891, was first to demonstrate the usefulness of the spectrographic method for instruments of medium size, and further, by means of a thorough and detailed description of methods, to enable an observer equipped with better instruments to carry on observations of this character; and possibly to give the method still further development. There was no need of our repeating these observations in the next few years with the same instrument, while we hoped that in the near future we should be able to extend them with more powerful optical means. Unfortunately the realization of this hope was deferred from year to year, and not until the present year did it again become possible for us to carry on regular observations at the Potsdam Observatory — this time, however, with instruments of the highest order of excellence.

Now, while the seed which we planted ten years ago has not hitherto received in Potsdam the nourishment which we could have wished, yet I have the pleasure of knowing that it has found nourishment in other places, so that it has thriven greatly, and has already blossomed beyond our highest expectations.

In 1890 and 1891 Keeler made his beautiful observations on the radial velocity of the brighter nebulae, by means of a grating spectroscope attached to the great refractor of the Lick Observatory. Fourteen nebulae were examined for motion, and the determinations of these motions are of remarkable accuracy, considering the great difficulty of the observations. The probable error of the result for each nebula, a result which is the mean of several observations, is on the average only $\pm$ 3.2 km. Of the fourteen nebulae, nine have a negative and five a positive motion, referred to the Sun. The average motion is 27 km and therefore, if we are justified in drawing a conclusion from so small a number of observations, is of the same order as that of the brighter stars. The greatest velocity — 65 km per second, is that of the well-known planetary nebula *G.C.* 4373, H IV 37. It exceeds by about 10 km that of *a Tauri*, which has the

greatest velocity of any of the brighter stars of the northern heavens.

While making these observations on nebulae, Keeler determined the radial velocities of *a Bootis*, *a Tauri*, and *a Orionis*. He found for the velocity of *a Bootis*, from nine measures during 1890 and 1891, a value of —6.8 km ± 0.3 km; for *a Tauri*, on three evenings, +55.2 km, and for *a Orionis*, on two evenings +14.0 km per second. The probable error of a single evening's observations, deduced from the results for the three stars, averages ± 1.8 km.

The Potsdam observations made from 1888 to 1890 give for these three stars the values: —7.6 ± 0.6 km, +48.5 km, and +17.2 km. A better agreement, considering the complete independence and entirely different character of the methods used (Keeler made his measures on the D lines), could hardly be expected. With the Lick refractor, which exceeds the Potsdam 11-inch refractor some eight times in light-gathering power, it has therefore been possible to determine the motions of the brighter stars, by direct observation, with about the same accuracy as with the Potsdam refractor by the spectrographic method.

About the end of the year 1891, the great Pulkova refractor of 76 cm aperture was provided with a spectrograph, built exactly according to the model of the Potsdam instrument, and like the latter provided with two Rutherfurd prisms. The instrument was so arranged that the prism box could be removed, and another, containing only a single prism, substituted for it when faint objects were to be observed with small dispersion. Since the refractor had been intended primarily for micrometric work, and only secondarily for spectroscopic, the telescope and observing chair had been designed mainly with reference to the convenience of the observer at the micrometer. For this reason Bélopolsky had to contend with many difficulties before he finally succeeded, after making a number of considerable changes in the telescope and the observing chair, in making spectrographic observations with accuracy. The very unsuitable climate made it necessary for him to abandon his original plan of observing

the motions of the fainter stars, a plan which may be regarded as an extension of the Potsdam observations. He therefore selected special objects from among the variable and double stars for his observing list, and made observations of their motions.

Among his valuable researches should be mentioned his investigations of δ *Cephei*[1] in which he demonstrates the existence of a periodical variation in the velocity of this star in the line of sight, which could be brought into good agreement with the period of its light curve, $5^d\ 9^h$. In the case of η *Aquilae*[2] he discovered a variation in the velocity which found an explanation in the period of light-variability of $7^d\ 4^h$, and finally he found a change in the velocity of α^1 *Geminorum*[3] with a period of $2^d\ 23^h.5$. It soon appeared that the line of apsides of this double star system was in rapid revolution, and from careful investigation the period of this motion was found to be 4 years 40 days.[4]

As the interesting double spectrum of the well-known variable β *Lyrae* was known, from spectrographic observations, to contain pairs of bright and dark lines, and since changes in the relative positions of these lines had been discovered which were connected in some way with the period of variability of $12^d.9$, Pickering attempted to calculate the orbit of this hypothetical double star from measures of the distances between the double lines at different phases of the light period. He found a relative velocity of the components of 482 km per second, and a diameter of the orbit, assuming it to be circular, of 85.3 million km. I have more recently pointed out that the distance between the lines, and therefore the relative velocity of the components, has been taken to be too large, by not taking into account a probable overlapping of the lines; since the above value leads to an enormous mass (150$\odot$) of the system. Bélopolsky has published

[1] This JOURNAL, **1**, 160. *A. N.*, **140**, 17.

[2] *Mem. Spettr. Ital.*, **26**, 101.

[3] *Bull. Acad. St. Petersbourg*, **4**, 341. This JOURNAL, **5**, 1.

[4] *Mem. Spettr. Ital.*, **26**, 101, and **28**, 103.

an extensive investigation[1] of *β Lyrae*, based on observations made with the Pulkova refactor, in which are contained many interesting details concerning the changes which take place in the pairs of bright and dark lines during the period of the star's variability. From his measures of the hydrogen line $H\beta$ he found an orbital velocity of 89 km per second, a semiaxis of 15 million km, and hence a mass of the system of the same order as that of the Sun.

In 1894, while examining a large mass of observations of *β Lyrae* made with a small single prism spectrograph attached to the photographic refractor of the Potsdam Observatory,[2] I found clearly a certain agreement between the relative displacement of the lines and the period of light variability, but not of the simple character assumed by Béloposky. The variation in the distance between the bright and dark line $H\zeta$, especially, seemed to correspond to a period much longer than the period of the light variation, so that a system of two bodies was no longer adequate to explain the phenomena observed in the spectrum. Now since it has been shown by Myers that the light curve is very satisfactorily represented under the assumption of a double star system, we are compelled to introduce explanations which depend upon phenomena of a physical nature.

Bélopolsky[3] thereupon, in 1897, resumed his investigations and from them there is no doubt that we have come nearer to a decision as to the nature of *β Lyrae*. In his measures he disregarded the bright lines altogether, and restricted himself to the dark magnesium line $\lambda\, 448\, \mu\mu$, which has no companion emission line, and thus freed them from the influence of a partial overlapping. By so doing he was able to obtain results which were referable to the case of a simple double star without further complication. He found for the orbital velocity 178 km per second, for the semiaxis of the orbit 31.9 million km, for the

[1] *Bull. Acad. St. Petersbourg*, N. S., 4, 341. Mélanges mathem. et astr., Vol. VII, Livr. 3, 1893.

[2] *Sitz. d. K. Akad. d. W. Berlin*, 1894, 115.

[3] *Mem. Spettr. Ital.*, **26**, 135; also Tikhoff, *ibid.*, **26**, 107.

distance between the two components 47.5 km, and for their masses respectively 9 and 18 times the Sun's mass.

Bélopolsky has also found variable motion in λ *Tauri*, ζ *Geminorum*, and θ *Ursae Majoris*,[1] so that his contribution to the discovery of spectroscopic binaries consists of seven objects.

It is of interest to note here the great velocity of —70 km per second, referred to the Sun, which Bélopolsky has found for ζ *Herculis*.[2] Campbell,[3] at the Lick Observatory, has confirmed this observation and obtained a value of 70.3 km, while Deslandres,[4] at Paris, obtained a value about 10 km smaller. In this connection I may remark that Campbell has found in η *Cephei* the greatest known velocity of a star in the line of sight (—87 km per second). This velocity would probably be reduced somewhat if we were to take into account the motion of the solar system. Assuming the coördinates of the apex of the Sun's way to be $a = 267^\circ$ and $\delta = + 31^\circ$, and the velocity of the Sun to be 17 km per second, we get for the absolute component of the velocity in the direction of the Sun, —74 km per second for η *Cephei*, —54 km for ζ *Herculis*, and —51 km for the nebula *G.C.* 4373.

In publishing his observations of ζ *Herculis*, Deslandres made a remark concerning the Paris telescope of 1.2 meters aperture, to which his spectroscope was attached, from which it appears that the stability of the telescope is hardly what it should be for such delicate investigations; and, possibly because the mirror did not unite perfectly all the rays on the slit, the exposure had to be made 50 per cent. longer than with the Pulkova refractor, under presumably less favorable atmospheric conditions. In these unsuitable conditions may also be found the reason why so few observations of stellar motion by Deslandres have been made known. He has published beautiful threefold enlargements of the original spectrograms[5] of the four stars a *Aurigae*, β *Aurigae*, a *Canis Majoris*, and γ *Pegasi*, which are specially

[1] *A. N.*, **145**, 281; also **149**, 239, and **151**, 39.

[2] *A. and A.*, Feb. 1894. *A. N.*, **133**, 257.

[3] This JOURNAL, **8**, 157.

[4] *C. R.*, **119**, 1252.

[5] *Spécimens de Photogr. astronomiques. Obs. de Paris*, 1897.

remarkable for the great length of spectrum which his apparatus is capable of defining sharply. With the exception of a series of observations of *a Aquilae*, nothing further is known to me of work on the motions of stars at the Paris Observatory.

At the suggestion of Poincaré of Paris, Deslandres has, on the other hand, made spectrographic researches on the motion of the planets, and on the rotation of *Jupiter*,[1] which are of importance. The investigations showed, in accordance with Poincaré's assumptions, that in the case of a body which shines by diffuse reflected light, the displacement of lines in its spectrum depends not only on the motion of the body with reference to the observer, but also on the motion of the body with reference to the source of light by which it is illuminated.[2] The observations on the rotation of Jupiter have been repeated and confirmed by Bélopolsky.

The beautiful results which Keeler obtained by the spectrographic method at the Allegheny Observatory on the rotation of the ring system of Saturn, should be mentioned in this connection. By these observations it was shown that Saturn's rings consist of separate small bodies which revolve about Saturn in obedience to Kepler's laws, and cannot be regarded as a rigid body, thus furnishing a practical confirmation of the conditions demanded by theory.[3] These interesting observations have been repeated by Campbell, Bélopolsky, and Deslandres.

The first attempts to prove the truth of Doppler's principle, by showing that there is a displacement of the lines in the spectrum of the edge of the Sun near the equator which corresponds to the known equatorial velocity obtained from observations of Sun-spots, were made by me twenty-nine years ago. In the

[1] *C. R.*, 120, 417. [2] *A. N.*, 139, 241.

[3] Keeler used in these observations a spectrograph composed of three simple prisms, which produced a total deviation of 180°. From a few original negatives of the solar spectrum and several excellent negatives of planetary spectra, which he kindly sent me, I was able to see that the apparatus is very similar to the Potsdam spectrograph of 1888, both as regards dispersion and resolving power. The dispersion of Keeler's apparatus is about one twentieth greater than that of the Potsdam spectrograph, while the resolution of close lines is almost exactly the same in both instruments. Perhaps Keeler's apparatus is slightly superior to the Potsdam apparatus in this respect.

course of time these observations have been frequently repeated with improved instruments. Dunér, in Lund, has undoubtedly carried out the most thorough investigation of the rotation of the Sun in different zones, by means of spectroscopic methods.[1] He was induced to make these observations because, in the beginning of the eighties, the trustees of the Lars Hjertas Minne endowment expressed a desire to have the spectroscope used for a a careful investigation of the question whether the wave-length of light really varies directly as the velocity of the source of of light, as required by the Doppler-Fizeau principle, and had expressed their willingness to grant sufficient money for a suitable spectroscope.

To give some idea of the results of his observations, which were made during the summer months of the years 1887, 1888, and 1889, I have inserted the following table of the means for various heliocentric latitudes, with their probable errors:

Heliocentric latitude	Velocity in km.	Number of observations	$\xi \cos \phi$	ξ
0°.4	1.98 ± 0.013	107	14°.14	14°.14
15.0	1.85 0.013	104	13.19	13.66
30.0	1.58 0.014	104	11.31	13.06
45.0	1.19 0.014	106	8.48	11.99
60.0	0.74 0.012	107	5.31	10.62
74.8	0.34 0.013	107	2.45	9.34

Shortly before the completion of Dunér's researches there appeared two extended investigations by Crew[2] on the same subject, which led to the result that the absorbing layer of the Sun rotates with a uniform angular velocity, while Dunér's observations cannot be harmonized with the assumption of a constant angular velocity, as is shown by the table, in which the angular velocity (ξ and $\xi \cos \phi$) is given in the two last columns. This is not the place to discuss the difference between the results obtained by the two observers; for it was merely my purpose to mention the beautiful observations which have been

[1] N. C. Dunér, "Recherches sur la Rotation du Soleil," *Nova Acta Reg. Soc. Sc. Ups.* Series III.

[2] Henry Crew, "On the Period of the Rotation of the Sun as Determined by the Spectroscope," *Am. Jour.*, 35, 151, and 38, 204.

made in this field, and to show what great accuracy can be given to spectroscopic observations of motion when there is sufficient light. The observations of Dunér agree very well with the law of the rotation of the Sun derived from the observations of Sun-spots.

With regard to the above-mentioned desire of the trustees to ascertain whether the change in the wave-length of light is proportional to the velocity of the luminous source, Dunér's observations have shown that, within the errors of observation, the simple form of Doppler's principle is valid, and the influence of possible higher terms is not recognizable.

Through the generosity of Mr. D. O. Mills, Professor Holden, at that time Director of the Lick Observatory, was able to have a spectrograph constructed for the Observatory about the middle of the nineties, to be used exclusively for determining stellar motions in the line of sight. In the October 1898 number of the ASTROPHYSICAL JOURNAL Professor Campbell has given an extended description of this instrument, which, mainly through his endeavors, has become the most noted instrument of its time.

Attached to the great 36-inch refractor of the Lick Observatory, used under the most excellent atmospheric conditions, and in the hands of a circumspect and careful observer, the "Mills spectrograph" has in the last few years achieved surprising results. The spectrograms taken with it possess, on the average, a sharpness which excels that of even the best spectrograms taken with the Potsdam apparatus of 1888, and the resolving power of the instrument is much greater. Through the kindness of Messrs. Keeler and Campbell I received last year two original negatives, one of γ *Andromedae*, and the other of η *Pegasi;* so that I was able to thoroughly assure myself of their excellence. They are marked as being above the average exellence, and their measurement is a real pleasure; yet even with such plates great care and long experience are required to obtain uniformly consistent results.

With spectra which contain many lines, and which differ considerably from the solar spectrum, the use of a solar spectrum in

measuring the plates cannot well be avoided. In many spectra, on the other hand, especially those with strong iron lines, the stellar and comparison spectra can be compared directly by means of Hartmann's interpolation formula.[1] The amount of the shifting of a known line in the star spectrum can be exactly determined even if there is no identical line in the comparison spectrum; for instance, the lines of hydrogen and clèveite gas can be compared with iron lines in the comparison spectrum.

On Campbell's plates the probable error of the determination of the distance between a stellar line and the corresponding comparison line may be taken as ± 1.2 km, and since from ten to twenty measurable lines may easily be found in the spectrum of a star of class II, the probable error of the mean of the measures on a single plate may be reduced to about $\pm \frac{1}{3}$ km. Slight changes in the instrument during the exposure, slight differences in the observer's habit of measurement, minute distortions of the film, and other unavoidable sources of error have here to be taken into account. They are recognizable in the fact that the probable error of the final result, deduced from the mean of several plates of the same object, is larger than that which the probable error of a single plate, determined from the agreement of the different lines, would lead one to expect. According to Campbell's observations of the brighter stars, the probable error of a single plate is however somewhat less than ± 1 km. Campbell's detailed description of the Mill's spectrograph, and the opportunity I have had of examining some of the spectrograms made with it, have been of great service to me, since the new spectrographs for the large Potsdam refractor, which have been made from my designs by Toepfer, of Potsdam, were just at that time so nearly completed that they could be subjected to a preliminary test. The construction of these two spectrographs was begun in 1897. One of them has three simple prisms and a total deviation of 180°, while the other has only one simple prism. Dr. Hartmann, whom I had commissioned to make the adjustments and tests of the optical parts of

[1] *Publ. des Astrophys. Obs. zu Potsdam*, **12**, No. 42.

these spectrographs, has devoted all his energies to obtaining the highest degree of excellence,[1] and it is a pleasure to be able to say that the new spectrograph with three prisms for the Potsdam 80 cm refractor, will not be inferior to the Mills spectrograph.

At the present time a series of observations of the radial velocities of stars down to the fifth magnitude is being made at the Lick Observatory. So far Professor Campbell has made two or more observations on about 300 stars, and has found 16 of them to have a variable velocity, thus bringing the number of binaries discovered by spectrographic methods up to 28. I shall merely give here a list of the spectroscopic binaries discovered by Campbell.[2]

Star	Period
η *Pegasi*	2¼ years
χ *Draconis*	9⅓ months
o *Leonis*	14½ days
ζ *Geminorum*	Unknown
ι *Pegasi*	Over 10 days
θ *Draconis*	Over 9 days
ϵ *Librae*	Unknown, several months
β *Capricorni*	Unknown, long
h Draconis	Undetermined
λ *Andromedae*	About 20 days
ϵ *Ursa Minoris*	A few weeks
ω *Draconis*	Unknown
α *Ursa Minoris*	3.9 days and a second longer period.
α *Aurigae*	3½ months
ϵ *Sagittarii*	A few weeks
β *Herculis*[3]	Unknown, one year?

It is worthy of remark that several stars have a period of many months, and that η *Pegasi* has a period of 2¼ years. Thus the gap which formerly existed between the spectroscopically discovered and the visual double stars, with respect to the lengths of their periods of revolution, has been filled up.

When we consider the large number of variable stars of the *Algol* type which have been discovered photometrically during

[1] For particulars on this investigation, and the methods used in testing, see Dr. Hartmann's paper in this and the next numbers of this JOURNAL.

[2] This JOURNAL, 8, 9, 10.   [3] *A. S. P.*, 12. 39, 1900.

the last decade, and for which the assumption appears to be justified that their variation is a consequence of duplicity ; when we further consider that these stars can be recognized as variable only when the line of sight makes an extremely small angle with the plane of the orbit, and that for the spectroscopically discovered stars, also, this angle cannot be supposed to be very great, we cannot repress our astonishment at the rapidity with which the number of stars thus discovered has grown.

Among the double stars discovered at the Lick Observatory, *Polaris*, which has a double period of motion in the line of sight, is of particular interest, since we are here led to infer the existence of three bodies. The discovery of the short period is an admirable proof of the excellence of the observations, since the variation from the mean velocity is only ± 3 km.

Of particular interest, also, is the discovery of the periodic doubling of the lines in the spectrum of *a Aurigae*, which consists of two superposed spectra uniting at times to form a spectrum strongly resembling that of the Sun.[1] Although *a Aurigae* was frequently observed at Potsdam during the years 1888–1891, this peculiarity of its spectrum escaped us, and the explanation of the fact that most of our spectrograms of *a Aurigae* are ill-defined, and the lines quite extraordinarily broadened, was only given later, by the investigations of Campbell. The days on which the spectrograms obtained were good and sharp, are those on which the two spectra were superposed, and our early observations have now at least given us the means of arriving at a more accurate value of the period ($104^d.1 \pm 0^d.2$). By means of observations with the new Potsdam refractor and

[1] From Campbell's observations the deduction can also be made that both stars, one having a spectrum similar to the solar spectrum, and the other a spectrum containing the hydrogen and the stronger iron lines, are of nearly the same mass, since their displacements from the mean position are about equal. Assuming the period to be $104^d.1$ and the maximum relative velocity to be ± 30 km, and assuming provisionally that the orbit is nearly circular, with its plane in the line of sight, I have found, for the combined mass of the stars, $m + m_1, = 2.3\odot$ and for their distance, 85.3 million km.

spectrograph, we have been able to confirm Campbell's observations, both of *α Aurigae* and *α Ursae Minoris.*[1]

When we consider that researches on the motions of stars in the line of sight have been begun and are being carried on with good results by Newall, in Cambridge, and by Lord, at the McMillin Observatory in Ohio; that in Meudon a double refractor of the same size as the Potsdam instrument has been set up, and provided with a spectrograph, with which Deslandres[2] has already succeeded in showing that *δ Orionis* is a star with variable velocity; that further, the largest instrument in the world, Yerkes refractor at Williams Bay, will also be used for this purpose, and that Gill, with the double refractor of the Observatory at the Cape of Good Hope, will extend the same researches to the southern heavens, we may confidently expect that our knowledge of the stellar system to which we belong will be increased as much in the new century upon which we are entering, as our knowledge of the solar system has been increased in the century just past. The energy with which some of the largest observatories in the world are participating in the work is most encouraging, for the amount of work to be done has grown most unexpectedly during the last decade, through the discovery of so many stars having a variable velocity in the line of sight.

In closing this review, I should not wish to leave unmentioned the fact that in one case the application of the Doppler-Fizeau principle can no longer be regarded as valid ; I refer to the interpretation of the pairs of bright and dark lines which are found in the spectra of the new stars. When this peculiarity of the spectra of new stars was discovered, it was very natural that the relatively strongly displaced emission and absorption lines should be ascribed to the spectra of two bodies, whose motions in the line of sight were oppositely directed. Corresponding to the great displacement of the lines, velocities were

[1] It should be added here that the discovery of the duplicity of the lines of α *Aurigae* was made independently, and almost simultaneously, by Newall, of Cambridge, England.

[2] *C. R.* 130, No. 7.

arrived at which, in comparison with the mean velocity of other heavenly bodies whose motions have been spectroscopically determined, must be characterized as perfectly enormous — more particularly since it was improbable that the whole motion of the bodies should be in the line of sight.

But since in the stars which have appeared more recently, *Nova Normae* and *Nova Carinae*, the same pairs of bright and dark lines were found, in which, as in the spectrum of *Nova Aurigae*, the emission line lay on the less refrangible side; and since similar lines can be observed in the spectra of *β Lyrae* and *P Cygni* (the Nova of 1600); and particularly since no change in the distance between the bright and dark lines could be detected during the whole of the first apparition of *Nova Aurigae*, and the lines remain unchanged in position in the spectrum of *P Cygni* — while in *β Lyrae*, though they vary during the star's period, the changes are never so great as to reverse the positions of the dark and bright lines — doubts as to the applicability of Doppler's principle to such cases appeared to be more and more fully justified.

The assumption that we are here concerned with phenomena of a purely physical nature gained a firmer basis through the researches of Humphreys and Mohler, Eder, and Wilsing,[1] on the changes produced in spectral lines by high pressure. It was found that under high pressure pairs of bright and dark lines could be produced in metallic spectra, in which the emission line always lay on the less refrangible side.

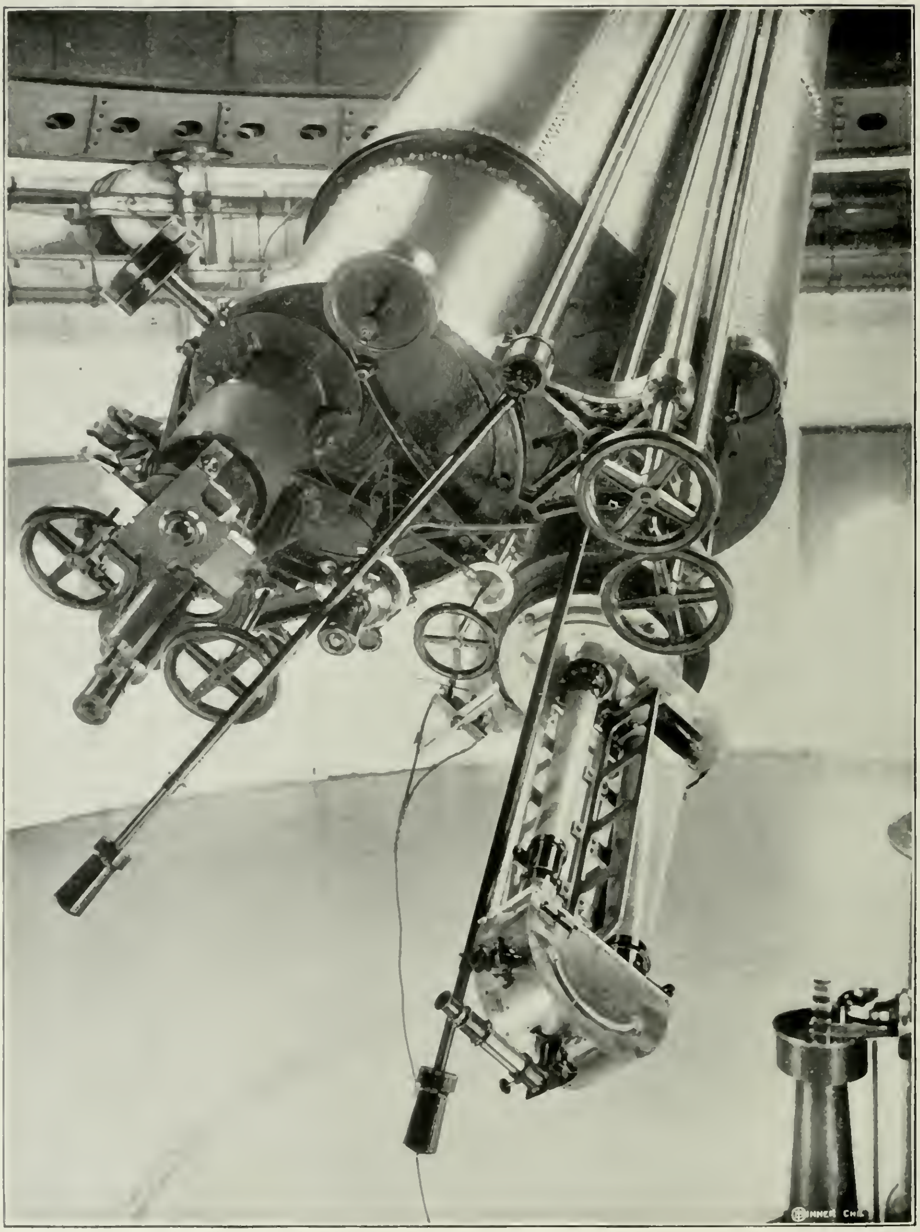
These observations are to be regarded as only first beginnings; but doubtless through them a wide field of highly interesting research is opened, in which the astronomer may hope for the zealous support of the physicist. The veil which has enveloped anew our knowledge of the nature of the temporary stars will certainly be lifted, when experiment shall yield results which harmonize with the phenomena observed in them; and not until then will the time come to frame hypotheses respecting

[1] "Ueber die Deutung des typischen Spectrums der neuen Sterne." *Sitz. d. K. Akad. d. W. Berlin*, **24**, 425. 1899.

the origin of the abnormal conditions of pressure in their atmospheres.

It is quite possible that, as observations of motions in the line of sight become more and more refined, the conditions of pressure in the atmospheres of the stars can no longer be neglected, although it may be assumed that the pressure in the atmospheric layers from which the light comes to us is neither subject to great variations for any individual star, nor varies widely for different individuals. Even if this should not be true, the means of arriving at accurate results for stellar motion, and at the same time of gaining information concerning the conditions of pressure in the stellar atmospheres, is to be found in the use of different metals for comparison spectra.

PLATE IX



NEW POTSDAM SPECTROGRAPH (III)

DESCRIPTION OF THE SPECTROGRAPHS FOR THE GREAT REFRACTOR AT POTSDAM.

By H. C. Vogel.

As I have already mentioned in the preceding article, two new spectrographs have been constructed for the Potsdam 80 cm refractor, and I shall give here a preliminary description of some further details of these instruments, which will serve to make more intelligible the illustrations in Plates IX, X, and XI.

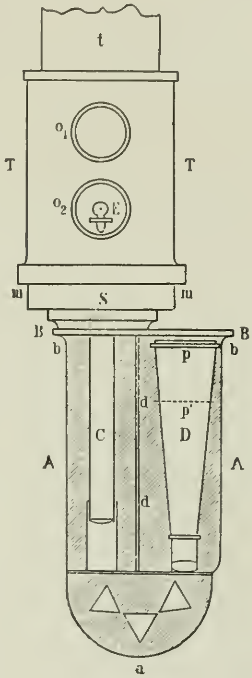


Fig. 1.

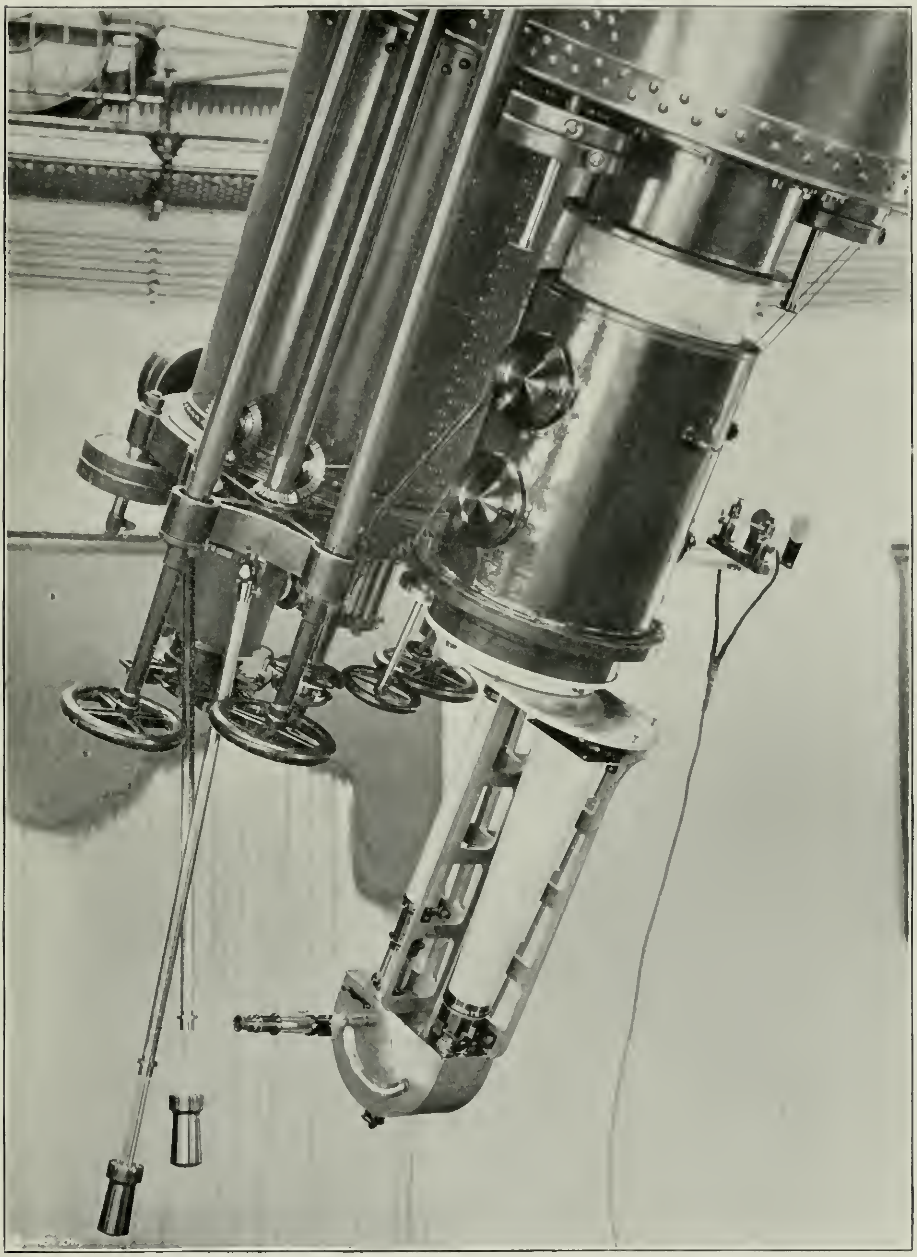
In the construction of the larger spectrograph with three prisms, I have departed somewhat from the previously chosen form, in order to obtain greater stability by firmly uniting collimator, prisms, and camera by means of a single plate. The steel plate *A* (see the schematic diagram in Fig. 1) has a thickness of 7 mm and a length of 78 cm. The width at *bb* is 41 cm, while at *a* the plate is rounded, to correspond with the arc formed by the prisms. This plate is rigidly connected with the elliptical base-plate *BB*, which is 10 mm thick. To guard against lateral flexure, the plate *A* is provided with three ribs, one, *dd*, in front, 12 cm deep and 5 mm thick, and two, on the back, 8 cm deep and 5 mm thick. The steel plate *BB* is secured by strong screws to the cylindrical casting *T*, which is rigidly connected with the draw-tube *t*, at the eye-end of the photographic telescope. *C* is the collimator, *D* the camera, and *p* the plate-holder. The camera *D* can be exchanged for another, which has an objective of shorter focal length. When this second camera is

used, its objective is of course as close as possible to the last prism, and the plate-holder has the position p'. The slit S projects somewhat into the cylindrical piece T. It has the same construction as the slits in all our spectrographs; namely, one movable jaw, which is pressed against the fixed jaw by a spring, and is separated from the latter by turning a micrometer screw. The jaws are made of platinum-iridium. The arrangement for photographing comparison spectra in juxtaposition to the star spectrum, and for limiting the length of the slit, has been described by Dr. Hartmann.[1] I should add that the small plates or screens have been placed at a small distance in front of the slit, in order that their boundaries may be sharply defined on the photograph, which is not the case when they are placed in contact with the slit-plate. At mm is an arrangement allowing the apparatus attached to T to be rotated around the optical axis of the refractor. The angle of rotation can be read to 1′, and in this way the slit can be accurately placed in any desired position angle.

In the cylinder T, which has a length of about 57 cm and a diameter of 36 cm, there are four apertures, closed by means of covers (in the diagram, only o_1 and o_2 can be seen). These apertures serve to introduce Geissler tubes, or other apparatus which it may be necessary to place in front of the slit. At E is the (electric) source of light, the rays from which are thrown on the slit by a mirror placed, inside the cylinder T, in the optical axis of the refractor and the collimator, when the comparison spectrum is to be photographed. Dr. Hartmann has found that the arc light is advantageous for producing the comparison spectrum. The electric plant for turning the dome, opening the shutter, and moving the observing chair furnishes a current which suffices for the arc light, and which is easily led from the observing chair to the instrument. An exposure of about one minute produces a sufficiently intense comparison spectrum when (and this is now always done here) the light is rendered diffuse by interposing a translucent screen between the source of

[1] *Zeitschrift für Instrumentenkunde.* 20, 57, February 1900. This JOURNAL, July 1900.

PLATE X



NEW POTSDAM SPECTROGRAPH (III)

light and the slit. The effect of this screen is to prevent entirely any accidental displacement of the spectral lines, in the case of a possible imperfect adjustment of the cone of rays from the source of light with respect to the optical axis of the collimator.

The collimator objective, by Steinheil, has a focal length of 48 cm; $\frac{A}{F}=\frac{1}{15}$. The equivalent focal length of the camera objective (an anastigmat by Zeiss, of Jena) is 56 cm; $\frac{A}{F}=\frac{1}{14}$. The second camera objective (a cemented triple lens by Steinheil) has a focal length of 41 cm; $\frac{A}{F}=\frac{1}{10}$.

The prisms were made by Steinheil of very white Jena glass *O*. 102, ($n=1.6744$ for $H\gamma$). The refracting angle of each is $63^\circ\ 27'.5$, the length of the refracting edge (height of the prism) is 40 mm. The lengths of the refracting faces of the three prisms are respectively 60 mm, 65 mm, and 70 mm, and the total average length of path, when the prisms are in the position of minimum deviation for the $H\gamma$ rays, is 103 mm. The anastigmat lens of the first camera makes it possible to obtain a uniformly sharp spectrum from b to K, but the strong curvature of the lenses causes a somewhat greater loss of light with this camera than with the second objective of 41 cm focal length.

For the sake of lightness, numerous openings are left in the steel plate *AA* and its strengthening ribs. The steel parts were finished with a fine matt surface and then nickel-plated. The box which encloses the prisms is made of thin sheet nickel, and is highly polished, in order to protect the prisms as much as possible from radiant heat, and during the observations the apparatus is further protected by enveloping it in cloth. A telescope for observing the image of the star in the slit, formed by light reflected from the first surface of the first prism, is provided, as in the earlier apparatus, and provision is also made for so connecting this telescope with the prism box, that the image

reflected from the first face of the second prism — that is, the star spectrum formed by the first prism — can also be observed. In the prism box is a thermometer, the bulb of which projects into the box, while for further determinations of temperature there is a thermometer at the lower end of the tube of the refractor, and another near the great objective, both within the tube. They can be illuminated by incandescent lamps, and are easily read from the eye-end of the telescope by appropriate systems of lenses.

The weight of the spectrograph (which has been designated by the numeral III), is 31 kg.

The construction of the apparatus I, which is provided with only a single prism, will be readily understood from the illustration. The focal length of the collimator objective is 53 cm; $\frac{A}{F}=\frac{1}{15}$. The focal length of the cemented triple camera objective is 72 cm; $\frac{A}{F}=\frac{1}{18}$. Both objectives are from the shops of C. A. Steinheil's Sons, Munich, while the prism, which has a refracting angle of 60°, a height of 45 mm, and faces 61 mm long, was made by Zeiss, of Jena. This apparatus gives a sharp spectrum from D to N, corresponding to a difference of wave-length of 230 $\mu\mu$. The weight of the apparatus is 20 kg.

The mountings of both instruments were made in the most satisfactory manner by the mechanician, O. Toepfer, of Potsdam.

The fine groups of lines in the solar spectrum, which Campbell, in his description of the Mills spectrograph, has cited as a test for such instruments, are perfectly resolved with the same angular slit-width.

In order to permit a more accurate judgment of the capabilities of the apparatus to be made, with respect to the resolution of close lines on a photograph of the solar spectrum, I give the following table of results:

NEW POTSDAM SPECTROGRAPH (I)

Region above K

λ	Δ	
λ 3921.563	0.132	Very distinctly divided; spectrum not quite sharp.
3921.695	0.160	
3921.855		

Region between H and K

λ	Δ	
λ 3935.965	0.156	Widely divided
3936.121		
3938.439	0.113	Distinctly divided
3938.552		
3948.818	0.107	Well separated
3948.925	0.114	
3949.039		
3952.465	0.084	Very weak lines; divided
3952.549		
3952.754	0.096	Indications of resolution
3952.850		
3956.476	0.127	Well separated
3956.603		

Between H and K all lines in Rowland's atlas of the solar spectrum can be seen.

Below H

λ	Δ	
λ 4056.135	0.086	Only four lines can be recognized; the first two are not resolved, the others distinctly
.221	0.124	
.345	0.150	
.495	0.106	
.601		
4074.835	0.112	Resolved
.947		
4076.644	0.148	Very well divided
.792		
4078.515	0.116	Not completely resolved, though duplicity can be distinctly recognized.
.631		
4092.431	0.116	Resolved though partly blended together
.547	0.118	
.665	0.156	
.821		

Between G and $H\gamma$

λ	Δ	
λ 4311.058	0.088	Not divided
.146	0.182	Divided
.328		

Between G and $H\lambda$

λ	Δ	
λ 4311.608	0.066	Not divided
.674	0.206	Divided
.880		
4314.248	0.133	Divided
.381	0.098	Not divided
.479		
.964	0.174	Divided
4315.138	0.124	Not divided
.262		
4320.907	0.212	Completely divided
4321.119		
4339.617	0.265	Widely divided; easy pair
.882		
4344.451	0.219	Distinctly divided
.670		
4351.930	0.153	Indications of resolution
4352.083		
4408.364	0.218	Well divided
.582	0.101	Not divided
.683		
4530.910	0.213	Three lines just recognizable
4531.123	0.204	
.327		
4533.133	0.286	Distinctly double
.419		
4534.139	0.201	Indications of the companion
.340		
4535.741	0.138 Not div'd	Appear like double line
.879	0.215 Divided	
4536.094	0.128 Not div'd	
.222		
4540.672	0.208	Divided
.880		
4556.063	0.243	Divided
.306		
4565.688	0.154	Indications of resolution
.842		
4607.510	0.321	Well divided
.831		
4613.386	0.158	Apparently single
5.44		

All the tests which have been made so far with respect to the stability of the apparatus have given very satisfactory results, so that there can be no further doubt that the Potsdam instrument will yield results equal to those obtained with the Mills

spectrograph. On account of the longer collimator of the Mills spectrograph, corresponding to the greater focal length of the Lick refractor, this instrument has an advantage, characterized chiefly by the shorter exposure times required, in that the slit can be opened more widely than that of the Potsdam instrument, for equal resolution of the systems of lines in the spectrum. On the other hand, the anastigmat lens of the longer camera, and the fact that the 80 cm objective of the Potsdam refractor is achromatized for the chemically active rays, permit a much greater length of sharply defined spectrum to be photographed with the Potsdam apparatus, a circumstance which is of great importance for investigations which are not specially restricted to the determination of motions in the line of sight.[1]

Taking absorption into account, the ratio of brightness for the Lick and the Potsdam refractors may be taken to be 11:9, or, when the correcting lens, which does not reduce the aperture, is used with the Lick refractor, 7:6. With respect to brightness the Lick instrument has therefore no material advantage, if we leave out of consideration the favorable atmospheric conditions on Mt. Hamilton. But in changing from a refractor of 5.4 m focal length to one of 12 m, we have already had ample opportunity to appreciate how weighty are these very advantages of transparent and quiet air with respect to the decrease in the requisite times of exposure. The observer with the Mills spectrograph on the Lick telescope will therefore have an advantage, particularly in consequence of superior atmospheric conditions; and will always have it. I am, however, convinced that in choosing an instrument with the relatively short focal length of 12 m, we have obtained that which is best suited to our own atmospheric conditions, and have secured the greatest possible efficiency.

In further comparison of the instruments in question I have compiled the following table, in which are given the values corresponding to a displacement of 0.25 mm (one revolution of the measuring apparatus used at Potsdam and Mt. Hamilton):

[1] The Lick refractor has recently been provided with a correcting lens for uniting the chemically active rays on the slit of the spectrograph. Its use is, however, attended by some further loss of light.—EDITORS.

	μμ	km
Potsdam spectrograph (1888)	0.324	224
Mills spectrograph.....	0.3143	217
Potsdam spectrograph III (1898), camera 1......... ...	0.2546	176
Potsdam spectrograph III, camera 2....................	0.344	238
Potsdam spectrograph I	0.675	466

For the derivation of the formula for spectrograph III Dr. Hartmann has made the following measures on the solar spectrum:

R	Wave-length (Rowland)	Wave-length (computed)	O.—C.
—0.224	4654.74	4654.76	—0.02
+17.220	4590.13	4590.13	0.00
34.180	4531.80	4531.83	—0.03
51.576	4476.22	4476.22	0.00
65.888	4433.39	4433.38	+0.01
86.404	4376.11	4376.10	+0.01
100.000	4340.63 $H\gamma$	4340.61	+0.02
108.706	4318.82	4318.83	—0.01
136.228	4254.50	4254.50	0.00
156.776	4210.53	4210.53	0.00
178.468	4167.44	4167.45	—0.01
194.764	4137.16	4137.14	+0.02

The formula

$$\lambda = 3279.50 + \frac{[7.762974]}{(R + 595.550)^{\frac{5}{3}}} ,$$

computed from these data represents, as the last two columns of the table show, all the observations to within the limits of error of measurement. The region of the spectrum here represented includes the entire range of 50 mm, symmetrical with respect to $H\gamma$, which is measurable at once with the screw of the new measuring apparatus.

A formula for spectrograph I has already been published.[1] Expressed in the terms of the units here adopted it is,

$$\lambda = 2232.8 + \frac{[5.819004]}{R - R_0} ,$$

and represents accurately the part of the spectrum between λ 3709.6 and λ 5183.7.

[1] *Pub. Astroph. Obs. Potsdam*, 12, No. 42.

REMARKS ON THE CONSTRUCTION AND ADJUSTMENT OF SPECTROGRAPHS. I.

By J. Hartmann.

At the suggestion of Professor H. C. Vogel I made a series of special investigations some two years ago, the results of which have been adopted for the construction and adjustment of two spectographs which are to be employed in connection with the large refractor for the photography of stellar spectra. Since the apparatus, which has now been completed for some time, has proved to be extremely efficient and reliable, I desire to communicate here some of the results of those investigations which may also be of service elsewhere in the construction of spectroscopic apparatus.

I. CHOICE OF OBJECTIVES FOR COLLIMATOR AND CAMERA.

Hitherto the objectives for the collimator and camera of a spectroscope have commonly been selected from types of lenses already existent; for the collimator objective a telescope objective is usually chosen, and a similar lens for the camera, or else one of the various systems of photographic objectives now found in commerce.[1] But the properties of all these lenses only partially satisfy the demand which they must fulfill in spectrographs, and in many cases the efficiency of the instrument could be increased by the construction of objectives especially calculated for this purpose. In now giving a few brief remarks on the essential properties of these lenses, I hardly need to remind the reader that only especially clear and transparent kinds of glass can be employed in making the lenses, and their thickness is to be limited to a minimum.

[1] Extended investigations have been published by others as to the best choice of apertures and focal lengths. See Wadsworth, "The Modern Spectroscope." This Journal, 3, 321, 1896; Keeler, "Elementary Principles Governing the Efficiency of Spectroscopes for Astronomical Purposes." *Sidereal Messenger*, 10, 431, 1891.

The demands made upon the collimator objective and the camera objective are decidedly different.

The collimator objective must render exactly parallel all the rays of different wave-lengths which proceed from the slit, that is, from points lying very near to the principal optical axis. If an *astronomical* objective is employed, *i.e.*, one computed only for points on the axis and for parallel incident light, it must therefore be as free as possible from spherical aberrations, and must be especially well achromatized if it is desired to photograph a considerable extent of spectrum with the constant length of collimator; on the other hand a large angular aperture is not necessary.

But for the camera objective a large angular aperture is necessary, in addition to as complete suppression of spherical aberration as possible, while the achromatism comes in as a secondary consideration. By giving the photographic plate a definite inclination to the axis of the camera it is possible to bring the sensitive film simultaneously into the focus of rays of different wave-length, even when their focal lengths are *not* equal, but only increase uniformly with the deviation of the rays in question. Now a uniform increase of this sort is quite as well possible for a lens which is not achromatic as for one which is achromatized for a point of the spectrum considerably distant from the region which is then to be photographed.

For a photographically corrected lens the favorable case may occur that the blurring of the image is compensated by the residual difference of focus for the rays of different wave-length, which usually produce the secondary spectrum. As is well known the focus of achromatic objectives composed of two kinds of glass reaches a minimum for some spectral ray lying in the middle of the portion achromatized. If the photographic plate is now placed vertically to this ray, all the other rays falling on the plate will have to cover longer paths from the objective to the plate, and since the focus is longer for these, as already mentioned, we may by properly achromatizing the objective accomplish the result of uniting these lateral rays precisely on the plate.

It is a very difficult problem for opticians to construct a lens fulfilling the above conditions—angular aperture of about 20°, absence of spherical aberration with the greatest possible aperture, and, for given dispersion, position of all the foci on a straight line not too greatly inclined to the axis.

Permit me to point out a theorem which results when a simple plano-convex lens from the same glass as the prism is chosen for the camera objective.

If we let r be the finite radius of curvature of a plano-convex lens, n be the index of refraction for any desired ray in the common material from which the lens and prism are made, A be its deviation and b the angle of the prism, and finally F be the focal length of the lens for the rays in question, then for the prism at the minimum deviation we shall have the well-known relation

$$n = \frac{\sin \frac{1}{2}(A+b)}{\sin \frac{1}{2} b} = \sin \frac{A}{2} \cot \frac{b}{2} + \cos \frac{A}{2} .$$

If we introduce this expression in the formula applying to the plano-convex lens

$$F = \frac{r}{n-1} ,$$

we shall obtain the following relation not involving n:

$$F = \frac{r}{\sin \frac{A}{2} \cot \frac{b}{2} - \left(1 - \cos \frac{A}{2}\right)} .$$

This is the polar equation of the focal line of the system in question, the pole being placed at the optical center of the lens. The term $1 - \cos \frac{A}{2}$ vanishes for small values of A, and since

$$F = \frac{r}{\sin \frac{A}{2} \cot \frac{b}{2}}$$

is the equation of a straight line which runs parallel to the incident ray at a distance of $r \tan \frac{b}{2}$, it follows that the foci of the

system will lie very nearly in a straight line for not too large deviations, so that considerable stretches of spectrum can be rendered simultaneously sharp by inclining the plate-holder to the axis of the camera.

I show the relations for a prism of 60° in Fig. 1. *O* is the optical center of the camera lens, whose radius of curvature is

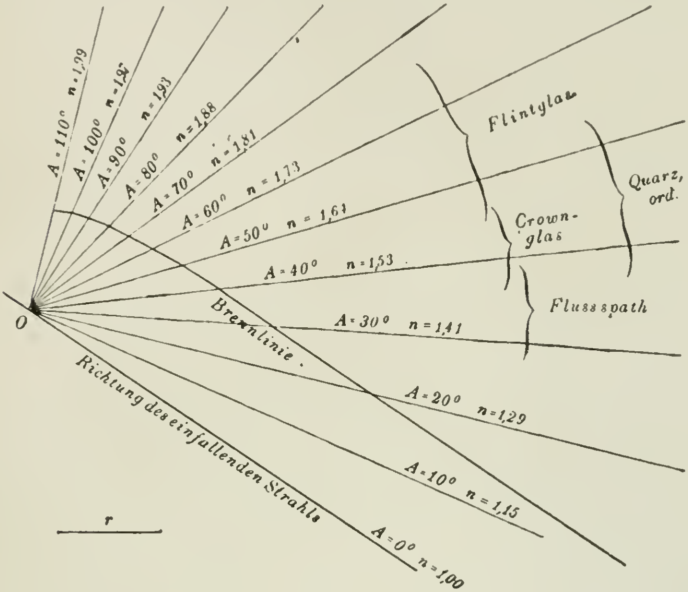


Fig. 1.

represented by the distance *r*. It will be seen that the focal line approximates a straight line quite closely for all deviations which will practically occur. The relation here derived will be especially useful in research with a quartz spectrograph, having a simple quartz lens for its camera objective, since it gives directly the necessary focus and inclination of the plates for every desired angle between collimator and camera.

II. RELATIVE LIGHT-POWER OF SIMPLE AND COMPOUND PRISMS.

Various means are available for giving a definite dispersion to a parallel beam of given section, such as emerges from the

objective of the collimator of the spectroscope. We may employ systems of simple prisms of either large or small refracting angle, different varieties of compound prisms, and finally diffraction gratings. Of all these dispersion systems that one should be selected which involves the least loss of light while yielding the dispersion sought, the other conditions prescribed by the particular purpose of the spectroscope being also taken into consideration.

As far as simple prisms are concerned, it was shown by Pickering[1] as early as 1868 that the most favorable form of the prism is that at which the incident beam is completely polarized. The corresponding angle of the prism will vary according to the index of refraction of the ray which is to pass through the prisms at the angle of minimum deviation. The following values are obtained:

Index	Angle of prism
1.50	67° 22'.8
1.55	65 39.4
1.60	64 0.6
1.65	62 26.2
1.70	60 55.8

The most favorable angle for the prism, therefore, lies between 60° and 67° for the ordinary kinds of glass, and a prism of 60° angle may be regarded as a very suitable one in all cases. Pickering's beautiful investigation does not appear to have been correctly understood by all, as the assertion is still met with in many treatises that the greatest possible refracting angle is always advantageous in respect to light-power where simple prisms are employed.

Another investigation[2] of the brightness of the spectrum produced by simple prisms has been given by Krüss. He gives the proof that the adjustment of the prisms for the minimum of deviation, which is so important for the purity of the spectrum, is also most favorable for its brightness.

[1] "On the comparative efficiency of different forms of the spectroscope." *Am. Jour.* (2) 45, 301, 1868.

[2] "Ueber Spektralapparate mit automatischer Einstellung" *Zeitschrift fur Instrumentenkunde*, 5. 181, 1885.

No thorough investigations on the loss of light in compound prisms[1] seem to have been made, but it is generally assumed that they are decidedly superior in light-power to simple prisms. The reason assigned for this is that a compound prism, in which the ray has to pass but twice between air and glass, can be so constructed as to give the same dispersion as two simple 60° prisms, in which a large loss of light by reflection occurs four times at the boundary of air and glass. The gain in light accomplished in this way is overestimated, however, and the calculation shows that this gain may be entirely cancelled by the loss of light due to the surrender of the angle of incidence which is specially favorable for a 60° prism, as well as by the loss, especially for the rays of shorter wave-length, caused by the large increase in the thickness of the glass.

While it is possible in the case of a simple prism to express the relation between the brightness of the spectrum, the dispersion, and the refracting angle by a not too seriously complicated formula, the corresponding expression in the case of the compound prism becomes entirely confused. It is not possible to express by a single formula the intensity of spectrum given by a compound prism of any given kind of glass, with given refracting angle and given dispersion, in such a way that we could be in position to find the most favorable conditions for brightness by a discussion of such a formula. I have, therefore, calculated rigorously the dispersion and light-power for a number of different kinds of compound prisms, as well as for systems of simple prisms, for comparison; and I communicate the numerical results below, as they may serve as a basis for others elsewhere, wherever the kind of glass employed does not differ too strongly from that on which my calculation is based.

I first summarize all the necessary formulæ, the derivation of which is so simple that I need not give it here.

[1] It may be remarked in passing that the widely adopted designation of the ordinary compound prism, in which a prism of strong dispersion with a large refracting angle is placed between two acute angled prisms of slight dispersion, as "Rutherfurd prisms" lacks a historical basis. Browning is responsible for the idea and the first actual construction of such prisms. Rutherfurd suggested a less useful quintuple prism. *Monthly Notices*, 31, 203, 1871.

Let us call

Q the diameter of the incident beam,
J the angle of incidence,
R the angle of refraction,
A the deviation of the ray,
D the dispersion for a difference of wave-length of $1 \mu\mu$,
S the length of face of the prism,
B the length of base of the prism,
c the coefficient of absorption of the glass for 1 mm thickness.
n the index of refraction for the ray passing at the minimum of deviation,
dn, dJ, dR, the variations of the corresponding quantities for a difference of wave-length of $1 \mu\mu$,
b the refracting angle for the simple prism,
i the refracting angle of flint glass for the compound prism,
a the refracting angle of the crown glass for the compound prism,
m the number of the prisms.

For compound prisms all the data which refer to the exterior (crown) prisms are given the subscript a, those referring to the interior prisms the subscript i, while J and R receive the subscripts 1 and 2 for the simple prism, and 1 to 4 for the compound prism for the passage through the separate surfaces.

FORMULÆ FOR THE SIMPLE PRISM.

Path of the rays:
$$\left(J_2 = R_1\,,\ R_2 = J_1\right)$$
$$R_1 = \frac{b}{2}$$
$$\sin J_1 = n \sin R_1$$

Dimensions:
$$S = \frac{Q}{\cos J_1}$$
$$B = 2S \sin \frac{b}{2}$$

Deviation:
$$A = 2J_1 - b$$

Dispersion:
$$D = dR_2 = \frac{2}{n} \tan J_1\, dn - dJ_1$$

Light power: $$H = \frac{c^B - 1}{B \log c}(H_1 + H_2),$$

where $$H_1 = \frac{1}{2}\left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2(J_1 + R_1)}\right)^2$$

and $$H_2 = \frac{1}{2}\left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2(J_1 + R_1)\cos^2(J_1 - R_1)}\right)^2.$$

H_1 and H_2 are the intensities of the light polarized perpendicular and parallel to the refracting edge, the intensity of the incident light being called unity.[1]

FORMULÆ FOR THE COMPOUND PRISM.

Path of the rays: $(J_3 = R_2,\ R_3 = J_2;\ J_4 = R_1,\ R_4 = J_1)$

$$R_2 = \frac{i}{2}$$

$$\sin J_2 = \frac{n_i}{n_a}\sin R_2$$

$$R_1 = J_2 - a$$

$$\sin J_1 = n_a \sin R_1$$

Dimensions: $$S_a = \frac{Q}{\cos J_1}$$

$$B_a = \frac{S_a}{\cos J_2}\sin a$$

$$S_i = \frac{S_a}{\cos J_2}\cos R_1$$

$$B_i = 2S_i \sin\frac{i}{2}$$

Deviation: $$A = 2(J_1 - R_1 + J_2 - R_2)$$

Dispersion: $$D = dR_4 = \frac{2\cos R_1 \sin R_2}{\cos J_1 \cos J_2}dn_i - \frac{2\sin a}{\cos J_1 \cos J_2}dn_a - dJ_1$$

Light power: $$H = \left(\frac{C_a^{B_a} - 1}{B_a \log c_a}\right)^2 \left(\frac{C_i^{B_i} - 1}{B_i \log c_i}\right)(H_1 + H_2),$$

where $$H_1 = \frac{1}{2}\left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2(J_1 + R_1)}\right)^2 \left(\frac{\sin 2J_2 \sin 2R_2}{\sin^2(J_2 + R_2)}\right)^2$$

and

$$H_2 = \frac{1}{2}\left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2(J_1 + R_1)\cos^2(J_1 - R_1)}\right)^2 \left(\frac{\sin 2J_2 \sin 2R_2}{\sin^2(J_2 + R_2)\cos^2(J_2 - R_2)}\right)^2.$$

[1] The derivation of these well-known formulæ of H_1 and H_2 may be found, for instance, in Winkelman's *Handbuch der Physik*, Bd. 2, Abth. I, 751.

My computations refer to the glasses from which the compound prisms were made for the spectrograph described in the *Publicationen des Astrophysikalischen Observatorium*, 7; it should be especially noted that the flint glass is very transparent, and is very slightly colored. These two glasses are from the works of Schott & Co., in Jena, and are designated in their price list as O.102 (heavy silicate flint), and O.144 (boro-silicate crown). In order to determine the differential quotients of the indices of refraction $\frac{dn}{d\lambda}$, the following values of the indices for three rays were taken from the price list mentioned (July 1886):

	O.102	O.144
D	1.6489	1.5100
F	1.6626	1.5156
H	1.6744	1.5201

From these figures the following dispersion formulæ were derived for the two kinds of glass:

$$\text{For O.102, } n = \frac{1.6122 + 13.91}{\lambda - 210.2}$$

$$\text{For O.144, } n = \frac{1.4934 + 6.85}{\lambda - 176.5},$$

from which follow the values for $\lambda = 4340$:

$$\text{For O.102, } dn = -0.000277\, d\lambda$$

$$\text{For O.144, } dn = -0.000103\, d\lambda\,.$$

I. SIMPLE PRISMS.

b	*S*	*B*	*m*	*A*	*D*	log *H*
30°	38.8mm	20.1mm	1	21°21′.8	32′.8	9.9148
			2	42 43.6	65.6	9.8302
			3	64 5.4	98.5	9.7461
			4	85 27.2	131.3	9.6625
			5	106 49.0	164.1	9.5795
45	45.6	34.9	1	34 41.9	57.0	9.8906
			2	69 23.8	113.9	9.7849
			3	104 5.7	170.9	9.6829
			4	138 47.6	227.8	9.5844
			5	173 29.5	284.8	9.4891

I. SIMPLE PRISMS—*Continued.*

b	*S*	*B*	*m*	*A*	*D*	log *H*
55	55.2	51.0	1	46 16.6	83.2	9.8578
			2	92 33.2	166.4	9.7274
			3	138 49.8	249.6	9.6079
			4	186 6.4	332.8	9.4980
			5	231 23.0	416.0	9.3961
60	64.0	64.0	1	53 41.5	104.5	9.8262
			2	107 23.0	208.9	9.6737
			3	161 4.5	313.4	9.5397
			4	214 46.0	417.9	9.4203
			5	268 27.5	522.4	9.3117
63	72.3	75.5	1	59 3.7	123.3	9.7952
			2	118 7.4	246.6	9.6213
			3	177 11.1	369.8	9.4723
			4	236 14.8	493.1	9.3414
			5	395 18.5	616.4	9.2226
65	80.2	86.2	1	63 13.7	140.6	9.7647
			2	126 27.4	281.3	9.5694
			3	189 41.1	421.9	9.4046
			4	252 54.8	562.6	9.2598
			5	316 8.5	703.2	9.1272

II. COMPOUND PRISMS.

i	*a*	S_i	B_i	S_a	B_a	*m*	*A*	*D*	log *H*
80°	8°	98.8mm	127.0mm	87.4mm	17.2mm	1	68°48'.8	186'.4	9.6908
						2	137 37.6	372.9	9.4132
	10	83.3	107.1	71.9	17.7	1	61 44.6	153.4	9.7488
						2	123 29.1	306.8	9.5172
	12	74.4	95.6	62.7	18.5	1	56 6.7	133.7	9.7797
						2	112 13.5	267.4	9.5720
	15	66.2	85.1	54.0	19.8	1	49 14.5	114.9	9.8047
						2	98 29.1	229.8	9.6162
	20	58.7	75.5	45.8	22.2	1	40 12.9	96.3	9.8228
						2	80 25.9	192.5	9.6481
90	15	101.9	144.1	79.2	32.7	1	67 30.7	195.6	9.6666
						2	135 1.4	391.2	9.3591
	20	77.4	109.4	56.7	30.9	1	53 43.6	141.0	9.7538
						2	107 27.2	282.0	9.5165
	25	67.5	95.4	47.2	31.8	1	44 9.7	117.2	9.7815
						2	88 19.5	234.5	9.5665
100	20	137.2	210.3	92.9	59.2	1	75 43.7	271.4	9.5196
						2	151 27.4	552.7	9.0770
	25	95.5	146.3	60.8	47.9	1	59 42.9	180.8	9.6758
						2	119 25.8	361.5	9.3642

The coefficients of absorption were found by taking the means of the measures by H. C. Vogel, Müller and Wilsing,[1] whose observations were made on varieties of glass almost identical, but not the same as the glass of the above prisms. For light of the wave-length 4340 I have taken the absorption coefficient for a thickness of 100 mm as 0.53 and 0.72 respectively. Hence we get for 1 mm thickness of glass

For 0.102, $\log c = 9.9972428$,
For 0.144, $\log c = 9.9985733$.

With these optical constants the computations were made for the following prism systems, for an incident beam of 35 mm section, and with the ray $H\gamma$ at the minimum deviation. The simple prisms are computed for the flint glass 0.102.

I have graphically represented the values of D and $\log H$ in Fig. 2, in order to give a clear survey of the relation between the dispersion and the light-power of the different systems of prisms. The curves for systems of simple prisms are given by the continuous line, for compound prisms by the broken line. The appended numbers are the refracting angles in degrees.

We can see at a glance that, aside from the very slight dispersion given by only a single prism of angle less than 60°, the simple prism with a refracting angle of from about 59° to 64° gives the greatest intensity for every dispersion, inasmuch as the points of their curves always lie highest. For instance, if a dispersion of 250″ is desired for $d\lambda = 1\ \mu\mu$ in the region of $H\gamma$, this can be obtained in the following ways:

Two simple	prisms	of	about	63°	angle	give	an	intensity of	0.413
Three "	"	"	"	55°	"	"	"	"	0.405
Two compound "	"	"	"	80° and 13°	"	"	"	"	0.391
Four simple	"	"	"	47°		"	"	"	0,368
One compound "	"	"	"	121°		"	"	"	0.362
Two "	"	"	"	90° and 23°	"	"	"	"	0.355
Five simple	"	"	"	41°		"	"	"	0.327

In a similar way the appropriate system of prisms and their light-power may be read off from the curve for every other

[1] This JOURNAL, 5, 75, 1897.

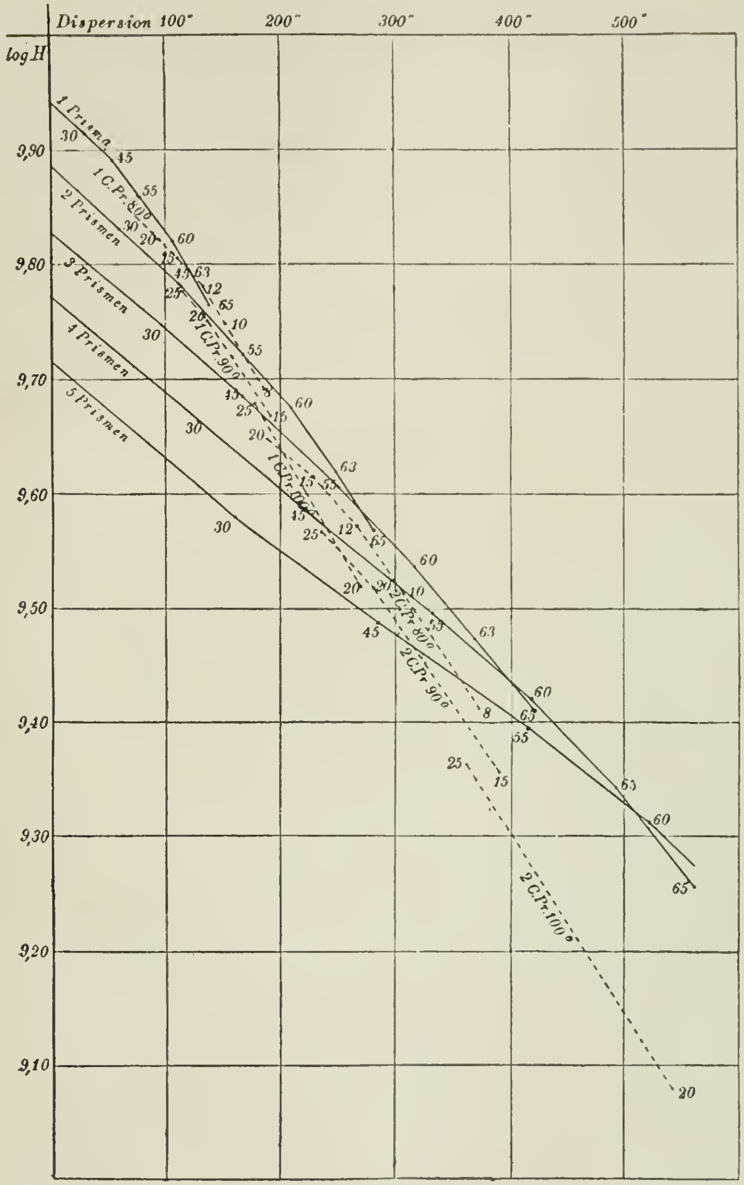

Dispersion 100"
200"
300"
400"
500"
log H
9,90
9,80
9,70
9,60
9,50
9,40
9,30
9,20
9,10
1 Prisma
1 C.Pr. 80°
2 Prismen
3 Prismen
4 Prismen
5 Prismen
1 C.Pr. 90°
1 C.Pr. 100°
2 C.Pr. 80°
2 C.Pr. 90°
2 C.Pr. 100°

Fig. 2.

dispersion. I must especially emphasize here the fact that compound prisms, although they may equal the light-power of simple prisms for faint dispersion become increasingly unsuitable for very high dispersions. Thus, if we draw a sort of common tangent to all the curves of the simple prism, this will come out distinctly less steep than the corresponding line for the compound prism. This is related to the fact that the light is completely polarized in the most favorable form of simple prisms, so that more than the half of its intensity can never be lost by the reflections. If a system of such prisms has almost completely polarized the light, the loss of light by reflection on increasing the number of prisms is almost zero. We cannot avail ourselves of this favorable circumstance in the case of compound prisms, since with them there are always two different angles of incidence.

Although it might be possible that the light-power of compound prisms should turn out a little better for some other point in the spectrum, it is clearly evident from the above that compound prisms are by no means superior in brightness to simple prisms in the region of the most active photographic rays. It will be necessary to prove by computation, ultimately with the employment of different kinds of glass, whether in a given case the compound prism does bring an increase of light, which certainly cannot be large. If we further consider that variations of temperature must always produce tensions of the hard-cemented compound prisms which affect the sharpness of the image, and further that the surface errors have a greater effect at the large angles of incidence which occur at the flint glass of the compound prisms, we shall conclude that such prisms can by no means be employed to advantage in the construction of spectrographs for faint objects, especially when the apparatus must be used in very different temperatures, as is the case for stellar spectrographs. Since very heavy and strongly yellow kinds of flint glass are not infrequently used for increasing the dispersion of compound prisms, I will remark here that such prisms are not at all permissible for spectrographs, since flint glass of that sort

completely cuts off a large part of the photographically active rays.

As a result of the above investigations there can be no doubt that only simple flint prisms of about 60° angle could come in question in the construction of the new stellar spectrographs for the Astrophysical Observatory. Since the dispersion of the former spectrograph for the determination of stellar motions in the line of sight (397.4 for $1\mu\mu$) had proven very satisfactory, a similar dispersion was planned for the larger of the new instruments, which is designated as spectrograph No. III. The graphical representation now shows at once that this condition could be met in two ways with an almost exactly equal intensity of light—either by three prisms of about 64° or by four prisms of about 59° angle. For the former the deviation of the ray would amount to about 184°, for the latter 208°, and since a very simple form of construction can be made for 180° deviation, a system of three prisms of the flint glass 0.102 was selected for spectrograph No. III. The refracting angle was now so computed that the ray $H\gamma$ suffers a deviation of exactly 180°; this condition required an angle of 63° 28′, and the firm of C. A. Steinheil's Sons accomplished the problem of giving the three prisms this angle with a highly praiseworthy accuracy. The investigation of the prisms yielded the following values for the refracting angles:

Prism	No. 1	63°	28′	17.13
"	No. 2	63	26	55.47
"	No. 3	63	27	26.37

The index of refraction for $H\gamma$ has precisely the same value for the three prisms, 1.67435, so that when placed at the minimum of deviation for $H\gamma$ the light-power of the system would reach its maximum at a refracting angle of 61° 42′. We see that in this spectrograph this most favorable angle is very nearly taken, and the slight difference in that value is entirely without effect.

[*To be continued.*]

THE VELOCITY IN THE LINE OF SIGHT OF ϵ *LEONIS.*

By W. H. Wright.

In this Journal, **11**, 307, Mr. W. S. Adams announced the variable velocity of ϵ *Leonis*(1900, $\alpha = 9^h\ 40^m.2$, $\delta = +\ 24°\ 14'.1$). The announcement was made on the basis of fifteen spectrograms secured with a spectroscope attached to the forty-inch refractor of the Yerkes Observatory.

Below are given the results of the measurements of all the spectrograms of this star obtained with the Mills spectrograph. At the time of receiving Mr. Adams' article the first five had been secured. As they showed no range which might not be due to accidental errors of observation, the remaining three observations were made for the purpose of confirming the variation.

		Gr. M. T.	Velocity
1897	April 8	$16^h.3$	+5 [1]
1898	Jan. 26	20 .4	+4.4
1898	Feb. 3	20 .2	+4.2
1898	Feb. 8	20 .1	+6.9
1900	Jan. 16	23 .8	+5.4
1900	May 24	16 .8	+5.0
1900	May 28	17 .0	+4.8
1900	May 30	16 .8	+4.9

It will be seen that the measurements give no evidence of variable velocity. The agreement of the determinations cannot be due to a purely fortuitous distribution of the observations between two points on the velocity curve having equal velocity, as will be seen by comparing the intervals between epochs of

[1] Rough measurements of a poor plate, not to be used in the final discussion of the star's motion.

observation with the period which Mr. Adams assigns—two and a quarter days.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
June 2, 1900.

NOTE.—In the absence of Mr. Adams, who made the observations and measurements of ϵ *Leonis* during the usual winter vacation of the undersigned, it seems proper that a statement should be given as to the accordance of measures made by Mr. Adams upon spectrum plates of other stars considered to be of uniform radial velocity. As far as known, the instrumental conditions were precisely the same as for the spectrograms of ϵ *Leonis*. The dates are those on which the spectrograms were taken.

	γ *Andromedae*	
1899	Sept. 27	—13.8 km
	Nov. 28	—15.1
	Dec. 7	—14.6
	Dec. 23	—16.9
	α *Arietis*	
1899	Oct. 6	—20.5
	Dec. 23	—20.7
	ϵ *Aurigae*	
1899	Nov. 28	—0.2
	Dec. 6	+0.1
	Dec. 7	+0.3
	α *Canis Minoris*	
1899	Nov. 3	—6.9 km
	Nov. 28	—4.2
	γ *Leonis*	
1899	Dec. 7	—39.2
	Dec. 23	—40.3
	α *Lyrae*	
1899	Oct. 18	—19.6
	Oct. 18	—18.1
	α *Ursae Majoris*	
1899	Dec. 7	—11.5
1900	Jan. 28	— 9.8

EDWIN B. FROST.

ERRATA.

VOL. XI, April 1900. The formula $m_p = [7.20386] PV^3$, which appears in the text on p. 249, should terminate the second footnote on that page. On p. 250, line 8, of Addendum, *for* "$m = 6$" *read* "$m > 6$."

Minor Contributions and Notes

MEASUREMENT OF PHOTOGRAPHIC INTENSITIES.[1]

The comparison of various celestial and terrestrial sources of light has for many years been a subject for investigation by the officers of this Observatory. One of the first comparisons of the light of the Sun and Moon was made by Professor George P. Bond in 1861 (*Mem. Amer. Acad.* 8, N. S., 298). Comparisons of the light of the solar corona, of the Moon, and of twilight, with a standard candle were made by the writer in 1870. A comparison of various artificial lights was made by Professor W. H. Pickering, as his graduating thesis in 1879 (*Proc. Amer. Acad.* **15**, 236). Since 1887, all the photographs obtained with the principal instruments of this Observatory have had an image of a standard light photographed on them for purposes of comparison. In 1887, when Mr. W. H. Pickering was placed in charge of the Boyden Department, an important part of his work was a quantitative determination of photographic intensities. This investigation was completed in 1891, and the results are published in the *Annals*, **32**, chapter I. The relative brightness of 10,498 stars, for the wave-length 430, was determined by Mrs. Fleming and was published in 1890 in the Draper Catalogue (*Annals*, **26** and **27**.) An independent determination of the total photographic brightness of the stars was undertaken in 1896 by photographing stars out of focus. As none of these investigations served to determine certain constants, which seemed to the writer to be important, a more extensive investigation was assigned to Mr. Edward S. King, under whose direction the photographs are taken at Cambridge. This work has been modified and extended by Mr. King, so that it now includes the following researches, which he describes below:

In 1896, a monthly test of plates was inaugurated, which was of a routine character. A complete determination, not only of the sensitiveness of the emulsion, but of the constancy of the light and of the developer employed, is obtained by the expenditure of one 8×10 plate per month (*Photogram*, **6**, 261). The means of the results of

[1] *Harvard College Observatory Circular* No. 50.

these tests, extending over a long period of time and necessarily subject to all the accidental errors due to season or other factors, will probably show the effect of the age of the plate on its sensitiveness, whether the image is affected by the interval between the exposure and the development, and other facts of similar interest.

About the same time with the above, the Director suggested that an extensive investigation be undertaken of the photographic measurement of various sources of light. The work was to be systematized into a routine, which should occupy the least time in making the observations. By extending the period over years and by performing each experiment under the most diverse circumstances, the final results should be of great accuracy. Preliminary to this work, an apparatus was constructed for making photographic wedges in which equal intervals of the scale corresponded to equal differences in stellar magnitude. The general design was the same as that described in the *Annals*, **26**, 6, except that the triangular aperture was replaced by an aperture bounded by a logarithmic curve. The part of the curve extending to minus infinity, but enclosing a finite area, is represented by admitting light through an additional aperture of equivalent area. This apparatus having a range in aperture of five magnitudes, is useful for measuring the intensity of surfaces directly, or of bright objects, when the aperture is covered with porcelain. It is also of value in comparing the sensitiveness of plates, and for studying the relation of the darkening of the film to the exposure, and to the intensity of the light.

The object of this investigation is to determine the photographic intensity of various sources of light upon a uniform scale. This scale will be that of the Meridian Photometer, in which α *Ursae Minoris* has the magnitude of 2.15, and one unit corresponds to the ratio of 2.518, whose logarithm is 0.400. Luminous points may be compared directly with stars. In the case of surfaces, the light emitted by a circle having a diameter of one minute of arc is employed. In general, different portions of the same photographic plate are exposed for a given time to the sources of light to be compared, and the darkening measured with a photographic wedge.

Observations of surfaces will include, intensity of the sky at different distances from the Sun; the sky in the zenith during twilight, on clear and cloudy days, on dark and moonlight nights; comparisons of blue sky with cumulus cloud; intensity of Milky Way, Aurora, and

Zodiacal Light. For this purpose pinhole cameras are used with various apertures. For very faint lights, apertures subtending an angle of 60° or more are used. When possible, automatic devices are employed, both to make the exposure and to shift the plate.

For measuring luminous points, the light concentrated by a lens illuminates the plate placed at various distances from the focus. In this manner, bright stars, the planets, and the Moon at different phases and altitudes, may be compared with *α Ursae Minoris* or an artificial light placed at a distance. It is also necessary to determine the effect of any slight fogging before, after, or during the principal exposure. The total and local absorption of the lens will also be measured. The same objects are also compared directly by pinhole cameras. In using a pinhole camera to measure points differing greatly in brightness the duration of the exposure must be varied. This necessitates a study of the darkening of the plate with relation to time. The comparison is made, in all cases, with a light, producing an equal darkening in the same time. Another factor may be introduced by inclining the plate. An angle of 60° is equal to a diminution of one half, or about three quarters of a magnitude. The exact amount corresponding to the inclination used, must be determined by experiment.

The determination of the light of the Sun is a difficult problem. The light may be reduced by a distant lens of short focus, and compared with the standard light similarly reduced, but at less distance. Another way is by a combination of lenses like a telescope, which enlarges or reduces objects, as we look through one end or the other. The absorption of the instrument is thus eliminated. Two plates of glass, placed so as to give multiple reflections, will also afford a wide angle of exposure, as well as a means of comparing lights of great difference in intensity. Pinhole cameras having the aperture covered with a porcelain plate can be constructed so as to give a range of 15 to 20 magnitudes, which is sufficient for comparing the Sun and the Moon. A second pinhole camera placed in front will cut off the light of the sky from the aperture. The absorption and the diffusing power of the porcelain plate must be measured independently. In all the preceding plans the sensitiveness of the plates to different colors affects the results. By combining a slit spectroscope with the cameras just described, a comparison of the light of the Sun and the Moon may be made in different parts of the spectrum, corresponding to given wavelengths.

The comparison of the spectrum of the various sources of light possesses many advantages. The work is placed on such a basis that the results are freed from the troublesome questions of absorption, sensitiveness of the plate, etc., and are rendered directly comparable with those that may be obtained at different times, by other observers, under widely differing circumstances. In fact, the results should be the same as those obtained by the eye, the bolometer, or in any other manner that may be devised. All these measures may be made either with a telescope and objective prism, or with a slit spectroscope combined with a pinhole camera or telescope. With the objective prism the spectra are made of the same width, either by interposing a cylindrical lens, by moving the plate at the same equable rate for each exposure, or by throwing the image out of focus. In the latter case, a rectangular aperture should be placed over the prism, so as to give width with a minimum loss of definition. With the moving plate, a variation of light is obtaind by covering one half of the prism at a time, along a line perpendicular to its edge.

The standard light used is an ordinary Argand gas burner shining through a small hole. The star α *Ursae Minoris* is made the ultimate standard to which all work is referred, since the standard light is compared with it before and after each monthly test of the plates. In addition to the wedges for measuring the density of photographic plates a polarizing photometer is used for comparing surfaces with each other.

Early in 1900, while this work was in progress, Mr. W. H. Pickering, in preparing to observe the Solar Eclipse of May 28, desired to select a suitable plate and developer for the work, and accordingly undertook the following investigation, which he describes below:

A suitable standard of light has long been wanted in photography. Artificial sources usually give even more uncertain results photographically than they do visually, because a slight variation in temperature will effect the blue end of the spectrum even more than the red end. In 1893 it occurred to the writer to employ as a primary standard of actinic intensity the radiation of a star shining directly upon the photographic plate, without previously passing through or being reflected from any medium, except our atmosphere. Since this standard is too faint for general use, a secondary one to be standardized from it has been devised. The light from the star is condensed through a simple plano-convex lens of 8.2 cm aperture and short focus, and is focused on a small circle of ground glass 0.5 cm in diameter. This is placed 3 cm in front of the sensitive plate, which is exposed to it through a small square aperture measuring 0.2 cm on a

side, cut in a blackened brass plate. The constant of this instrument was determined, and it was found to give about thirty times the light of the direct radition of a star upon the photographic plate. With twenty minutes exposure *α Ursae Minoris* darkened the plate sufficiently to produce the "Sensitive Tint," that is to say the tint where a small variation in the light is most noticeable. The secondary standard, however, is not sufficiently brilliant for ordinary purposes, and a tertiary one has therefore been devised. This consists of a box 30 cm in length, one end of which contains an aperture 5 cm in diameter, covered by a piece of ground glass, and the other end carries the sensitive plate and the blackened brass plate described above. Just inside the ground glass may be placed diaphragms ranging from 0.04 to 4.0 cm in diameter. At a distance of about 200 cm beyond the ground glass is placed an Argand gas burner. This apparatus gives ample light for all photographic purposes, and the plates exposed in it may always be standardized when necessary by an additional exposure to the secondary standard.

The three pieces of apparatus above described are adapted to various investigations. Thus the photographic brightness of the Moon, and of the brighter stars and planets, may be measured with accuracy by employing different apertures in front of the lens of the secondary standard, and measuring the brightness of the various images obtained by means of a photographic wedge, or a series of standard squares of varying density. It is very important to vary the aperture rather than the time of exposure, since the results obtained in the latter case would have to be corrected by the "Time Correction" (*Annals*, 32, 20). The secondary standard also enables us to express the sensitiveness of a plate, in terms of universal application throughout all time. Thus the Seed plate No. 27 is capable of being appreciably darkened when exposed for ten minutes to a source equal to 300 times the brightness of *α Ursae Minoris*. This amount of light is equal to about thirty times the brightness of a star whose photographic magnitude is 0.0. The logarithm of this number, 1.5, may be conveniently used to represent the sensitiveness of the Seed plate. By the use of the secondary and tertiary standard, together with an ordinary photographic telescope, we may make a study of the brightness of nebulæ, comets and other luminous surfaces.

From these various investigations it is hoped that we shall obtain a scale of photographic intensities with which all sources of light may be compared and to which they may be referred.

EDWARD C. PICKERING.

May 9, 1900.

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